ESR and Bypass Capacitor Self Resonant Behavior
How to Select Bypass Caps

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Introduction

It's no news that the band pass requirements for power systems on PCB's are increasing and that power supply impedance requirements are getting tighter. Bypass capacitor fabrication and assembly techniques are improving and pushing higher the normal self-resonant frequencies we have to deal with. ESR's (equivalent series resistance) are decreasing, pushing further down the floor of the power supply impedance curve.

All this has created increased debate as to how to take advantage of this higher self-resonant requency and lower ESR. One argument is that lower ESR is thoroughly beneficial. Another is that, while lower ESR lowers the impedance at the minimum points, it also increases it at the maximum ("anti-resonant") points, and therefore lower ESR is not necessarily beneficial. Some argue for system designs that incorporate a well defined number of high quality (precise self-resonant frequencies and low ESR) capacitors with carefully chosen self-resonant frequencies. Others argue for more general quality bypass capacitors with SRF's (self-resonant frequencies) well spread across the frequency range of interest.

And here is a point to ponder. In the past, with large numbers of capacitors spread all over our boards, "anti-resonant" peaks have not generally been regarded as an issue. How did we get away with that for so long?

Here are some "truths" that can (and will) be demonstrated in this paper:

1. As ESR goes down, the troughs get deeper and the peaks get higher.
2. The minimum impedance value is not necessarily ESR (or ESR/n, where n is the number of identical parallel capacitors); it can be lower than that!
3. The impedance minimums are not necessarily at the self resonant points of the bypass capacitors.
4. For a given number of capacitors, better results can be obtained from more capacitor values, with moderate ESRs, spread over a range than with with a smaller set of capacitor *values*, with very low ESRs, at even well-chosen specific self resonant frequencies.

Self Resonant Frequencies

Assume a simple capacitor with capacitance C, inductance L, and equivalent series resistance (ESR) equal to R. The inductance should be considered from the practical sense — i.e. not only the inherent inductance associated with the capacitor physical structure itself, but also the PCB pads and attachment process, etc. The impedance through this capacitor is:

$$Z = R + jwL + 1/jwC, \text{ or}$$
$$Z = R + j(wL - 1/wC)$$

where w is the angular frequency:

$$w = 2 * Pi * f$$

Resonance occurs, by definition, when the j term is zero:

$$wL = 1/wC$$
$$w^2 = 1/LC$$
$$w = 1/Sqrt(LC)$$

The impedance through the capacitor at resonance is R.

Effects of multiple capacitors

Assume we have n identical caps, as above. The equivalent circuit of the n identical capacitors is the single capacitor whose values are

$$C = nC$$
$$L = L/n$$
$$R = R/n$$

The impedance of this system is now

$$Z = R/n + j(wL/n - 1/wnC)$$

The resonant frequency of this system is, again, where the j term goes to zero, or where

$$wL/n = 1/wnC$$

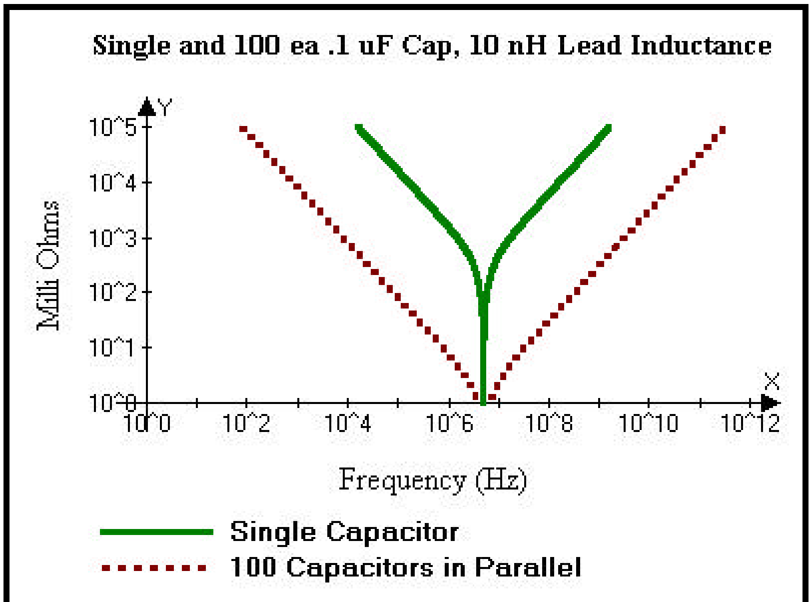


Figure 1.
Difference in frequency response between a single capacitor and n parallel capacitors.

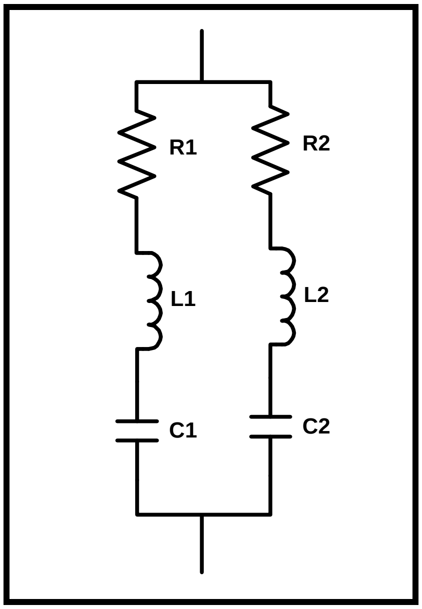


Figure 2.
Two capacitors in parallel

which results in exactly the same self-resonant frequency as before. Paralleling capacitors does not change the self-resonant frequency, but it effectively increases the capacitance, reduces the inductance, and reduces the ESR compared to a single capacitor. The resulting impedance response curve tends to "flatten out" compared to a single capacitor, see Figure 1.

Historically, on circuit boards, circuit designers have used a large number of bypass capacitors of "the same" value (the reason for the quotes will become evident later!). The advantage of this process has been the increased C and the reduced L and R that results.

