Photons and Neutrinos as Electromagnetic Solitons

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Abstract

Photons and neutrinos are modeled as oscillatory states of the electromagnetic field confined within a local domain, the motion of which is governed by Maxwell's equations. The size and shape of the domain are limited by the relativistic principle of causality; i.e., the interval between events within the domain is timelike. These localized, soliton waves are called "wavicles."

The solutions of Maxwell's equations are eigenstates of the intrinsic (spin) angular momentum with eigenvalues k$\hbar$, k *being an integer or half-integer. The causally limited domain is a circular ellipsoid of length* λ *(the wavelength) and circumference* kλ. *The solutions possess helicity, which for* k = *1 correspond to left or right circularly polarized light, and for* k = $\frac{1}{2}$ *to the neutrino and the antineutrino.*

This wavicle model of the photon correlates very well with many of the experimental properties of light. It predicts how light is transmitted through apertures: We report a confirmatory measurement of the photon's diameter for microwaves. Multiphoton phenomena are predicted to occur above the observed intensity thresholds. The production of multiphoton wavicles in stimulated emission explains the occurrence of photon bunching. The wavicle also explains the directional and polarization properties of a helical microwave antenna.

The photon wavicle is the physical basis of the Heisenberg uncertainty principle; i.e., the wavicle is the quantum of action. The product of the wavicle's length and momentum is its relativistically invariant action k$\hbar$. *This action is necessarily involved in any observable process in which the wavicle is totally absorbed or emitted.*

Key words: photon, neutrino, wave-particle duality, Maxwell's equations, multiphoton, relativistic causality, spin angular momentum, Heisenberg uncertainty principle, Planck's constant

1. INTRODUCTION

The wave-particle duality exhibited by photons and other elementary particles is commonly accepted as a classical paradox that is inherent in quantum mechanics.[1] On the other hand, it is believed that the coexistence of wave and particle properties may be reconciled through a description in terms of hidden variables.[2] These two beliefs differ in philosophical outlook. The former maintains that it is futile to be concerned with quantities that are not observable because of the Heisenberg uncertainty principle, whereas the latter asserts that the elements of nature should be capable of objective description regardless of experimental measurements. The issues and history of this controversy are discussed in a tribute to de Broglie.[3] A recent proposition is that the state of a system *in between measurement events* may have simultaneously well-defined values of noncommuting observables.[4]

The dichotomy of the wave-particle duality arises because particles are regarded as pointlike objects having both position and momentum. This concept of particle is necessarily dichotomous, because its position and momentum are subject to the minimum uncertainties asserted by the Heisenberg uncertainty principle. In contrast, the theory presented here regards an elementary particle as electromagnetic energy contained within a finite domain. Hence its position is essentially uncertain within the linear size of the domain, and its momentum is distributed throughout the domain. This leads to the idea that the minimum quantum of action arises in interactions of the particle (observable processes), because the particle cannot transfer its distributed momentum in less time than is taken for it to traverse the length of its own domain. To emphasize its non-point-like nature we refer to this kind of particle by Eddington's term "wavicle."

Our wavicle concept of a particle is a pure field model. The particle's entire energy and action are derived from the oscillatory/rotational motion of its localized field. For the photon (at least) the field must be the electromagnetic field. This qualitative concept led us to consider the nature of the angular momentum and of the electromagnetic field and the relationship between them.

In Newtonian mechanics angular momentum is represented by a second rank, three-dimensional, skew-symmetric tensor, and because such a tensor has just three nonzero elements, it may alternatively be regarded as an axial (pseudo) vector. In relativistic mechanics angular momentum is also a second rank, skew-symmetric tensor,[5]-[9] but being four-dimensional, it has six nonzero elements. These six components may be regarded as two Newtonian (three-component) axial vectors: the orbital and spin angular momenta. This separation into spin and orbital vectors is, however, not gauge invariant,[9] and hence it is an approximation; the total angular momentum represented by the six-component tensor field is physically significant and mathematically correct.

The electromagnetic field has the same mathematical structure as that of angular momentum. When Maxwell's equations are written in tensor form the six electric and magnetic field components appear as the nonzero elements of a four-dimensional, second rank, skew-symmetric tensor.[8],[10] Regarding the electromagnetic field as two three-vectors (the electric and magnetic fields) is similar to the view of angular momentum as being composed of orbital and spin vectors; it appeals to human three-dimensional visualization, but the corresponding skew-symmetric tensors are fundamental.

