Omweso: The Royal Mancala Game Of Uganda
- A General Overview Of Current Research

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Brief introduction to the rules of Omweso

Omweso is a Mancala game popular in the East African country of Uganda, with major tournaments being held in the capital city of Kampala in the Kingdom of Buganda (see Figure 1).

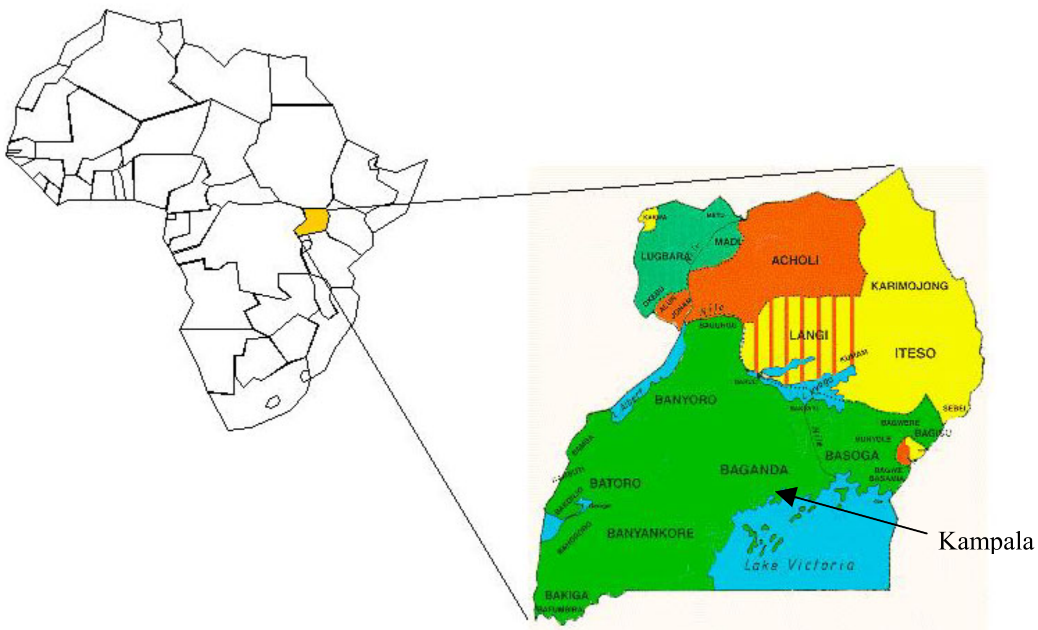
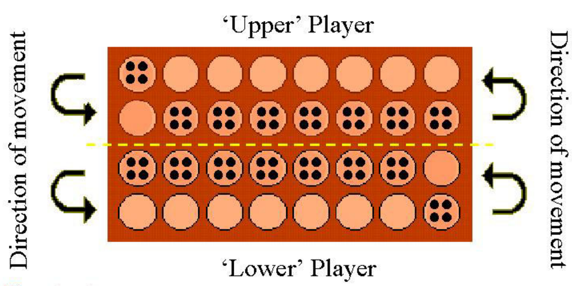


Figure 1 - The Peoples Of Uganda

Unlike the mancala games played on the West coast of Africa, Omweso is a 're-entrant' game. That is to say that all the seeds remain in play – captured seeds are re-entered onto the winners side of the board. Unlike the game of "Bao" played on the Swahili coast of East Africa, in Omweso the players start with all 64 seeds in play, and set-up freely at the start.

The board

- 32 hole Board with 64 seeds



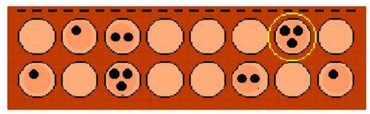
- To start:
  - each player sets up his/her 32 seeds in holes anywhere on his/her side of the board
  - a suggested beginners set-up is shown above

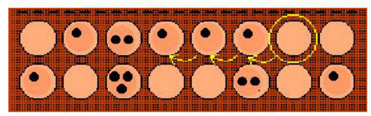
The objective of the game is to capture opponent's seeds until he is unable to move, or to gain a knockout by 'Cutting off both his heads' (that is to capture seeds from both ends of his board in one move).

Nsimbi notes that in the past games were slower, with players thinking for many minutes to find the right move. Modern play, however, requires very fast play, with only 3 seconds thinking time per turn. The referee counts “Omu”, “Ebiri” and if the player does not choose a hole to start his move he loses his turn to other player...

How to play: Rule 1 - sowing seeds

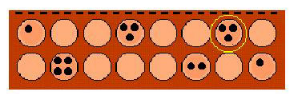
- Choose a hole to sow from on your side of the board
- Pick up all the seeds therein
- Sow them on anti-clockwise
- If the last ends in an empty hole end of your turn
- Singletons cannot move

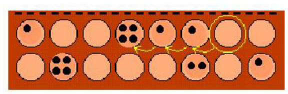


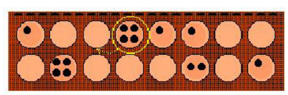


How to play: Rule 2 - relay sowing

- If the last is not sown into an empty hole, then …
- Pick up all the seeds in that hole …
- And sow again!
- (and again and again until an empty hole is found...)



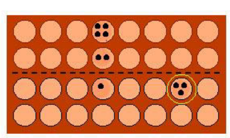


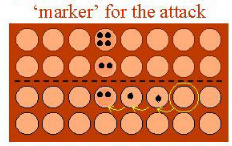


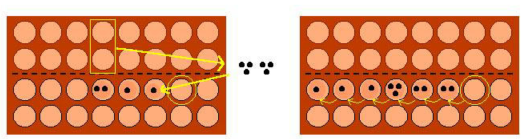
How to play: Rule 3 - capture

a) Pairs of occupied holes are vulnerable

b) Lower’s inside hole opposite is occupied - this acts as a ‘marker’ for the attack





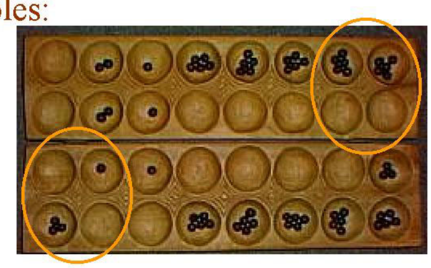


c) Upper’s 6 seeds are captured...