Parallel Capacitors

Take the case of two parallel capacitors, shown in Figure 2. Let's let R1 = R2 = R in order to simplify the arithmetic. (This assumption does little harm and greatly helps the intuition!) Let us also assume that:

C1 > C2
L1 > L2

which means that Fr1 (the self-resonant frequency of C1) is lower than Fr2. Now:

X1 = wL1 - 1/wC1　　　X2 = wL2 - 1/wC2
Z1 = R + jX1　　　Z2 = R + jX2

The combined impedance through the system is:

$$Z = \frac{1}{\frac{1}{Z1} + \frac{1}{Z2}} = \frac{(R + jX1)(R + jX2)}{2R + j(X1 + X2)}$$

From this, we can derive the real and imaginary terms of the impedance expression:

$$\mathrm{Re}(Z) = \frac{R\left[2\left(R^2 - X1X2\right) + \left(X1 + X2\right)^2\right]}{4R^2 + \left(X1 + X2\right)^2}$$

$$\mathrm{Im}(Z) = \frac{\left(X1 + X2\right)\left(R^2 + X1X2\right)}{4R^2 + \left(X1 + X2\right)^2}$$

Further, we derive that the magnitude and phase of the impedance term are:

$$|Z| = \sqrt{\mathrm{Re}(Z)^2 + \mathrm{Im}(Z)^2}$$

$$\Theta = Tan^{-1}\left(\frac{\mathrm{Im}(Z)}{\mathrm{Re}(Z)}\right)$$

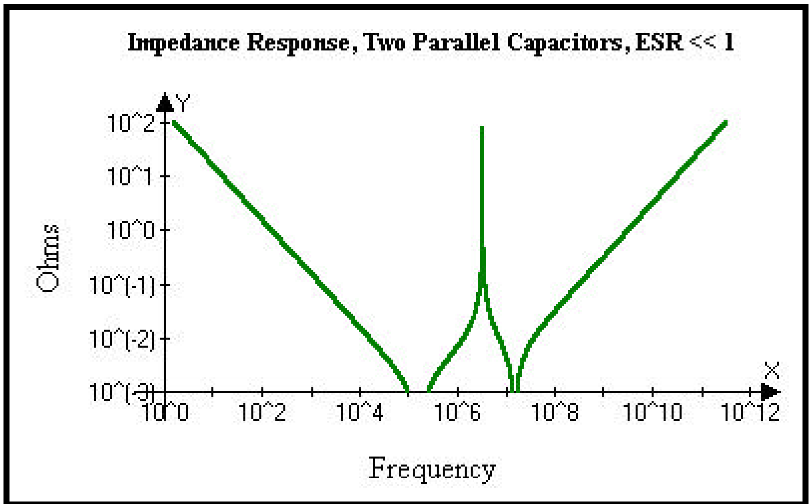


Figure 3
Impedance curve for two capacitors in parallel

The curve of impedance as a function of frequency is shown in Figure 3. It is instructive to look at this curve, and the real and imaginary terms of the impedance expression formula together.

Let Im(Z) Equal Zero

Resonance occurs when the imaginary term is zero. This is also the point at which the phase angle is zero. The impedance at that point is simply the real part of the impedance expression.

The imaginary term for Z goes to zero under two conditions:

$$X1 = -X2$$
$$R^2 = -X1X2$$

The first condition would represent the "pole" between the self-resonant frequencies of the two capacitors if R were zero. Since R > 0, there is not a "true" pole for any real value of frequency. But X1 equals –X2 when the reactance term of C1 is inductive (+) and increasing, the reactance term for C2 is capacitive (-) and decreasing, and where the two reactance terms are equal. This is the "anti-resonance" point that occurs at a frequency between Fr1 and Fr2.

Assuming R is small, the second condition can only occur where either X1 or X2 is small. X1 is small near Fr1 and X2 is small near Fr2. X1 and X2 must be of opposite sign, since R^2 must be positive. Therefore, these resonant points must be ***between*** Fr1 ad Fr2, and they must ***not*** be equal to Fr1 or Fr2 (unless, in the limit, R = 0).

The system resonant frequencies are not necessarily the same as the capacitor self resonant frequencies unless ESR is zero.

It can further be shown that at this point, where the imaginary term is zero and $R^2 = -X1X2$, the real term, and thus the impedance itself, ***simply reduces to R.***

Impedance at Fr1

At Fr1, the self-resonant frequency of C1, X1 = 0. It can be shown that:

$$\Theta = \text{Tan}^{-1}(RX2/(2R^2+X2^2))$$

If X1 = 0, then X2 must be negative (capacitive, under the conditions we have been assuming) so

$$\Theta < 0$$

Only in the limit where R = 0 does Θ go to zero.

The magnitude of impedance at the point where X1 = 0 can be shown to be:

$$|Z| = R\sqrt{\frac{R^2 + X2^2}{4R^2 + X2^2}}$$

This is ***less than*** R for any value of R > 0. In the limit, it is equal to R for R = 0 and equal to R/2 if R>>X2.

The results are exactly symmetrical if we are looking at Fr2, the point where X2 = 0.

The minimum value for the impedance function is at a frequency other than the self resonant frequency of the capacitor and less than ESR when two capacitors are connected in parallel. Further, the minimum value declines as X2 gets smaller, or, as the self resonant frequencies of the capacitors are moved closer together, or, as the number of capacitors increases. This point is illustrated in Appendix 3.

Impedance at "Anti-resonance"

If we let X1 = -X2, then Im(Z) goes to zero, by definition. This is the "anti-resonant" point between Fr1 and Fr2. At this point, it can be shown that:

$$Z = \frac{R}{2} + \frac{X^2}{2R}$$

For small values of R, this is inversely proportional to R and can be a *very large* number if R << 0. This is why there is concern about very high impedances at the "anti-resonant" point. If R, on the other hand, is only in the range of .1 or .01, then this number might be more manageable.

But consider this. If Z equals (approximately) R at the minimum, under what conditions is Z *also equal to R* at the maximum? Under those conditions, the impedance curve will be (at least approximately) *flat*! It turns out that Z equals R if:

R = X1 = -X2

We can achieve a (relatively) flat impedance response curve if we position our capacitor values such that, at the "anti-resonant" points, X1 = -X2 = ESR.

This has a very significant consequence. As ESR gets smaller, then, for a flat impedance response, X1 and X2 must be smaller at the anti-resonant points. This means that Fr1 and Fr2 must be closer together. And *THIS* means, that ***as ESR gets smaller, it requires more capacitors to achieve a relatively flat impedance response!*** This point is highlighted graphically in Appendix 4.

General Case Analysis

As we add more values for C, the algebra associated with these kinds of analyses gets *very* difficult. We at UltraCAD wrote our own program so we could look at various capacitor configurations and see what happens in a more "real world" situation.