In so far as physically distinct properties usually have mathematically distinct representations, we conjecture that the electromagnetic field is essentially angular momentum in four-dimensional space-time. This idea is applied to modeling of the structure of elementary particles. An elementary particle is regarded as a packet of electromagnetic action (angular momentum) localized in space-time, the motion of which is governed by Maxwell's equations. The field's motion within the packet is necessarily nonrectilinear, and hence it generates a local metric that manifests itself as the inertia (mass, momentum) of the particle. A similar, pure field model of elementary particles has been developed by Jennison.[11]

In this article this general concept of a wavicle is elaborated for particles moving at the speed of light, being developed most fully for the photon. The free-field Maxwell equations are especially tractable in the case of particles moving at the speed of light, and so for our first application of the wavicle theory we choose to model light-speed particles, and in particular the photon. Preliminary accounts of this work have appeared in conference proceedings.[12],[13]

2. THEORY OF ELECTROMAGNETIC WAVICLES

2.1 Concepts of the Photon

Several dynamical models of the photon have been proposed dating back to the work of Thomson.[14]-[20] More recent theories include the half-wave model of Honig[18] and the three-wave model of Kostro.[21],[22]

Our own work was initiated by an analysis of Einstein's photon-defining equation

$$E = h\nu \text{ or } E\tau = h \tag{1}$$

by Wadlinger,[23] a synopsis of which is as follows:

> *Units Analysis.* Since the units of the period τ are s/cycle, then Planck's constant h must have the dimensions of J · s/cycle for the units of E to be simply J. Thus, since h is the action carried by a single photon [the essence of Eq. (1)], its units are also J · s/photon, and hence we must equate *photon* with *cycle*.
>
> It follows that the photon acts for one cycle of the oscillation; i.e., it acts for a time period $\tau = 1/\nu$, and its effective length is the wavelength $\lambda = c/\nu$.
>
> This conclusion is, of course, contrary to the Copenhagen philosophy that one cannot know what one cannot measure, the basis of which is the Heisenberg uncertainty principle
>
> $$\Delta E \Delta\tau \geqq h, \tag{2}$$
>
> which expresses a minimum for the product of the uncertainties ΔE and $\Delta\tau$ (intrinsic experimental errors) in simultaneous measurements of the energy E and time τ. This experimental limitation (2) must not be confused with (1), which expresses a precise relationship between the photon's energy E and its period of oscillation τ. Contrary to the Copenhagen philosophy we shall see that nonmeasurable properties, such as the photon's length, do have predictive value (Secs. 3.1 to 3.3).
>
> The conclusion from the units analysis[23] that the photon is one wavelength long is compatible with the particlelike behavior exhibited in the photoelectric and Compton effects.

Our original objective was to produce a model of an isolated photon, for while single photons are not directly observable, they are believed to be of common occurrence; e.g., the emission of a single photon when an atom decays from an excited state to a lower state. Atomic line spectra show that the emitted photon must have a well-defined energy and frequency, and that the process must take place within a time τ, that is shorter than the average lifetime of the excited state. Hence the length and radial breadth of the photon can be no larger than τc. The lifetimes of excited states are typically a few orders of magnitude larger than the period of oscillation of the emitted photon; this is consistent with the deduction from the units analysis[23] that the photon's size is λ.

This photon size is supported by Mandel's result: The number of photons within a volume V is only meaningful if the linear dimension of V is larger than the wavelength of any occupied mode of the field. This implies that a photon cannot be localized within a length less than its wavelength, thus suggesting that the linear size of the photon is of the order of its wavelength. A relativistic argument based upon packets of de Broglie waves also leads to the conclusion "that the energy of a photon lies largely within one wavelength."[24]

Einstein,[3] Planck,[25] and Lewis[26] (who coined the term "photon") believed that the photon is a localized packet of electromagnetic energy. Our objective is to quantify their qualitative concept.

Our basic concept is that a photon is electromagnetic energy moving at the speed of light within a volume that is limited in extent both along and perpendicular to its direction of propagation. This qualitative model is in accordance with simple observations on rays of light.