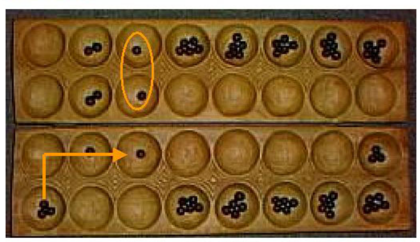
d) … and re-enter onto the winner’s side of the board AS IF FROM THE ORIGINATING HOLE

How to play: Rule 4 - reverse capture

- Reverse capture is allowed from leftmost 4 holes:



- Clock-wise movement from these special holes - BUT ONLY IF CAPTURE IS POSSIBLE:



There are two ways to win in street play:
1. Normal - the loser is immobilised and cannot move (only has empty holes and singleton seeds)
2. Emitwe-Ebiri - 'cutting off at 2 heads'. Capture of both extreme pairs of holes in one move

"*Okukoneeza*" is a special tournament rule: if a hole has 3 seeds during the opening before first capture 2 seeds into next hole, final seed into next but one
Winning by Akakyala - capturing seeds from the loser in two separate moves before the loser has even made his first capture of the game.

Speculation On The Origins Of Omweso

I would like to continue by posing a question relating to the origins of Omweso: What is the historical relationship between Bao[3], played to the East along the Swahili coast, Gabata[4] to the North, and Igisoro[5] to the West? Which came first? How did the rules spread and evolve?

Uganda is a landlocked country found between Lake Victoria and Lake Albert in Central Africa. The country was a British colony created with its core the central kingdom of the Ganda tribe, and today remains fragmented by 17 different languages. This fragmentation of peoples and their interactions is of great interest to anthropological research into the origins of the mancala board games of the area (see Figure 1 - The Peoples Of Uganda).

Firstly, we must note that the peoples of the Inter-lake area were extremely isolated until modern times. They were discovered by outsiders at a very late stage of Victorian exploration, just 140 years ago. In contrast, Europeans first discovered Australia in 1606 or even earlier according to some accounts.

Secondly, there was great insularity and lack of trading between the Interlake kingdoms after 1884. Speke was the first European to discover the source of the Nile at the Ripon Falls at the north end of lake Victoria in 1862, and was friends with the Kings of the Baganda, Karagwe and Banyoro tribes. Mutesa was the 30th King of Buganda, thus dating the kingdom to perhaps the early sixteenth century. Although Mutesa was a regarded as a wise and progressive king, showing a great deal of interest in the ways of the Europeans, he maintained the traditional 'band of steel' around his kingdom, and was brutal in the treatment of his own people. When Speke gave him rifles as a present, Mutesa ordered a court page to shoot someone in the outer court to demonstrate the effect of one of the gun. His son, Mwanga ascended the throne in 1884 and he opposed all foreign presence, including the missions. In fact, the murder of Bishop Hannington in October 1885 was a direct result of his taking the 'easier' land route through Busoga from the East, rather than sail across Lake Victoria landing directly at the Royal piers. King Mwanga was afraid of strangers approaching from enemy kingdoms, and passing through all his own people before the royal audience took place. Hannington lost his life because he did not take seriously the paranoid hostility of Mwanga towards the surrounding kingdoms. Mwanga's use of slave labour appears to have been an internal issue, with his sub-chiefs being granted permission to 'harvest' their own people, or raid other sub-chiefs' populations. The great oral historian, Jan Vansina, has dated Arab slave incursions in the Upper Congo basin around 1850-60, in the area to the West of the Rwandan Kingdom, but these do not appear to touch upon the Rwandese or Urundi Kingdoms.[6] Mwanga's rule was chaotic and the long-distance trading declined because of ten years of civil wars in from 1888 between the king and his chiefs, and between Muslim and Christian factions. In 1897 Mwanga left the capital and was exiled by the British. By this time slave trading from Zanzibar had been outlawed and the inland Arab trade routes did not resume.

Finally, I note that there is great similarity between the Omweso-like games of Southern DRC/Zaire (Nsumbi), Rwanda (Igisoro), Sudan (Ryakati) and Western Ethipoia (Baré). These games vary to the extent that they have different reversing holes (e.g. Igisoro) or that they allow both players to start the first move simultaneously (e.g. Ryakati). But all require a capture by landing on a marked hole on the inside row opposite two occupied opponent's holes. In this respect they are quite different from Bao as played on the Swahili coast, and inland somewhat in a band from Lake Victoria to Malawi (see Figure 2: Geographical relationship to other games). Above this coastal area we find the 2 row game of "Bosh" in Somalia, and southwards we find Moruba-like games, such as Tsoro and Njombwa.[7]

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[3] Russ, L (1995) "The Complete Mancala Games Book" New York: Marlowe
[4] Russ, Ibid
[5] Frey, C (1998) "Le Francais au Burundi" Cedex, EDICEF
[6] Vansina, J "Paths in the rainforests" (1990) London : James Currey
[7] Russ (Ibid)

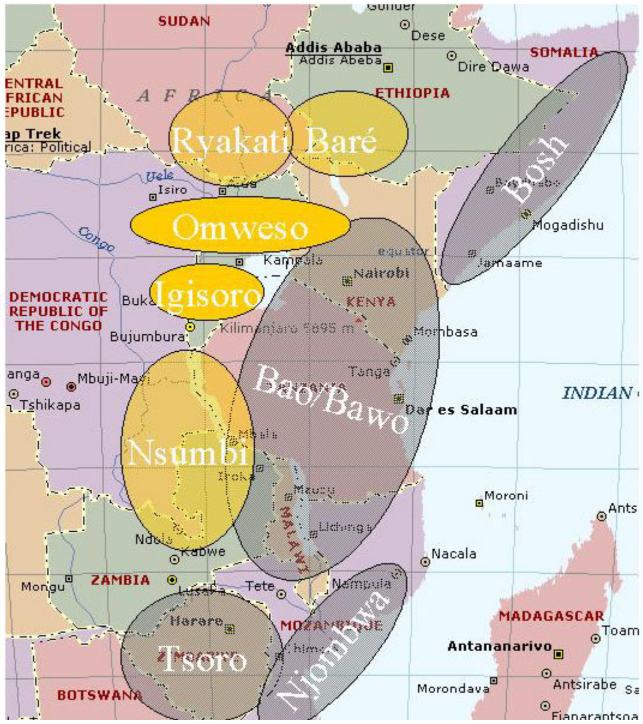

Figure 2: Geographical relationship to other games[8]

Another ‘strong’ variant of Omweso is the Rwandese game of Igisoro, which has reversing rules that make for complex play.