The program is both elegant and inelegant at the same time! It is elegant in that it actually works, works easily, and it gets to an answer! It is inelegant in that it reaches an answer by "brute force" calculations that can take a fair amount of time in a complex case. And, it does not solve for exact maximum and minimum impedance values (and frequencies) but gets only arbitrarily close (but as you will see below, close enough).

The program operates in two modes, (1) internally selected capacitor values and (2) user supplied values. Using the first mode, there must be at least two capacitor values, .1 uF and .001 uF. Inductance associated with these two values are 10 nH and .1 nH, respectively. If additional capacitors are used, their capacitive and inductive values are spread logarithmically over this range. The user enters ESR separately, which is assumed constant for all values of capacitance. The specific program code looks like this:

```
‘  user has entered nvalues, number of capacitor values
‘  user has entered nsame, number of caps of same value
For i = 1 To nvalues
C(i) = (0.1 * ((0.01) ^ ((i - 1) / (nvalues - 1)))) * 10 ^ (-6)
L(i) = (10 * ((0.01) ^ ((i - 1) / (nvalues - 1)))) * 10 ^ (-9)
Next i
For i = 1 To nvalues
Ctotal = Ctotal + C(i) * nsame
Next I
```

Note: Although this approach might, in fact, lead to an optimal distribution of capacitance values, this technique was not chosen for that purpose, and that property is not claimed for this distribution. The computer needed some rule for selecting capacitor values; thus was simply the rule chosen.

Appendix 1 shows the first set of results. Three capacitor values were chosen, .1, .01, and .001 uF. Ten capacitors of each value were assumed. The inductance and the self-resonant frequency associated with each capacitor value are shown in the individual tables. The conditions under the three analyses shown in Appendix 1 were identical except that ESR is different for each case, being 0.00001, 0.001, and .1 Ohms, respectively.

The top portion of each output gives the general input conditions; the middle portion gives the calculated capacitance and related inductance value, and the self-resonant frequency for each capacitor. The bottom portion of each table provides the results. It provides each (approximate) turning point frequency in the impedance curve, whether that turning point is a minimum or maximum point, and the value of the impedance function at that point. It also provides the phase angle of the impedance function at that point.

For very low values of R, the phase angle changes *very rapidly* as it passes through zero (which it does near (but not necessarily exactly at) each turning point.)

Note from the results how dramatically the maximum and minimum values of impedance depend on R. Also, note how, when R is small, the minimum point actually begins shifting outside the self resonant point of some capacitors.

The results from Appendix 1 are shown in graphical form in the appendix.

Appendix 1 illustrates 30 capacitors, 10 each for three values of capacitance. What if, instead, we selected 30 individual capacitors spread evenly across the same range? Appendix 2 illustrates the results, and it tabulates them for approximately half the frequencies —– because of the way the capacitor values are selected, the results are symmetrical for the higher frequencies in the table.

The results are dramatically better in Appendix 2 than in Appendix 1 (middle table) for the same number of capacitors (30) and same ESR! The peaks and valleys are 40.2 and 0.0001 Ohms, respectively for 10 each of 3 values, and only 1.0 and 0.001 Ohms, respectively, for 1 each of 30 values! This suggests that very acceptable results can be achieved with:

1. a smaller number of capacitors
2. spread across a range of values, with
3. a nominal, but not exceedingly low ESR.

For the same number of capacitors and value for ESR, best results are obtained by spreading the capacitance values across a range rather than groups of capacitors around a given value.

This may explain why we have not had many problems in the past. Historically, we have used bypass capacitor values with wide tolerances, therefore spread broadly across a range, and with only moderate ESR values, just what this analysis suggests is optimal.

Achieving a Smooth Response

As suggested above, we can achieve an (approximately) flat frequency response if we place the self resonant frequencies of the capacitors close enough so that the following relationship applies at the anti-resonant frequency:

$$R = X1 = -X2$$

Appendix 3 illustrates what happens as we continue to increase the number of capacitors to what we sometimes see on our boards. Capacitor values are selected so that the self-resonant frequencies are optimally spaced between 5 MHz and 500 MHz. Three cases are shown, 100 capacitors, 150 capacitors and 200 capacitors, all with ESRs of .01.

Of particular interest is that, for each case, *the highest impedance values are lower than the lowest impedance values for the case before, at every frequency!* This demonstrates that the minimum impedance is, indeed, below ESR, and that as the capacitor values become closer together, the peaks drop dramatically.

```
1,67,4,.01
1,1,1.1,.001
20,.01,.9,.001
1,.0009,.00005,.00001
```

Input file for calculator mode2 operation. Data is for:

1 ea 67 uF caps with 4 nH inductance and .01 ESR
1 ea 1.0 uF caps with 1.1 nH inductance and .001 ESR
20 ea .01 uF caps with .9 nH inductance and .001 ESR
The fourth line simulates a plane with .0009 uF capacitance.

Figure 4
Input file illustration

Further, note that 200 capacitors with ESR of .01 and with self-resonant frequencies placed optimally between 5 MHz and 500 MHz provide a virtually flat impedance response curve at 5 milliohms or less! Even the case with 150 capacitors results in a very flat impedance response curve.

Appendix 4 shows what happens when we use the same 150 capacitors as shown in Appendix 3, but lower their ESR to .001 Ohms. The results are dramatically worse! This confirms what was stated above, that as ESR declines, it takes *more* capacitors to achieve a given response function!

User Supplied Input Values

In operating mode 2, the user may enter up to 500 sets of capacitor data. Each set of data (one record) consists of four items of information (fields). The information, in this order, includes:

The number of capacitors with these parameters
Capacitance, in uF
Inductance, in nH
ESR, in Ohms

Records do not have to have unique values for capacitance. In fact, records need not even be unique.

Figure 4 illustrates a sample input file. It contains three records reflecting a total of 22 capacitors and one additional record simulating the capacitance of a plane.