2.2 Solution of the Wave Equation

We seek solutions of Maxwell's equations for a field moving at the velocity of light within a domain that is localized along the axis of propagation. Maxwell's equations can be transformed into d'Alembert's wave equation[27]

$$\partial^2\psi/\partial x^2 + \partial^2\psi/\partial y^2 + \partial^2\psi/\partial z^2 = 1/c^2 \times \partial^2\psi/\partial t^2, \tag{3}$$

where c is the velocity of light, and ψ is any one of the Cartesian electric and magnetic field components $E_x, E_y, E_z, H_x, H_y, H_z$. Solution of the wave equation (3) is attractive, because it only involves one of the six components, but the Maxwell equations are fundamentally definitive; once Eq. (3) has been solved for one component, the other five components are defined by it through Maxwell's equations.

For a wave traveling in the positive z direction the solution is a function of $u = z - ct$.[28] This relationship between z and t simplifies the four-variable partial differential equation (3) by separating (z, t) from (x, y). The separation is complete in the sense that there is no separation constant connecting the (z, t) equation with the (x, y) equation; i.e., the (z, t) equation

$$\partial^2\psi/\partial z^2 = 1/c^2 \times \partial^2\psi/\partial t^2 \tag{4}$$

is an identity for $\psi \equiv \psi(x, y, z - ct)$. The solution $\psi(x, y, z - ct)$ could have almost any dependence upon $u = z - ct$. However, we seek solutions that are oscillatory functions of the time t, and assuming the separable product form

$$\psi(x, y, z - ct) = f(x, y)S(u), \tag{5}$$

we find that $S(u)$ is either

$$S(u) = S_0 \exp(2\pi iu/\lambda) \tag{6a}$$

or

$$S(u) = S_0 \exp(-2\pi iu/\lambda) \tag{6b}$$

with λ being the wavelength of the oscillation. This form is characteristic of nonspreading, solitary waves,[29] which leads us, together with the following consideration, to keep only (6a) for a single value of λ.

The components of the electromagnetic field, such as E_x and H_x, are field amplitudes that are operationally similar to Schrödinger wave functions.[30] The first form for S [(6a)] is an eigenfunction of the operator for the z-component of linear momentum $[P_z = (\hbar/i)\partial/\partial z]$ with eigenvalue $+h/\lambda$; i.e., the de Broglie momentum in the positive z direction. This accords with $S(u)$ representing a wave moving in the positive z direction. The second form for $S(u)$ [(6b)] is an eigenfunction of P_z with eigenvalue $-h/\lambda$. However, the correct form for a wave moving in the negative z direction is $\exp[-2\pi i(z + ct)/\lambda]$.[28] Hence Eq. (6b) represents a negative energy state, which is physically untenable. Since an isolated particle should have a well-defined momentum in the z direction, we select a single value of λ in Eq. (6a) as being appropriate to the field of a single particle. Incidentally, a linear combination of $\exp[+2\pi i(z - ct)/\lambda]$ and $\exp[-2\pi i(z + ct)/\lambda]$ could represent positron-electron annihilation, where two photons are emitted moving in opposite directions.[31]

The separated (x, y) equation is simply Laplace's equation in two dimensions.[32] The rectilinear motion along the z-axis favors the use of cylindrical coordinates (r, φ) related to x and y by $x = r\cos\varphi$ and $y = r\sin\varphi$, while maintaining Eq. (3) as a differential equation for only one of the electromagnetic field components favors solving for the Cartesian components E_x and E_y, rather than the cylindrical components E_r and E_φ.[33],[34]

The solution of Laplace's equation in cylindrical coordinates yields the following form for the solution ψ of d'Alembert's equation (3):

$$\psi(r, \varphi, u) = (\alpha f + \beta g) \times (AP + BQ) \times S(u), \tag{7}$$

where $f = r^k$, $g = r^{-k}$, $P = \exp(ik\varphi)$, $Q = \exp(-ik\varphi)$, $S(u)$ is defined by Eq. (6a), and α, β, A, and B are the arbitrary constants of the general solution. The constant k (really k^2) is the separation constant that arises when r and φ are separated.[32] The most general solution of (3) will be a linear combination of terms of the form of (7) for different values of k, but as will be shown below, the field of an elementary particle is represented by (7) for a single value of k. Fields representing more than one particle may contain terms of the form of (7) for more than one value of k, but they are not considered any further in this article.

A sufficient condition for the physical requirement that the field be single-valued is that k be an integer, but as will be shown below, half-integer values of k also produce single-valued fields. In view of the occurrence of $\pm k$ in (7) we can restrict k to non-negative values without loss of generality.