Mugerwa maintains that Omweso was Mutesa’s favourite game[9] Can one then postulate a hypothesis that as Mwanga’s reign was a period of economic and cultural isolation, then any swapping or adoption of mancala rules between Baganda people and others must have taken place in and before Mutesa’s reign, before 1884? Further research is required in this area. By concentrating on the rule variants of 16 hole mancala is various villages from the Swahili Coast all the way to DRC (Zaire) some inferences could be draw. Such field-work outside of the major cities where people have lived in stable communities for hundreds of years should surely be the next step in tracing the evolution of the game.

[8] This diagram is not intended to be to scale, and the nomadic nature of the Pastoral cattle-herders in these regions will blur any simplistic approach in matching games rules to fixed territorial tribal borders.
[9] Mugerwa p17

Omweso's Complexity

State-Space Complexity

A formula for game state-space complexity:

$$\frac{(h-1+s)!}{(h-1)! \times s} \quad \text{minus } k$$

Where:
h = number of holes
s = number of seeds
k = number of illegal/improbable positions

An illegal position in chess might be where a King and Queen share the same square. An improbable position in Awari would include 47 seeds in one hole, and the remaining seed in another.

Table 2: State-space complexities of 'world' games

Game	Initial set-up	As play starts	Midgame	Endgame
Awari	1	Slowly rising	2.8×10^{11}	Falling
Bao	1	Very slowly Rising	10^{25}	-
Omweso	5.6×10^{23}	Rising quickly	10^{25}	-
Chess	1	Rising slowly	10^{50}	Falling
Go[11]	1	Rising quickly	10^{160} to ^{500}	Falling

In Omweso, each player has 32 seeds to set up, giving 7.5×10^{11} possible positions for the first player to set up, which can be countered by the other player in 7.5×10^{11} ways giving 5.6×10^{23} combinations[12]. In tournament play there are no illegal set-up combinations, and very few improbable ones.

The 'k' factor for mancala games is lower than in positional games since many pieces may share the same 'hole' in mancala. The fact that pieces are white or black in positional games, and have different attributes (e.g. knight, King etc.)which increases the combinations in those games. State-space complexity in Omweso rises rapidly after the first captures, and remains high throughout as the seeds become redistributed in large numbers quasi-stochastically from one player to the other.

Note that different games have different numbers of pieces in play at different stages. For example, in Bao seeds are introduced throughout the initial phase of the game, with captures are re-entered onto the board. In chess captured pieces are removed. In Go new pieces are added until the end of the game, when areas of stability appear in 'safe' areas. Therefore all these calculations have many implicit assumptions, and are only roughly comparable. De Voogt has pointed out that games with high theoretical state-space complexity, such as Songka may be less 'intricate' for humans as the outcomes are beyond mental calculation and require 'brute force' calculations of no finesse.[13] This implies that only games which are on the edge of human capacity for calculation and tactics and especially those that show high degrees of chaotic behaviour are 'interesting'.

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[11] Goldis, Bjorn (1999) "Towards Abstraction in Go Knowledge Representation"
[12] Mayega pp11-12
[13] De Voogt (1995) "Limits of the mind" Leiden, CNWS

Game-tree Complexity

Table 3: Game-tree complexities of 'Mancala' games

Game	Initial set-up	By endgame (no forced moves)
Awari	1	10^{32}
Bao	1	2×10^{34}
Omweso	5.6×10^{23}	(5.6×10^{23}) $\times (5 \times 10^{50})$ $= 2.8 \times 10^{74}$

Game-tree complexity can be calculated as:

$$i_1 \times i_2 \times (b)^p$$

Where:
i = branches in set-up of game for players 1 and 2
b = branches per move
p = plays in game length

Awari and Omweso do not have forced moves (except where in Awari one must 'feed' the opponent if all his holes are empty). However, in Bao forced moves are very common, perhaps 1/10 in master-level play[14], and more common amongst less experienced players who do not know how to avoid or take advantage of these situations. One must also take into account the increased choices a Bao player has in choosing where to re-enter captured seeds (left or right) in the initial stage of the game. In Omweso the player has no choice where to re-enter the seeds, but he/she has greater choice over where to begin sowing to make a capture.

The average in a sample game[15] from the Kampala 2000 tournament is 5.4 anti-clockwise possibilities plus the possibility of deciding to reverse capture in about 20% of sowings. Therefore the number of branches per player turn in Omweso is well above 6. However, when the other player is susceptible to a reverse capture, the possibility of further reverse captures often appears (say 50% of cases). An example of this is shown at one point in the sample game. A multiple-reverse capture is made by Semakula, where his initial 5 choices of move expand into 9 choices of outcome depending on whether he decides to reverse capture or not during his move.

If one assumes then that there are 7 branches per turn with an average game being 60 turns, then the branching complexity would become $7^{60} = 5 \times 10^{50}$

There are, however, many assumptions in the above calculations. For example, using an average for the branching complexity "b" will overstate the branches if b is sometimes low and sometimes high. For example, if a two turn game has 5 branches per turn, then the number of end positions is 25. However if the first turn is forced (i.e. b=1) and the second turn has, say, 9 possibilities, then only 9 outcomes exist. But the average of 5+5 is the same as 1+9…..

A comprehensive analysis of tournament games is required before one can come to any conclusion about the relevance of these sorts of calculations in relation to branching complexities. I have only analysed one game for this paper – a full transcription of the games is in progress, and a selection of the games in video format is available on CD-ROM where the seeds being played are easy to follow, even at the speed these players manage to achieve!

[14] Ibid p.158
[15] ddamba s vs semakula u - game 1 Kampala 2000 (at 2:20 minutes)

Mutational Complexity

De Voogt[16] has introduced the concept of 'mutational complexity' with respect to the number of changes on the board due to a single move. These moves change the nature of the board by changing the state-space not just in moving from one 'position' to another, as in Chess, but by changing the all the intervening numbers due to the sowing process.

The capturing re-entry rules in Bao lead to a preponderance of seeds in the front rows, and it is reasonably rare for players to 'takata' round the entire board (5 out of 72 moves in one example tournament game) In Aware there is no 'relay sowing' so the impact of any one move usually limited to changes in half-a-dozen or less holes. De Voogt estimates a change in 6 holes per move in Bao (including the hole from which the seeds have been removed).