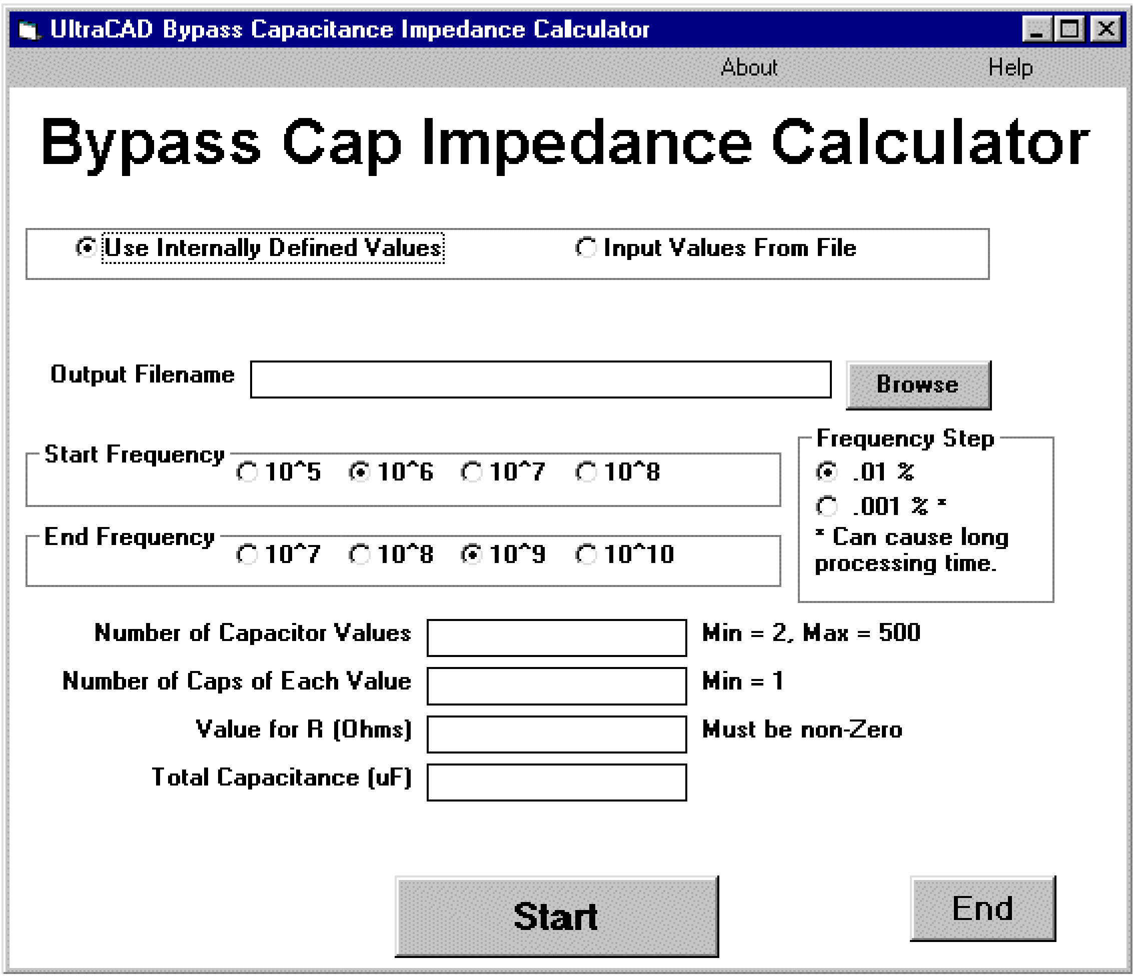
The output result from this input is shown in Appendix 5. Note in particular the sharp impedance peak caused by the anti-resonance between the bypass capacitors and the plane capacitance.

Bypass Capacitor Impedance Calculator

The calculator used in this analysis is available from UltraCAD's web site:

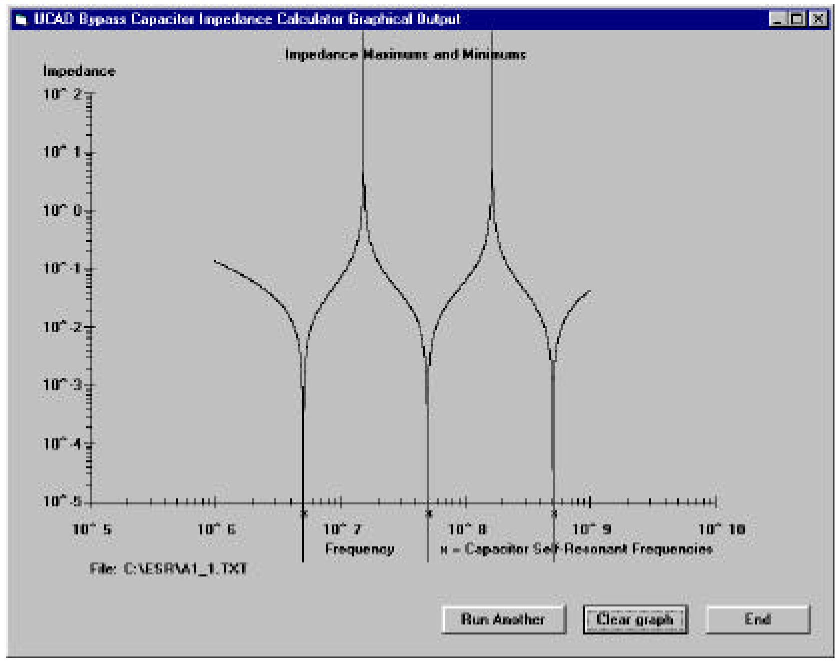
http://www.ultracad.com

The shareware version is limited to up to 3 each of up to 3 different capacitor values. It works in both modes described above. A license for the full function calculator is available for $75.00. Details and a mini-user's manual are available on the web site.



UltraCAD's Bypass Capacitor Impedance Calculator

Appendix 1
Effects of Varying ESR



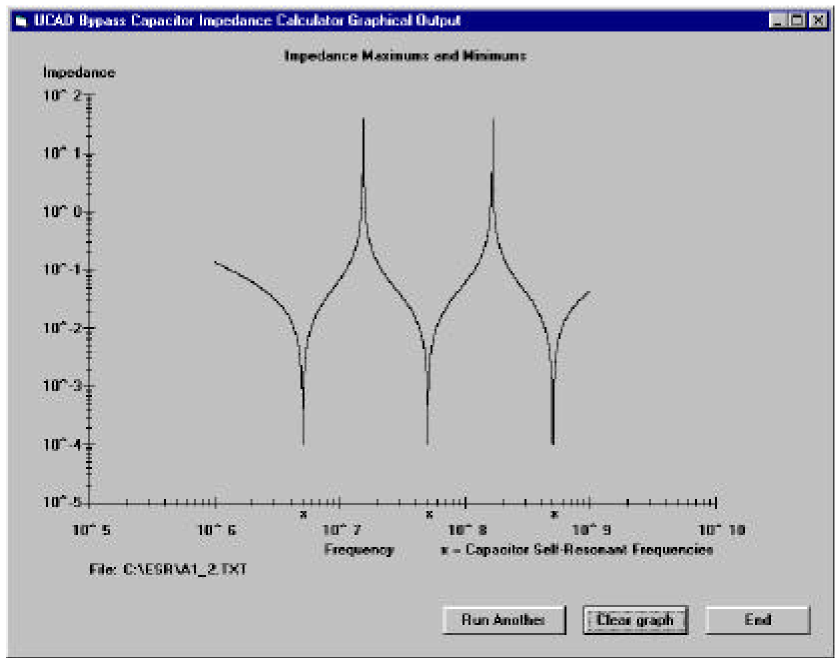
These three graphs correspond to the three (output) cases tabulated on the next page. They each model the case of:

