

Possible Worlds: A Fashionable Nonsense?

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“... as words copulate, possible worlds are brought
into being”.

G.Olsson,

In *Possible Worlds in*

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Sokal and Bricmont have written a book entitled *Fashionable nonsense* where they describe the abuse of science by post-modern philosophers: Lacan compares the phallus with the square root of -1 , Kristeva argues that the continuum hypothesis is crucial for the analysis of poetry and so on. They have perfectly shown how these intellectuals easily succeeded to impress people by talking about complicate scientific matters, in particular mathematical notions, they do not know much about themselves.

In the case of possible worlds, on the one hand we have a formal definition (FPW hereafter), made popular by Kripke (cf. our historico-philological remark at the end of the paper) and on the other hand an intuitive conception which goes back to Malebranche and Leibniz. Most of what the philosophers say about possible worlds is related to the latter and has no precise connection with FPW, but since there exists a formal definition of possible worlds, the layman can be impressed by this formal background and take seriously the most delirious discussions about possible worlds.

Sokal and Bricmont's book is also a more general attack against post-modern philosophy, and in particular against its relativism, according to which modern science has the same value as Ancient Mongolian Cosmology. The situation with possible worlds is even worse since we have here philosophers making an intimate combination of relativism with abuse of science. For example someone like David Lewis, not only defends relativism (according to him there are a plurality of possible worlds, as real as the real world - Goodman is more consistent, defending the same idea, he uses simply “worlds” rather than “possible worlds”) but bases his defense of it on abuse of science, using an alleged scientific notion of possible worlds. As we will see, FPW can in no reasonable way be used in a crucial way to support the existence of one thousand and one possible worlds or even of only one possible world other than the real world.

It is not really surprising that philosophers do not say much about FPW since these objects themselves are completely undetermined, and they can easily be eliminated from the formal definition of Kripke semantics. In fact their introduction hides the central feature of these semantics: the accessibility relation and the use of this relation in a definition by simultaneous recursion of truth for operators expressing necessity, possibility, belief, knowledge, obligation, etc. In the semantics of propositional modal logic, when possible worlds are dropped out, we just have valuations and the accessibility relation is defined directly between these valuations (in the case of Kripke semantics for intuitionistic logic, the relation is simply the natural partial order between valuations). Note that it is also possible to introduce possible worlds in the semantics of propositional logic, it is useless, but it gives a touch of fantasy to it. The difference between the standard classical bivalent semantics (or other semantics like many-valued semantics) and

Kripke semantics does not lie in the notion of possible worlds, but in the introduction of a relation between valuations.

In Kripke semantics without possible worlds, one may think that it is a good idea to introduce this terminology by calling valuations “possible worlds” because they are used to define the operator of possibility. But then why using the terminology “possible worlds” in Kripke semantics used to define other operators not related directly with possibility like intuitionistic negation, deontic operators, etc.? On the other hand if we call valuations possible worlds in all Kripke semantics, why not calling valuations possible worlds in the case of bivalent classical semantics or many-valued semantics?

In fact Wittgenstein in the *Tractatus* calls bivaluations, “truth-possibilities”, and he relates the basic semantic concepts of classical logic to modalities, saying that a tautology is necessary and a statement that can be false and true is possible (hence for him, validity=necessity, satisfiability minus validity =possibility). But Wittgenstein was against the introduction of modalities as logical operators at the same level as negation or implication.

Lukasiewicz also did not introduce possibility as a connective, but he introduced it as a third truth-value, developing henceforth many-valued logic. Lukasiewicz’s views on modal logic are not standard views, although he himself stucked until to the end of his life to the matrix approach, developing a four-valued semantics for modal logic this time introducing possibility and necessity as connectives (If one supports Lukasiewicz’s project, it is reasonable to call 3-valuations or 4-valuations of his semantics, possible worlds). The reason why Lukasiewicz’s views on modal logic are forgotten nowadays is mainly due to the emergence of Kripke semantics at the end of the 1950s rather than to the proof by Dugundji in 1930s that the standard modal logics of C.I.Lewis, S5, etc., cannot be characterized by finite matrices.