In Omweso the average number of changed holes in the sample game was exactly 3 per move for the first 9 moves whilst the players positioned themselves without laying themselves open. The next stage had an average of 7.1 moves per player turn as one player maintained a large position ready to strike, and thereafter 13.4 per turn. This count of mutational complexity does not however take into account lapping the board, in which many holes are changed more than once, and the additional changes to holes already visited that occur when captured seeds are re-entered onto the board. If these are counted, then in one single move Ddamba not only changes the state of 6 holes on his opponent's side of the board, but sows 47 times into the holes on his side of the board, changing their seed count in some cases 5 times.

The average number of holes changed across the sample Omweso game is 9.31 (or 12.4 if one double-counts holes sowed into more than once in a turn). The game was 26 moves per player, and 3:05min in length. This is about 3.6 seconds per player turn (including thinking and sowing time). The sample game was somewhat shorter in length than average (tournament games are usually 4-5 minutes long).

From these calculations I could conclude that Omweso is a more complex game than Bao. But more research is needed to define what we mean by 'complexity'. We need to move beyond simple combinatorial statistics into definitions of 'finesse', 'chaos' and non-computational complexities that human beings find difficult, such as 'mutational complexity' and 'memory complexity'. Other human characteristics such as dexterity and speed of play are pronounced in Mancala games, especially where thinking time is curtailed as much as it is in Omweso. Does this lead to a lessening of depth of play, or does it enhance a players use of subliminal 'zen-like' techniques and 'feel' for play that 'Go' dan-grade players exhibit?

[16] De Voogt p.158

Standard Set-Ups: 2 examples

There are many standard openings in Omweso I have gathered from my informants.

Here I would like to present two: one is a ‘junior grouping’, that is to say 8 seeds or less, and the other is the most extreme opening position the “twenty-three”.

In Omweso there is no restriction on setting up seeds, but in ‘street-play’ one generally only uses ‘junior groupings’. In tournaments, ‘senior groupings’ are allowed, but require very careful play:

Table 4: Position notation[17]

P	O	N	M	L	K	J	I	‘Upper’ Player
H	G	F	E	D	C	B	A	
a	b	c	d	e	f	g	h	
i	j	k	l	m	n	o	p	‘Lower’ Player

Example ‘junior grouping’ set-up

Table 5 shows Upper having laid out 'seven', and Lower has responded with a strong counter opening. The footnote in the presentation notes give some idea of the tactics that strong players will know either by rote or by quick calculation.[18]

Table 5: Example ‘junior grouping’

7	3	3	3	3	3	3	7
5		7	2	4	2	6	
							6

[17] Several alternative notations have been used. Shackell presented two different notations: one using anti-clockwise counting from 1-16 (Uganda Journal, 1934 vol.II) and an alphabetical notation omitting “o” and “l” (1935 vol.III). Both Russ and Zaslavsky use an anti-clockwise alphabetical notation. Ilukor uses Shackell’s anti-clockwise number notation to demonstrate the ‘magic numbers’ therein. The notation I am using here is one that has been used in Kampala by the Ugandans, so it has the advantage of compatibility with the notes of the experts, even though the Zaslavsky or Ilukor notations have some advantages.

[18] Notes on recommended play:

A. The best opening for the ‘Upper’ player is “B”, which moves the 3 seeds there into A, I and J. ‘Lower’ is best to respond with “d”, sowing into c and b.

B. Upper then plays the 3 in “C”
Lower plays “e”, landing in “a”, and then reversing with those 6 into “g” capturing the 2 now in “B” and “J”

C. Upper plays “H”, hoping for Lower to play c which will put on 2 heads….
Lower wisely plays “p”

Example ‘senior grouping’ set-up

Table 6 shows Uppers set up using the most extreme ‘senior grouping’: the “twenty-three”.
Table 7 shows recommended responses to the set-up by Upper:

Table 6: Upper sets up the 'twenty-three'

23							
	3	3	3				

Table 7: Lower responds to the "twenty three"

			3	3	3	3	
							20

Or:

		3	3	3	19		2
						2	

Or:

1	1	2	3	3	18		2
							2

Or:

		3	3	3	3	3	17

Or:

		3	3	3	3	3	
							17

See footnote [19]

Or:

				3	3	3	
							23

Junior groupings having a maximum of 5, 6, or 7 seeds are popular. Senior groupings of 17, 19, 20, 21, 22 and 23 seeds are seen often in tournament play. Openings, responses and analysis of the first dozen or so moves exist for these opening settings have been documented in manuscript form, but I have yet to collate, analyse or check all these.

[19] Nsimbi (1968) Diagram 8 illustrates exactly the same opening as ‘Uppers’ position in Table 5. My informants were familiar with Nsimbi’s work published in the vernacular. However, this opening (the 17) was also illustrated in the Uganda Journal in 1934 (Ref: Shackell 1934) which was unknown to them. This opening may therefore have been handed down the generations by play alone for more than 65 years.