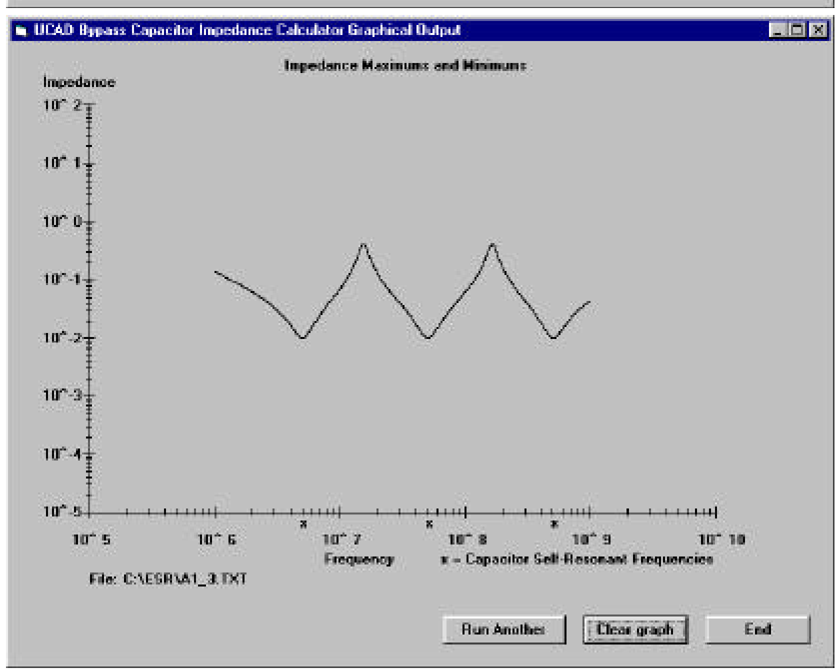
3 capacitor values,
chosen internally by the program, with
10 caps of each value.

The difference between them is that is that the ESR assumed for the caps varies. The assumed ESRs are:

Top:	.00001	Ohms
Mid:	.001	Ohms
Bot:	.1	Ohm

Note how lower ESR reduces the peaks and tends to "flatten" the curves somewhat.





Appendix 1 (Cont.)
Effects of Varying ESR

```
Initial Conditions
R (Ohms)                     =  0.00001
Number of Capacitor Values   =  3
Number of caps for EACH Value =  10
Total Capacitance            =  1.11    uF

 L nH                        C uF                        R             Resonant F (MHz)
10.00000                    .100000                      0.00001       5.033
01.00000                    .010000                      0.00001       50.329
00.10000                    .001000                      0.00001       503.292

Frequency (MHz)             Impedance      Turn          PhaseAngle(Rad)
5.0329                      .0000010       Min           -.8716
15.3599                     4003.5583008   Max           -4.7111
50.3292                     .0000010       Min           -1.5911
164.9127                    3936.4686108   Max           -11.501
503.292                     .0000010       Min           -.9432
```

```
Initial Conditions
R (Ohms)                     =  0.001
Number of Capacitor Values   =  3
Number of caps for EACH Value =  10
Total Capacitance            =  1.11    uF

 L nH                        C uF                        R             Resonant F (MHz)
10.00000                    .100000                      0.001         5.033
01.00000                    .010000                      0.001         50.329
00.10000                    .001000                      0.001         503.292

Frequency (MHz)             Impedance      Turn          PhaseAngle(Rad)
5.0329                      .0001000       Min           -.1728
15.36                       40.1688765     Max           -.6469
50.329                      .0001000       Min           -.1527
164.91                      40.1659130     Max           .9541
503.29                      .0001000       Min           -.1326
```