2.3 Solution of Maxwell's Equations

The form (7) is the solution of d'Alembert's equation (3), and each of the six components of the electromagnetic field will have this form, with different constants α, β, A, and B for each component. We designate these different component constants by subscript numerals: $C_1 \equiv E_x$, $C_2 \equiv E_y$, $C_3 \equiv E_z$, $C_4 \equiv H_x$, $C_5 \equiv H_y$, $C_6 \equiv H_z$, where C is any one of α, β, A, and B.

It is reasonable to assume that all six components have the same $S(u)$ factor, otherwise different components would be oscillating at different frequencies. With the assumption of a common $S(u)$ factor, substitution of the form (7) for H_z, H_y, and E_x into the Maxwell equation

$$\partial H_z/\partial y - \partial H_y/\partial z = \epsilon_0\partial E_x/\partial t \tag{8}$$

leads to

$$\begin{aligned}(ik/r)\{\alpha_6 f[A_6P(k-1) - B_6Q(k-1)] + \beta_6 g[A_6P(k+1)\\ - B_6Q(k+1)]\} = (2\pi i/\lambda)\{(\alpha_5 f + \beta_5 g) \times [A_5P(k)\\ + B_5Q(k)] - \epsilon_0 c(\alpha_1 f + \beta_1 g) \times [A_1P(k) + B_1Q(k)]\},\end{aligned} \tag{9}$$

where $P(k \pm 1) = \exp[i(k \pm 1)\varphi]$ and $Q(k \pm 1) = \exp[-i(k \pm 1)\varphi]$.

Now, since the powers of r, r^{k-1}, r^{-k-1}, r^k and r^{-k} are linearly independent, it follows that the left-hand side of (9) must be zero, and hence that $\alpha_6 = \beta_6 = 0$ (i.e., $H_z = 0$). By the same argument the right-hand side yields $H_y = \epsilon_0 cE_x$. Similarly, the Maxwell equation

$$\partial E_z/\partial y - \partial E_y/\partial z = -\mu_0\partial H_x/\partial t \tag{10}$$

leads to the conclusions that $E_z = 0$ and $H_x = -\epsilon_0 cE_y$. These results are summarized by

$$H_z = E_z = 0 \quad H_y = \epsilon_0 cE_x \quad H_x = -\epsilon_0 cE_y, \tag{11}$$

from which it follows that two more of Maxwell's equations

$$\begin{aligned}\partial H_x/\partial z - \partial H_z/\partial x = \epsilon_0\partial E_y/\partial t\\ \partial E_x/\partial z - \partial E_z/\partial x = -\mu_0\partial H_y/\partial t\end{aligned} \tag{12}$$

are automatically satisfied.[1]

The remaining four Maxwell equations[27] reduce to a single independent equation – because of Eq. (12), and because the components satisfy Laplace's equation. This single independent equation may be chosen as

$$\partial E_y/\partial x = \partial E_x/\partial y, \tag{13}$$

and on substituting the form (7) this becomes

$$\begin{aligned}\{\alpha_2 f[A_2P(k-1) + B_2Q(k-1)] - \beta_2 g[A_2P(k+1)\\ + B_2Q(k+1)]\}\\ = i\{\alpha_1 f[A_1P(k-1) - B_1Q(k-1)]\\ + \beta_1 g[A_1P(k+1) - B_1Q(k+1)]\}.\end{aligned} \tag{14}$$

Since the powers of $r, f = r^k$ and $g = r^{-k}$, are linearly independent, and since $P(k \pm 1)$ is linearly independent of $Q(k \pm 1)$, it follows that

$$\alpha_2 A_2 = i\alpha_1 A_1 \quad \alpha_2 B_2 = -i\alpha_1 B_1$$
$$\beta_2 B_2 = i\beta_1 B_1 \quad \beta_2 A_2 = -i\beta_1 A_1. \quad (15)$$

These relationships between the arbitrary constants of E_x and E_y may be simplified to

$$\alpha = \alpha_1 = \alpha_2 \quad \beta = \beta_1 = -\beta_2$$
$$A = A_1 = -iA_2 \quad B = B_1 = iB_2. \quad (16)$$

Hence, in summary, the general solution of Maxwell's equations for a free electromagnetic field moving at the velocity of light in the positive z direction, for a particular value of k, is

$$E_z = H_z = 0$$
$$E_x = (\alpha f + \beta g) \times (AP + BQ) \times S(u) = \mu_0 c H_y \quad (17)$$
$$E_y = i(\alpha f - \beta g) \times (AP - BQ) \times S(u) = -\mu_0 c H_x,$$

where $f = r^k$, $g = r^{-k}$, $P = \exp(ik\varphi)$, $Q = \exp(-ik\varphi)$, $S(u = z - ct) = S_0 \exp(2\pi iu/\lambda)$, and α, β, A, and B are the arbitrary constants of the general solution. The constant k determines the angular momentum of the field, as explained below.