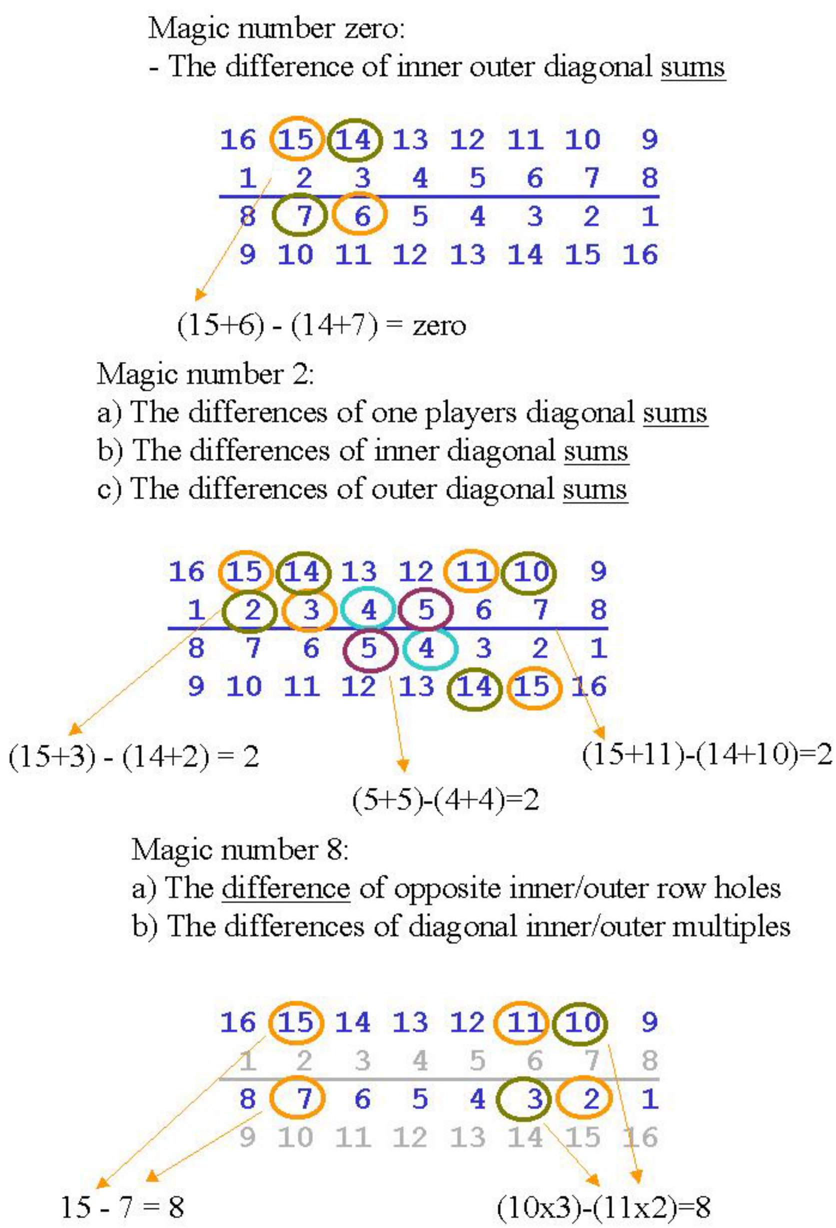
Magic Numbers

Professor Mayega wrote the first known Omweso computer program (written in Algol 60), and presented some statistical analysis in his 1974 paper, together with some work on applying matrix theory to the Omweso problem to create a goal-seeking program driven by a points scoring system.

In 1978 Professor Ilukor took a different tack by investigating the 'magic numbers' within Omweso rather than crunching away at traditional statistical and matrix bound techniques.

He made several simple observations which are illustrated in Figure 3. These observations help introduce us to a new form of mathematics that may solve many complex problems in Mancala, as we shall see in the next section.

Figure 3: Magic Numbers In Omweso



Magic number 9:
a) The sum of inner opposites
b) The differences of inner diagonal multiples

16	15	14	13	12	11	10	9
1	2	3	4	5	6	7	8
8	7	6	5	4	3	2	1
9	10	11	12	13	14	15	16

3 + 6 = 9 (7x3)-(6x2)=9

Magic number 17:
a) The sum of opposite outer row holes
b) The differences of outer hole diagonal multiples

16	15	14	13	12	11	10	9
1	2	3	4	5	6	7	8
8	7	6	5	4	3	2	1
9	10	11	12	13	14	15	16

.4+3 = 17 (2x16)-(1x15)=17

Magic number 25:
a) The sum of outer opposites
b) The differences of outer diagonal multiples

16	15	14	13	12	11	10	9
1	2	3	4	5	6	7	8
8	7	6	5	4	3	2	1
9	10	11	12	13	14	15	16

15 + 10 = 25 (12x14)-(11x13)=25

Never-Ending Moves – A Proof!

Trivial Never-Ending Moves

There is a category of position which I have called "Trivial never-ending moves" which may be based on triangular numbers such as those presented in Figure 4.

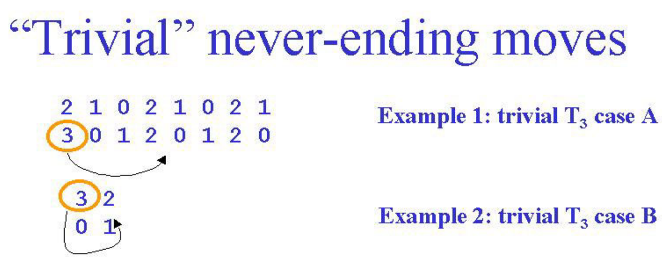


Figure 4: "Trivial" never-ending moves

These "trivial" settings have the following properties:

- One can see the repetition immediately without experimentation
- After each move the board is left in exactly the same state, except that the starting hole position is rotated
- Triangular numbers play a part with patterns of 4,3,2,1 appearing

Complex Never-Ending Moves

Of more interest, is a starting position that my informants in Kampala showed me that appeared to lead to apparently infinite looping (see Table 8).[20]

Table 8: "Hudson's 32" a 61,776 iteration never-ending game

3	2	3	2	3	2	3	1
2	2	1	2	1	2	1	2

Two players working in 4 hour relays in Kampala had failed to find an end-point or repetition in the game. On my return to London I wrote a Java program to:

- Calculate the number of iterations to the return position
- Check for any rotational symmetry in intervening moves

I also checked some other never-ending moves from the literature and one of my own invention. The results are shown in Figure 5.

[20] Mayega (1974) assumed in writing his Algol 60 program that *"infinite looping cannot occur, but it would be nice to have a formal proof or disproof of this statement"*. But Ilukor (1978) reported that players were well aware of *"infinite looping"* , and postulated his (incorrect) theory that there were only 4 such cases, all involving 25 seeds.

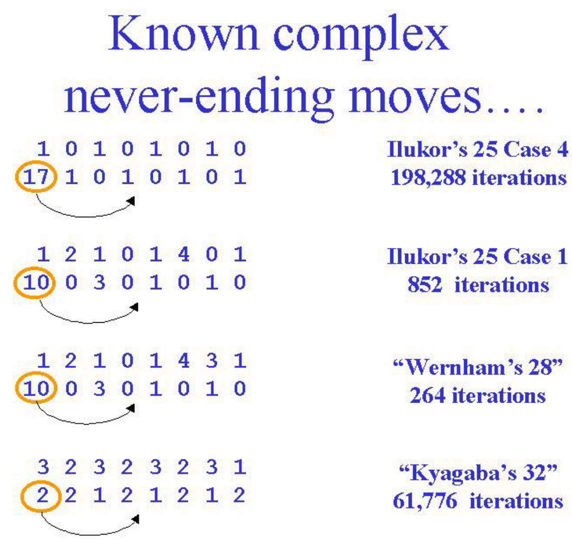


Figure 5: Known complex never-ending moves

Ilukor had a theory that all never-ending positions were related to his Magic Number "25". The examples here of my "28" and Kygaba's "32" disprove this idea.[21]

These complex positions have the following attributes:

- A large (200+) number of repetitions before the position appears again
- When the seeds appear again in the same sequence, the starting hole is in exactly the same position (i.e. no intervening rotation occurs)
- The number of iterations in these examples is always divisible by 4
- There are no obvious patterns occurring