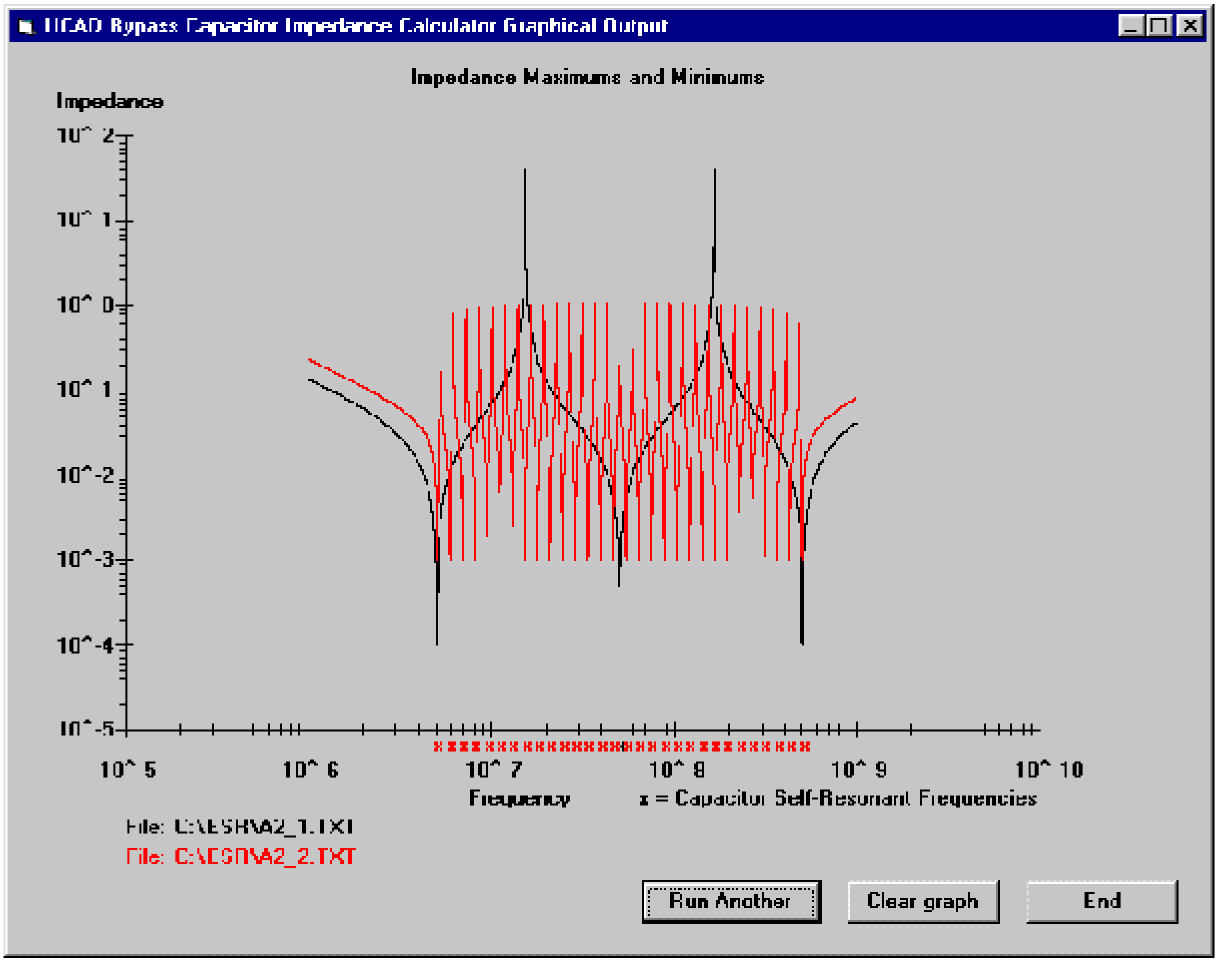
```
Initial Conditions
R (Ohms)                     =  0.1
Number of Capacitor Values   =  3
Number of caps for EACH Value =  10
Total Capacitance            =  1.11    uF

 L nH                        C uF                        R             Resonant F (MHz)
10.00000                    .100000                      0.1           5.033
01.00000                    .010000                      0.1           50.329
00.10000                    .001000                      0.1           503.292

Frequency (MHz)             Impedance      Turn          PhaseAngle(Rad)
5.0059                      .0099777       Min           -3.9414
15.368                      .4066792       Max           -1.1535
50.329                      .0099797       Min           -.0015
164.82                      .4066792       Max           1.1724
506.01                      .0099777       Min           3.9422
```

These results come from three runs using identical values for the capacitors except for their ESR. There are 10 capacitors of each value used in the analysis. The values for the capacitors are shown in the middle portion of each report. The bottom portion of the reports shows the minimum and maximum impedance values, the frequency (MHz) associated with that value, and the phase angle (in degrees) of the impedance expression at that frequency. The minimum and maximum frequency points are accurate to about .01%.

Appendix 2
Effects of Number of Capacitor Values



The black curve shows the impedance response from 10 each of three values for a total of 30 capacitors and 1.11 uF total capacitance. The red curve shows the results from the same number of capacitors (30), but with one each spread over the same range of values. Although the total capacitance is less (only .67 uF), the overall response is better. The output corresponding to the red curve is partially shown on the next page.

Appendix 2 (Cont.)
Effects of Number of Capacitor Values

Initial Conditions
R (Ohms) = 0.001
Number of Capacitor Values = 30
Number of caps for EACH Value = 1
Total Capacitance = 0.6752 uF

L nH	C uF	R	Resonant F (MHz)
10.00000	.100000	0.001	5.033
08.53168	.085317	0.001	5.899
07.27895	.072790	0.001	6.914
06.21017	.062102	0.001	8.104
05.29832	.052983	0.001	9.499
04.52035	.045204	0.001	11.134
03.85662	.038566	0.001	13.05
03.29034	.032903	0.001	15.296
02.80722	.028072	0.001	17.929
02.39503	.023950	0.001	21.014
02.04336	.020434	0.001	24.631
01.74333	.017433	0.001	28.87
01.48735	.014874	0.001	33.838
01.26896	.012690	0.001	39.662
01.08264	.010826	0.001	46.488
00.92367	.009237	0.001	54.488

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Frequency (MHz)	Impedance	Turn	PhaseAngle(Rad)
5.0327	.0009989	Min	-3.3536
5.2563	.6347305	Max	-3.6028
5.8989	.0009993	Min	-2.4119
6.2134	.7897249	Max	-2.5201
6.9142	.0009995	Min	-1.6806
7.3224	.8781140	Max	-1.76
8.1042	.0009996	Min	-1.2704
8.6172	.9346362	Max	-1.3348
9.4989	.0009996	Min	-1.3302
10.132	.9724241	Max	.0143
11.134	.0009997	Min	-.1913
11.907	.9987498	Max	-1.4751
13.05	.0009997	Min	-.6241
13.986	1.0171292	Max	-.4517
15.296	.0009997	Min	-.3987
16.423	1.0300747	Max	-.2126
17.928	.0009998	Min	-1.3008
19.28	1.0392816	Max	-.3874
21.014	.0009997	Min	-.3006
22.629	1.0456834	Max	.4032
24.631	.0009998	Min	.3959
26.557	1.0503213	Max	-.5864
28.87	.0009998	Min	.3725
31.162	1.0534425	Max	-.0349
33.838	.0009997	Min	-.2245
36.563	1.0554540	Max	.0406
39.662	.0009997	Min	.1786
42.898	1.0565857	Max	-.019
46.488	.0009997	Min	.2741
50.329	1.0569449	Max	.1568
54.488	.0009997	Min	-.1518