Who is able to seriously argue that S5 provides the best account of the notion of possibility? What have turned S5 and her sisters famous is Kripke semantics, otherwise they will probably have fallen in the shadow of history, as suggested by Mancosu. But obviously Kripke semantics is not limited to S5, modal logic or possible worlds, it is a powerful technical tool than has a wide range of applications, in particular in computation (note that the computation-oriented people who are using Kripke semantics talk rather about “states” than “possible worlds”). The account of possibility in terms of accessibility relation is neither better nor worse than the account of possibility in terms of a third value and maybe these two approaches have to be combined to build a good logic of possibility.

Philosophers don’t really discuss this question. Their discussions are not about some particular technical features of the definition of FPW, technical features like the nesting of modalities or the question to know if the accessibility relation is incestual or euclidean are not especially relevant for their debates. Their main focus is about the ontological status of possible worlds and related issues. An important distinction for them is the one between the real world and other worlds (proper possible worlds). This distinction is totally useless in the standard Kripke semantics for S5, etc. One can introduce a specific distinguished world supposed to represent the real world, but it is generally not needed, originally Kripke did so but the “real world” was later dropped out.

When the philosophers are talking about possible worlds, they are talking of objects which are supposed to be, at least metaphorically, “worlds”. But what is the relation between FPW and this metaphor? How can one seriously defend the idea that a FPW, or a valuation, represents a world? How is it possible to built a world with a couple of connectives and quantifiers? Formal logic at best represents the logical structure of a world. Modal logic does not offer any substantial world account. Some people have argued that possible worlds are not real worlds but fictitious worlds, that they can be seen as novels, however this idea is obviously absurd if we think of FPW: nobody, not even Oulipians, has written a novel using the language of modal logic.

Moreover philosophers have most of the time a narrow view on possibility, reducing it to non-contradiction. In fact Goodman argues that we must admit a multiplicity of worlds, that this is the

only way to give an account of the multiplicity of contradictory views, given by science, arts, etc. However paraconsistent logic can do the job. And in the face of Ockham's razor, it is perhaps better to introduce an additional operator (paraconsistent negation) than additional worlds.

Possible worlds are certainly fun, but if one wants to have fun, she can have it in the real world. In fact we think that one can have real fun only in the real world, this sounds logical. Furthermore there is a bunch of very good science-fiction books dealing with parallel worlds which are more fun than possible worlds philosophy. Carnap once wrote that philosophers like Hegel or Heidegger were bad poets, and that all their writings were nothing else but nonsense based on ill-formed syntactic constructions. Following Carnap's idea, we can say that people like Goodman and Lewis are bad science-fiction writers and that their writings is nonsense based on ill-formed mathematical constructions. Our assertion is less provocative than Carnap's since Goodman and Lewis are not considered first rank philosophers, and probably will never be so (at least in the real world). But since Sokal and Bricmont did not find useless to chase nonsense among second rank philosophers, we have indulged ourselves to do so in the third world of philosophy.

Historico-Philological Remark We have used here the terminology "Kripke semantics" since it is quite widespread. Since it is possible to do Kripke semantics without possible worlds, to avoid ambiguity, we didn't use the expression "Possible worlds semantics" which is generally used as a synonym to "Kripke semantics". In fact if it is commonly admitted that Kripke was the one who introduced possible worlds in Kripke semantics, most people think that several other logicians presented such kind of semantics before him, in particular Tarski himself in a paper with Jónsson, as claimed by Givant. For this reason it would be better to use another terminology, for example "Relational semantics", instead of "Kripke semantics". Some people who agree that Kripke is not the unique and true creator of Relational semantics, nevertheless insist on the fact that it is mainly Kripke's work which was really influential in the development of Relational semantics. And probably this is due in great part to the introduction by Kripke of the possible worlds metaphor (as it is known Carnap also made a reference to this expression and had the idea of possible worlds semantics for S5 but not of the full framework of Relational semantics). So in some sense this metaphor was useful and even crucial for the development of a logico-mathematical tool. There is nothing strange or wrong about it. We can find also many other examples in the history of mathematics, not to speak of physics or biology, where an imaginative word or symbol played a fundamental role in the development and popularity of a notion. The problem is the "boomerang effect": when the metaphor introduced in mathematics gets back to philosophers and sets their imagination into fire, then everything is possible...

Short Bibliography

The literature on possible worlds is larger than the size of the universe, therefore the reader cannot expect to find everything here. We are just giving some basic references directly relevant for the present discussion and that he may use as a take off for his odyssey through possible worlds.

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