Jonkers, Uiterwijk and de Voogt in last year's Colloquium[22] presented the starting position shown in Table 2. Wondering whether such positions can occur in a game of 16 hole Mancala. The answer is yes, but in Bao this can only occur in a real game if one player cannot capture at the start of his move and then plays "takata" unable to capture again in his move, and therefore continuing an un-ending move. This could occur with Ilukor's "Case 4" opening shown in Figure 5, where only one hole may start, and would only capture if the 'Lower' opponent had a front-row occupied where the 17th seed would land. This will seldom occur in Bao. However, in Omweso, if the 'Lower' player is invulnerable to capture, as he often is, then 'Upper' will sometimes find himself unknowingly starting a never-ending move. The tournament rules from Kampala recognise this with their rule for *"Ekyeso kyolutentezi oba ekitayalika" = "Non stop/unending move"*

> *"…the umpire shall give 3 minutes, and where the 3 minutes elapse, he shall order the game to be repeated."* [23]

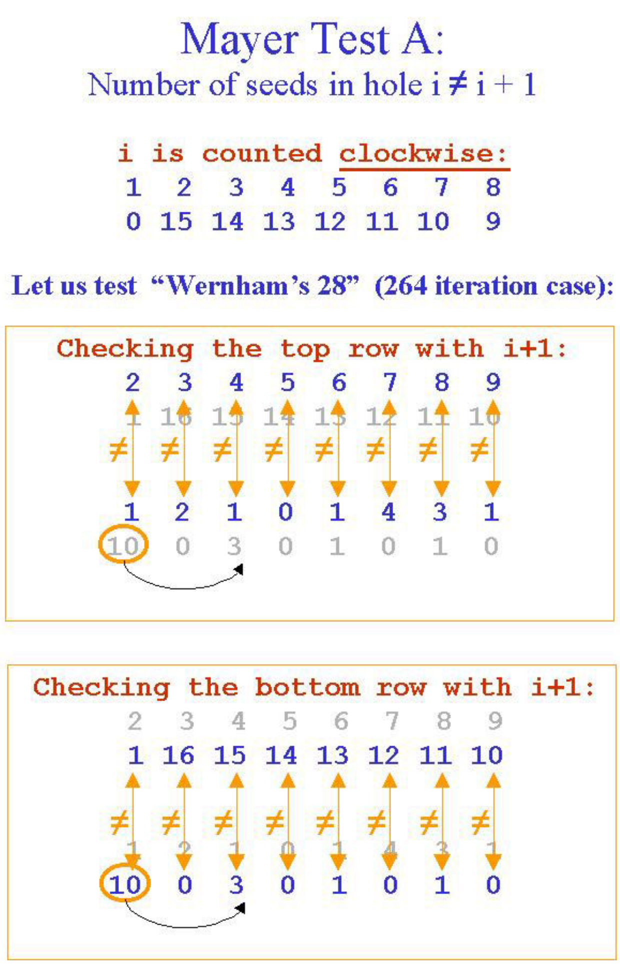
[21] Ilukor's Cases 2 & 3 do not work as presented in his paper, probably due to typo errors.
[22] "Mancala Games – Topics in Matematics and Artificial Intelligence" (2001) Donkers J, Uiterwijk J and deVoogt A
[23] "Amateeka Agafuga Omweso Mu Uganda Y.M.C.A." or "Rules Governing The Board Game "Omweso" In The Uganda YMCA", Uganda Ymca Mweso Council, Nakasero, Kampala 1999

Table 9: Jonkers, Uiterwijk and de Voogt's "40"

3	2	3	2	3	2	3	2
$\boxed{2}$	3	2	3	2	3	2	3

Steven Mayer[24] is Professor of Physics at the Milwaukee School of Engineering, Milwaukee, Wisconsin. I am thankful to him for providing a theorem for a test for 'complex never-ending moves' together with a mathematical proof. His full paper is attached as an Appendix to these presentation notes. I will give a brief non-mathematical overview of his theorem and proof here in Figure 6.

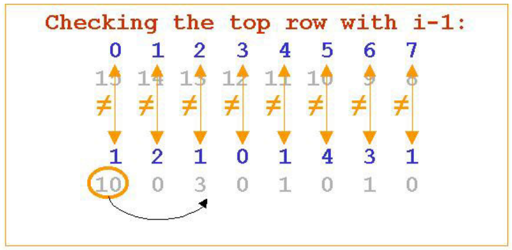
Figure 6 Mayer Tests A, B, C & D

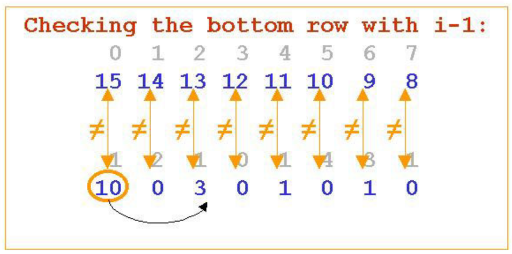


24 mayer@msoe.edu

Mayer Test B:

Number of seeds in hole i ≠ i - 1





Mayer Tests C & D:

Number of seeds in hole (i+j) ≠
either (Number of seeds in hole i) + j + 1
or (Number of seeds in hole i) + j - 1

Hole 6

```
 1  2  1  0  1  4  3  1
10  0  3  0  1  0  1  0
```

- Example
  if i = 4 and j = 2 then compare:
  - Hole 6 with 4 seeds
  - Hole 4 has zero seeds
    - Zero + j +1 = 3
  - 4 ≠ 3
  - Test C for i = 4 and j = 2 is OK
- Repeat for
  - all values of i from 0 to 15
  - and all values of j from 0 to 15

Mayer's full proof is provided in the Appendix to these presentation notes. Mayer starts by proving that Test A is violated at the end of a sowing if the last seed lands in an empty hole. He goes on to show that his four tests are mutually supportive:

- Test A can only be violated at the end of any move if Tests B, C or D are violated at the beginning of that move
- Test B can only be violated if Tests A or D were previously violated
- Tests C & D can only be violated if Tests A, B, C or D were previously violated

Conclusions

There are many areas of mathematical, historical and anthropological research still requiring work.

How did these games come to be played in so many areas by so many peoples who were for long periods of time isolated from outside influence? Why is Omweso so different from Bao, yet played with the same board configuration and number of seeds?

Some mathematical questions are still unanswered:

- Does Mayer's theorem cover all Complex cases?
- Is the iteration count always divisible by 4?
- Can we define a method to catalogue all never ending positions for n-hole mancala games?