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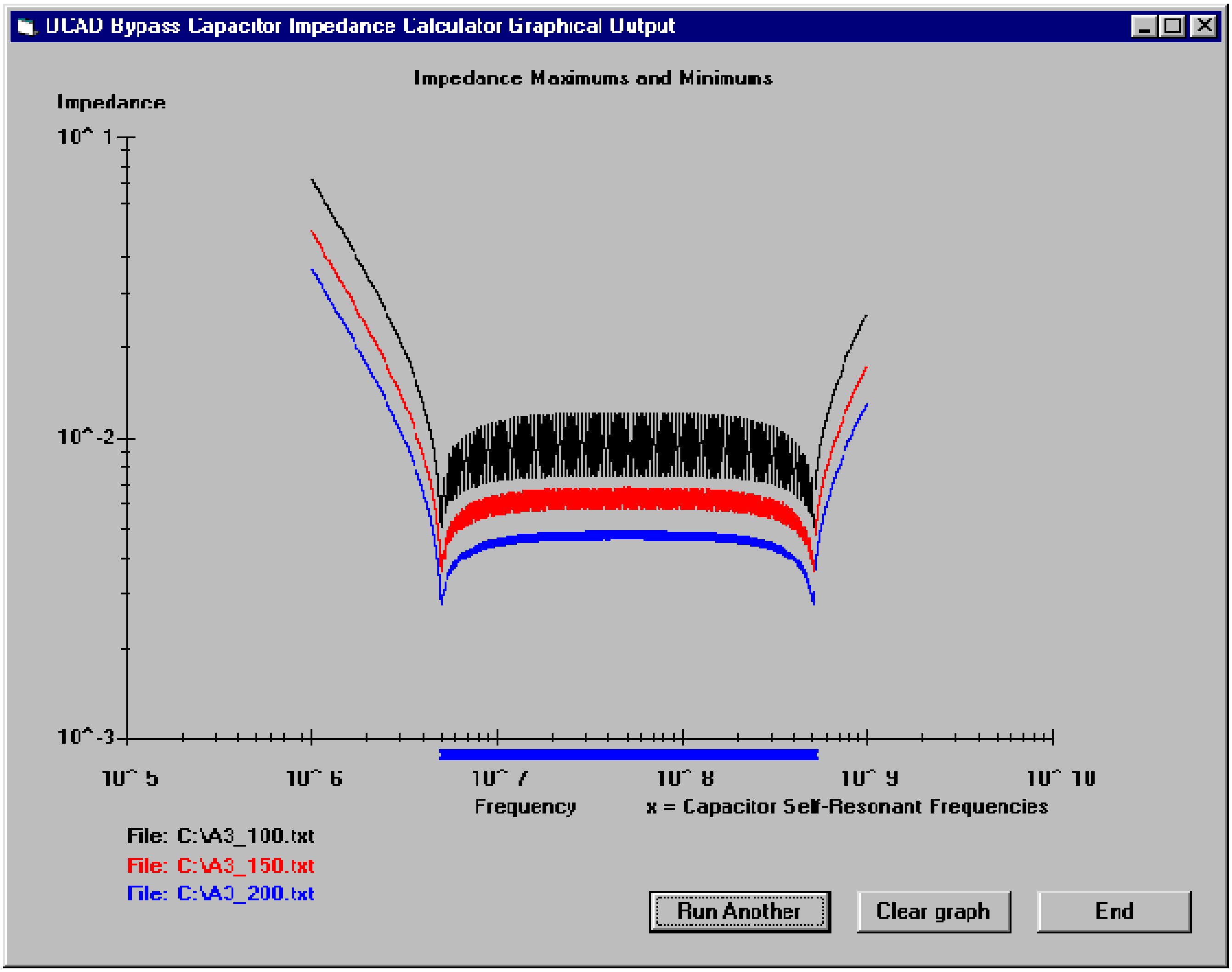
Appendix 3
Achieving a Smooth Response



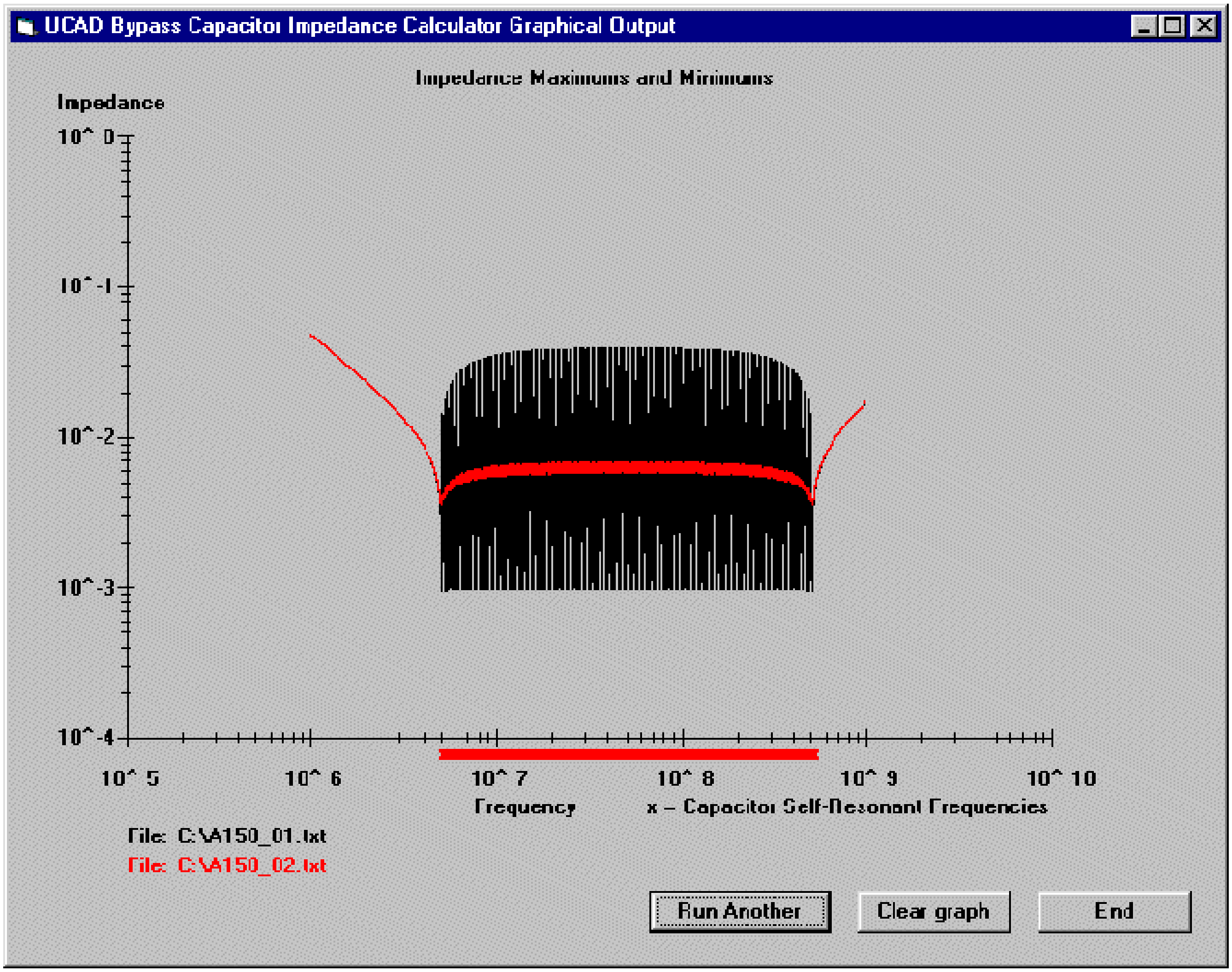
These curves show the impedance response from a number of capacitors optimally placed with self-resonant frequencies between 5 MHz and 500 MHz. In the center region, the impedance range is approximately