2.4 The Valid Values of k

Points of the wave (17) having the same phase angle δ are related by

$$2\pi u/\lambda \pm k\varphi = \delta, \quad (18)$$

the upper (+) sign being for $B = 0$ and the lower (−) for $A = 0$. If the amplitude is also constant, then we have the additional constraint $r = \text{const}$.

For a point of the wave moving forward at velocity c (i.e., $u = z - ct = \text{const}$), these equations imply that the phase point remains in the plane $\varphi = \text{const}$. A snapshot of the equiphase points (i.e., $t = \text{const}$) is a helix about the z-axis. For $z = \text{const}$ the trajectory of the phase point is a circle in the plane $z = \text{const}$, centered on the z-axis.

Differentiation of (18) with respect to t (with z const) produces the following expression for the angular velocity of the wave $\omega = \partial\varphi/\partial t$:

$$\omega = \partial\varphi/\partial t = \pm 2\pi c/(k\lambda) = \pm 2\pi\nu/k. \quad (19)$$

Thus the angular velocity ω is only equal to $2\pi\nu$ ($\nu = c/\lambda$ being the oscillatory frequency) when $k = 1$.

When the angle φ transits a full circle (i.e., $\Delta\varphi = 2\pi$), the u coordinate of the phase point must change by $\Delta u = \pm k\lambda$. Since φ is an angle, while u is a linear variable, both cases (+ and −) are possible regardless of whether $A = 0$ or $B = 0$. For a pair of equiphase points separated by this interval, the field must be single-valued, and since

$$\psi(r, \varphi + \Delta\varphi, u + \Delta u) = \psi(r, \varphi, u)$$
$$\times \exp[in(2\pi\Delta u/\lambda \pm k\Delta\varphi)], \quad (20)$$

the single-valued condition requires that

$$(2\pi\Delta u/\lambda \pm k\Delta\varphi) = n \times 2\pi, \quad (21)$$

where n is an integer.

Substitution of $\Delta\varphi = 2\pi$ and $\Delta u = \pm k\lambda$ yields

$$2k = n; \quad (22)$$

i.e., $k = n/2$ where n is an integer. The parameter k can take any integer or half-integer value $k = 0, \frac{1}{2}, 1, \frac{3}{2}, 2$, etc.

Alternatively, this result is derived by considering the general case $A \neq 0$ and $B \neq 0$. In this case ψ (specifically E_x) is proportional to $[AP(k\varphi) + BQ(k\varphi)] \times S(u)$. Consider the transformation ($\varphi \to \varphi + \Delta\varphi$, $u \to u + \Delta u$). If $\Delta\varphi = 2\pi$, the first factor $[AP(k\varphi) + BQ(k\varphi)]$ is invariant when k is an integer, and it changes sign when k is a half-integer (odd integer/2). For other fractional values of k (such as an odd integer/4) the transformed first factor is not a multiple of itself, and hence such values of k are obviously invalid. For ψ to be invariant under the transformation, the second factor $S(u)$ must itself be invariant when k is an integer, and it must change sign when k is a half-integer. Both cases (k integral and half-integral) are satisfied if Δu is related to $\Delta\varphi = 2\pi$ by the constant phase condition (18).

In summary: Because the field amplitude ψ is a product of two oscillatory factors, half-integer as well as integer values of k allow the field to be single-valued. Only integer values would be allowed if there were only one factor or if we thought of $\Delta\varphi$ and Δu as being independently variable, rather than being related by the constant phase condition. That wave functions may have geometric phase factors $[\exp(ik\varphi)]$ in addition to a time-dependent phase $[\exp(ik\omega t)]$ has been investigated (in a different context) by Berry.[35],[36] In Berry's work, as in ours, k may be half-integral as well as integral; i.e., the phase factor may change sign for a rotation through a full cycle of $\varphi = 2\pi$.[37]