I hope that this paper shows that Omweso poses a unique and interesting area of board games research that has been relatively neglected up to now by the academic community, and perhaps I shall stimulate further research by the presentation of this paper.

Theorem on Endless Moves - Appendix

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Notation:

On a 16-hole mancala board, let the hole from which the next iteration is to begin be "0" and number the remaining holes in a clockwise direction from 1 to 15. Let y_k denote the number of seeds in hole k.[26]

Theorem:

A move in 16-hole mancala is endless if it satisfies all of the following incongruencies for all values of i and j, with i running from 0 to 15 and j running from 0 to 15 - i:

y_i not congruent to $i + 1 \pmod{17}$ **A**

y_i not congruent to $i - 1 \pmod{17}$ **B**

$y_{(i+j)}$ not congruent to $y_i + j + 1 \pmod{17}$ **C**

$y_{(i+j)}$ not congruent to $y_i + j – 1 \pmod{17}$ **D**

Outline of Proof:

Assume A, B, C and D to be true for all permissible values of i and j at the start of one iteration. We will prove that they are true for all permissible valus of i and j at the start of the next iteration. Therefore they will hold for all iterations. Then incongruency A for the case $i = 0$ implies that the number of seeds in hole 0 can never be congruent to 1 (and therefore can never equal 1). We will establish that all of the incongruencies hold by assuming the negation of each one and showing that in each case it implies the negation of an incongruency in the previous iteration, which contradicts our assumption.

Notation:
Let A(i) refer to incongruency A for a particular value of i.
Let B(i) refer to incongruency B for a particular value of i.
Let C(i,j) refer to incongruency C for particular values of i and j.
Let D(i,j) refer to incongruency D for particular values of i and j.
Let x_k denote the number of seeds in hole k at the start of iteration 1.
Let y_k denote the number of seeds in hole k at the start of iteration 2.
Let ~ mean "is congruent modulo 17".

For example, if the arrangement of seeds is as follows:

12	5	6	0	2	11	9	0
2	6	3	3	5	0	1	8

and if the move begins by lifting the twelve seeds from the upper left-hand hole, then $y_0 = 12$, $y_1 = 5$, $y_2 = 6$, $y_3 = 0$, $y_4 = 2$, $y_5 = 11$, $y_6 = 9$, $y_7 = 0$, $y_8 = 8$, $y_9 = 1$, $y_{10} = 0$, $y_{11} = 5$, $y_{12} = 3$, $y_{13} = 3$, $y_{14} = 6$, $y_{15} = 2$

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[26] BW: This proof can be extended for an 'n' hole Mancala board, with the test and proof working to mod n+1.

Case 1: Suppose x_0 is a multiple of 16.

Then $x_0 = 16n$ for some n. Thus
$x_0 \sim -n$ (1)
$y_0 = n$ (2)
$y_k = x_k + n$ for $k > 0$ (3)

Suppose A(0) is violated on iteration 2. Then
$y_0 \sim 1$
$n \sim 1$ by (2)
$x_0 \sim -1$ by (1)
contradicts B(0) on iteration 1

Suppose A(i) is violated on iteration 2, i > 0. Then
$y_i \sim i + 1$
$x_i + n \sim i + 1$ by (3)
$x_i \sim i + 1 - n$
$x_0 \sim x_0 + i + 1$ by (1)
contradicts C(0,i) on iteration 1

Suppose B(0) is violated on iteration 1. Then
$y_0 \sim -1$
$n \sim -1$ by (2)
$x_0 \sim 1$ by (1)
contradicts A(0) on iteration 1

Suppose B(i) is violated on iteration 2, i > 0. Then
$y_i \sim i - 1$
$x_i + n \sim i - 1$ by (3)
$x_i \sim i - 1 - n$
$x_i \sim x_0 + i - 1$ by (1)
contradicts D(0,i) on iteration 1

Suppose C(0,0) is violated on iteration 2. This can never happen, since C(0,0) implies $0 \sim 1$.

Suppose C(0,j) is violated on iteration 2, j > 0. Then
$y_j \sim y_i + j + 1$
$x_j + n \sim n + j + 1$ by (3) and (2)
$x_j \sim j + 1$
contradicts A(j) on iteration 1

Suppose C(i,0) is violated. This can't happen since it implies $y_i \sim y_i + 1$

Suppose C(i,j) is violated on iteration 2, i > 0 , j> 0. Then
$y_{(i+j)} \sim y_i + j + 1$
$x_{(i+j)} + n \sim x_i + n + j + 1$ by (3)
$x_{(i+j)} \sim x_i + j + 1$
contradicts C(i,j) on iteration 1
D(0,0) can't happen.

Suppose D(0,j) is violated on iteration 2 with j > 0. Then
$y_j \sim y_0 + j - 1$
$x_j + n \sim n + j - 1$ by (3) and (2)
$x_j \sim j - 1$
contradicts B(j) on iteration 1
D(i,0) can't happen.

Suppose D(i,j) is violated on iteration 2, with i > 0 and j > 0. Then
$y_{(i+j)} \sim y_i + j - 1$
$x_{(i+j)} + n \sim x_i + n + j - 1$ by (3)
$x_{(i+j)} \sim x_i + j - 1$
contradicts D(i,j) on iteration 1

Case 2: x_0 is not a multiple of 16.

Then $x_0 = 16n + m$ for some n and m, where $0 < m < 16$. Thus
$x_0 \sim m - n$ (4)
$y_0 = x_{(16-m)} + n + 1$ (5)
$y_m = n$ (6)
If $0 < k < m$ then $y_k = x_{(16-m+k)} + n + 1$ (7)
If $k > m$ then $y_k = x_{(k-m)} + n$ (8)

To save space I will use boldface type to denote incongruencies that are hypothetically violated on iteration 2; I will use normal type to denote incongruencies that are consequently violated on iteration 1. I will omit cases where j = 0, since these are all vacuous.