100 Capacitors:	.007 to .012 Ohms
150 Capacitors:	.005 to .006 Ohms
200 Capacitors:	.0046

Note that each successive curve is below the prior curve *at every frequency*.

Note: The apparent "banding" or modulation pattern in the graph for 100 capacitors is caused by the interaction of the graphical program resolution and the screen resolution of the monitor from which this picture is taken.

Appendix 4
Another Illustration of the Impact of ESR



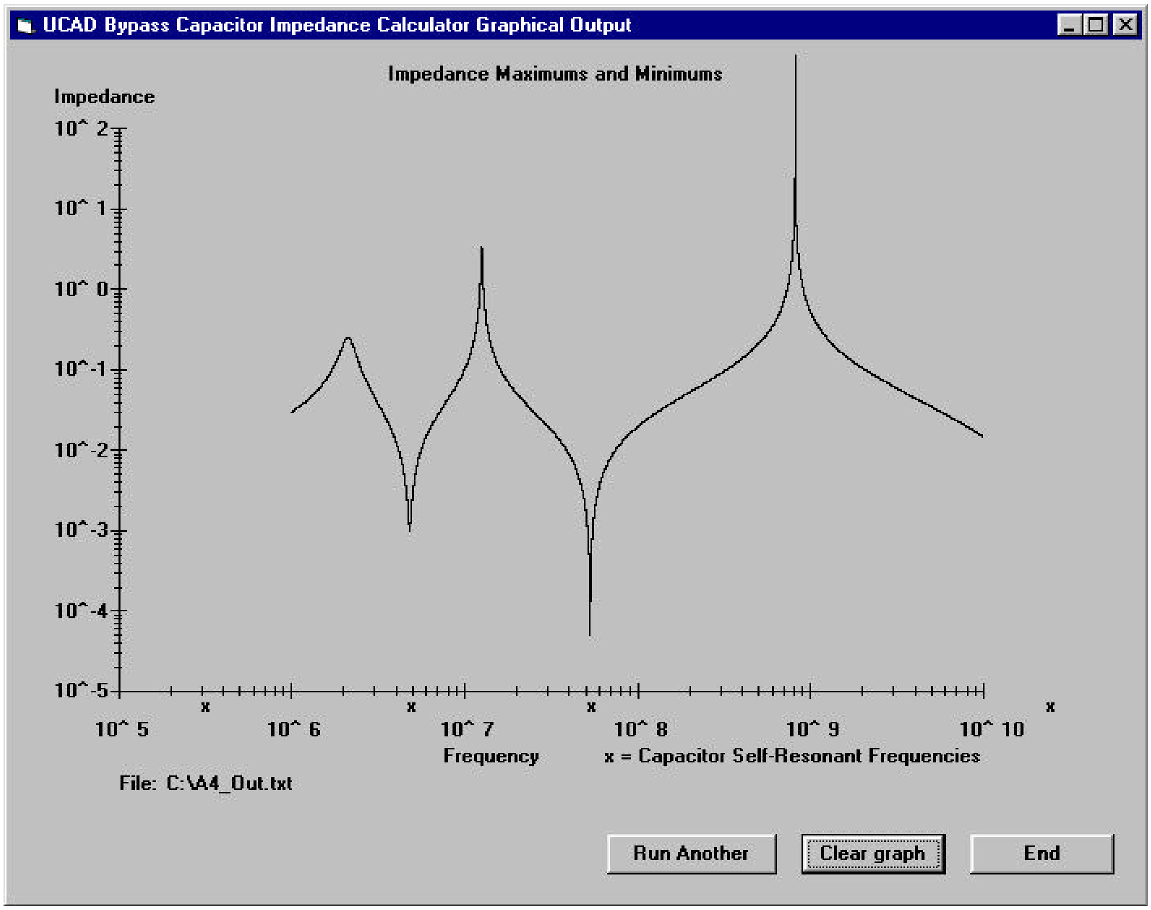
The red (second, or center, or gray) graph is the same data as the 150 capacitor model in Appendix 3. That was 150 capacitors, each with an ESR of .01. The larger, black graph shows the impedance curve with the same 150 capacitors, but each with an ESR of .001. The *average* impedance is (roughly) the same, but the impedance curve for the lower ESR capacitors is higher than the other curve for over half the frequencies in the range!

Note: As before, the apparent pattern in the graph for ESR = .001 is caused by the interaction of the graphical program resolution and the screen resolution of the monitor from which this picture is taken.

Appendix 5
General Case File Input Example

Input File

```
1,67,4,.01
1,1,1.1,.001
20,.01,.9,.001
1,.0009,.00005,.00001
```



Output File

```
Initial Conditions
Input filename                = C:\A4_in.txt
Output filename               = C:\A4_Out.txt
Number of Capacitance Values =  4
Total Capacitance             =  68.2009
```

Number	L nH	C uF	R	Resonant F (MHz)
1	04.00000	67.000000	.01	.307
1	01.10000	1.000000	.001	4.799
20	00.90000	.010000	.001	53.052
1	00.00005	.000900	.00001	23725.418

Frequency (MHz)	Impedance	Turn	PhaseAngle(Deg)
1.	.0300925	Min	60.77
2.1112	.2574331	Max	-8.0969
4.7989	.0009993	Min	.2803
12.518	3.3499338	Max	-.9026
53.052	.0000500	Min	.2375
812.38	817.2332967	Max	1.7471