A(0):
$y_0 \sim 1$
$x_{(16-m)} + n + 1 \sim 1$ by (5)
$x_{(16-m)} \sim -n$
$x_{(16-m)} \sim x_0 - m$ by (4)
$x_{(16-m)} \sim x_0 + (16-m) + 1$ because $16 \sim -1$
C(0,16-m)

A(i), 0 < i < m:
$y_i \sim i + 1$
$x_{(16-m+i)} + n + 1 \sim i + 1$ by (5)
$x_{(16-m+i)} \sim i - n$
$x_{(16-m+i)} \sim x_0 + i - m$ by (4)
$x_{(16-m+i)} \sim x_0 + (16-m+i) + 1$
C(0,16-m+i)

A(m):
$y_m \sim m + 1$
$n \sim m + 1$ by (6)
$-1 \sim m - n$
$-1 \sim x_0$ by (4)
B(0)

A(i), i > m :
$y_i \sim i + 1$
$x_{(i-m)} + n \sim i + 1$ by (8)
$x_{(i-m)} \sim (i-m) + m - n + 1$
$x_{(i-m)} \sim (i-m) + x_0 + 1$ by (4)
C(0, i-m)

B(0):
$y_0 \sim -1$
$x_{(16-m)} + n + 1 \sim -1$ by (5)
$x_{(16-m)} \sim (16 - m) + m - n - 1$ because $16 \sim -1$
$x_{(16-m)} \sim (16 - m) + x_0 - 1$ by (4)
D(0, 16-m)

B(i), 0 < i < m :
$y_i \sim i - 1$
$x_{(16-m+i)} + n + 1 \sim i - 1$ by (7)
$x_{(16-m+i)} \sim (16 - m + i) + m - n - 1$
$x_{(16-m+i)} \sim (16 - m + i) + x_0 - 1$ by (4)
D(0, 16-m+i)

B(m):
$y_m \sim m - 1$
$n \sim m - 1$ by (6)
$1 \sim m - n$
$1 \sim x_0$ by (4)
A(0)

B(i), i > m :
$y_i \sim i - 1$
$x_{(i-m)} + n \sim i - 1$ by (8)
$x_{(i-m)} \sim (i - m) + m - n - 1$
$x_{(i-m)} \sim (i - m) + x_0 - 1$ by (4)
D(0, i-m)

C(0, j), 0 < j < m :
$y_j \sim y_0 + j + 1$
$x_{(16-m+j)} + n + 1 \sim x_{(16-m)} + n + 1 + j + 1$ by (7) and (5)
$x_{(16-m+j)} \sim x_{(16-m)} + j + 1$
C(16-m, j)

C(0, m):
$y_m \sim y_0 + m + 1$
$n \sim x_{(16-m)} + n + 1 + m + 1$ by (6) and (5)
$x_{(16-m)} \sim (16 - m) - 1$
B(16 – m)

C(0, j), j > m :
$y_j \sim y_0 + j + 1$
$x_{(j-m)} + n \sim x_{(16-m)} + n + 1 + j + 1$ by (8) and (5)
$x_{[(j-m) + (16-j)]} \sim x_{(j-m)} + (16 - j) - 1$
D(j-m, 16-j)

C(i, j), 0 < i < m , 0 < j < m – i :
$y_{(i+j)} \sim y_i + j + 1$
$x_{(16-m+i+j)} + n + 1 \sim x_{(16-m+i)} + n + 1 + j + 1$ by (7)
$x_{(16-m+i+j)} \sim x_{(16-m+i)} + j + 1$
C(16-m+i, j)

C(i, m-i), 0 < i < m :
y_(i+m-i) ~ y_i + m – i + 1
y_m ~ y_i + m – i + 1
n ~ x_(16-m+i) + n + 1 + m – i + 1 by (6) and (7)
x_(16-m+i) ~ (16 – m + i) – 1
B(16-m+i)

C(i, j), 0 < i < m, m – i < j < 16 – i :
y_(i+j) ~ y_i + j + 1
x_(i+j-m) + n ~ x_(16-m+i) + n + 1 + j + 1 by (8) and (7)
x_[(i+j-m) + (16-j)] ~ x_(i+j-m) + (16 – j) – 1
D(i+j-m, 16-j)

C(m, j), j > 0 :
y_(m+j) ~ y_m + j + 1
x_(m+j-m) + n ~ n + j + 1 by (8) and (6)
x_j ~ j + 1
A(j)

C(i, j), i > m , j > 0 :
y_(i+j) ~ y_i + j + 1
x_(i+j-m) + n ~ x_(i-m) + n + j + 1 by (8)
x_(i+j-m) ~ x_(i-m) + j + 1
C(i-m, j)

D(0, j) , 0 < j < m :
y_j ~ y_0 + j – 1
x_(16-m+j) + n + 1 ~ x_(16-m) + n + 1 + j – 1 by (7) and (5)
D(16-m, j)

D(0, m):
y_m ~ y_0 + m – 1
n ~ x_(16-m) + n + 1 + m – 1 by (6) and (5)
x_(16-m) ~ (16 – m) + 1
A(16-m)

D(0, j), j > m :
y_j ~ y_0 + j – 1
x_(j-m) + n ~ x_(16-m) + n + 1 + j – 1 by (8) and (5)
x_[(j-m) + (16-j)] ~ x_(j-m) + 1 + 16 – j
D(j-m, 16-j)

D(i, j), 0 < i < m , 0 < j < m – i :
y_(i+j) ~ y_i + j – 1
x_(16-m+i+j) + n + 1 ~ x_(16-m+i) + n + 1 + j – 1 by (7)
D(16-m+i, j)

D(i, m-i) , 0 < i < m :
y_m ~ y_i + m – i – 1
n ~ x_(16-m+i) + n + 1 + m – i – 1 by (6) and (7)
x_(16-m+i) ~ 16 – m + i + 1
A(16 – m + i)

D(i, j), 0 < i < m , m – i < j < 16 – i :

y_(i+j) ~ y_i + j – 1
x_(i+j-m) + n ~ x_(16-m+i) + n + 1 + j – 1 by (8) and (7)
x_[(i+j-m) + (16-j)] ~ x_(i+j-m) + (16 – j) + 1
C(i+j-m, 16-j)

D(m, j), j > 0 :

y_(m+j) ~ y_m + j – 1
x_(m+j-m) + n ~ n + j – 1 by (8) and (6)
x_j ~ j – 1
B(j)

D(i, j), i > m , j > 0 :

y_(i+j) ~ y_i + j – 1
x_(i+j-m) + n ~ x_(i-m) + n + j – 1 by (8)
D(i-m, j)

End of Appendix

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[27] Found at Kimberley Public Library, South Africa by the Author

[28] The Basoga are a different tribe to the Baganda (see Figure 1). I have not had sight of this paper since it is missing from the SOAS library. I would welcome a copy if anybody can source it.

[29] Cited by Mayega. This was an undated offprint held at Makerere University at Kampala. I have not had sight of it, and would welcome a copy if anybody can source it.