2.5 The Domain of the Wavicle

One of our basic precepts about a wavicle is that it is a causally related entity; i.e., the interval between events within the wavicle must be timelike.[7],[10] The infinitesimal interval ds between a pair of contiguous events is expressed by

$$ds^2 = c^2dt^2 - dz^2 - dx^2 - dy^2, \quad (23)$$

and in cylindrical coordinates by

$$ds^2 = c^2dt^2 - dz^2 - dr^2 - r^2d\varphi^2. \quad (24)$$

In view of the rotational motion of the field, the finite interval between a pair of noncontiguous events Δs^2 must be computed along the geodesics of the nonrectilinear (helical) motion.[6] The geodesics are defined by the equal-phase condition (18) and $r = \text{const}$. Thus we have $dr = 0$, and from (19)

$$c^2dt^2 = (k\lambda/2\pi)^2 d\varphi^2. \quad (25)$$

Combining this relation with (24) we obtain the expression for a finite interval along a geodesic:

$$\Delta s^2 = [(k\lambda/2\pi)^2 - r^2]\Delta\varphi^2 - \Delta z^2. \quad (26)$$

This is the interval between a pair of points having the same phase angle and the same amplitude. For this interval to be timelike $\Delta s^2 \geqq 0$, and the domain of the wavicle is bounded by the null geodesics $\Delta s^2 = 0$.

Considering a plane $z = \text{const}$ ($\Delta z^2 = 0$) the radius r must satisfy the relation

$$r \leqq k\lambda/2\pi. \quad (27)$$

The equality in (27) defines the wavicle's maximum transverse radius:

$$r_{max} = k\lambda/2\pi. \quad (28)$$

At this radius the pair of causally related events unite into a single event, because the proper time between them is zero. That is, *all* phase points on a circle of radius r_{max} correspond to a single relativistic event. From (19) and (28) the tangential velocity of the rotating field at this radius, ωr_{max}, is equal to the velocity of light c.

The wave motion repeats itself when $\Delta t = \lambda/c$ (the period of the oscillation), and from (25) this period is equivalent to $\Delta\varphi = 2\pi/k$. Thus the interval between identical, repeating points of the wave is

$$\Delta s^2 = \Delta z^2 + (2\pi r/k)^2 - \lambda^2. \tag{29}$$

For a null geodesic $\Delta s^2 = 0$, this pair of points coalesces into a single relativistic event, and hence the wave is bounded by a null geodesic surface defined by

$$(2\pi r/k\lambda)^2 + (\Delta z/\lambda)^2 = 1. \tag{30}$$

For r = const this is the equation of a pair of points on an ellipse. If the origin ($z = 0$) is chosen to be at the center of the ellipse, then the locus of all such pairs of points is

$$(2\pi r/k\lambda)^2 + (2z/\lambda)^2 = 1. \tag{31}$$

This is the equation of an ellipse with the length of one axis (along the z-axis) being λ and that of the other axis (along a radius r in the $x - y$ plane) being $k\lambda/\pi$.

The wavicle's field is contained within the ellipsoid obtained by revolving the ellipse of Eq. (31) about the z-axis. The ellipsoidal surface is the locus of all null geodesics ($\Delta s^2 = 0$) between pairs of equiphase, equal amplitude points on it. The r-axis of the ellipse becomes the diameter of the cylindrical ellipsoid at $z = 0$; the circumference of the ellipsoid at $z = 0$ is $k\lambda$.

The ratio of the r-axis to the z-axis is k/π. For $k \leqq 3$ the r-axis is the minor axis of the ellipse, and the z-axis is its major axis. For $k \leqq 3$ the ellipsoid is prolate; for $k = 3$ it is almost spherical; for $k \geqq 7/2$ it is oblate.

Containment of the oscillating field within the finite region of space within the ellipsoid is consistent with Love's study of wave-motion with discontinuous boundaries.[38] The relationship between solutions of the wave equation (3) and non-Euclidean space-time metrics has been considered by Flint and Fisher.[39]

Notwithstanding Love's work[38] and our invocation of the relativistic principle of causality to limit the domain of the wavicle's field, we must concede that it leaves unresolved the radial dependence of the field components; i.e., the values of α and β in (7). Thomson was faced with the same problem.[17] Furthermore, soliton waves are described by the solutions of nonlinear wave equations[29],[40]-[43]; d'Alembert's linear equation (3) (used in this work) is a first approximation to the nonlinear sine-Gordon equation.[29] In this regard the present theory must be regarded as incomplete.

2.6 Angular Momentum Eigenfunctions

In view of the cylindrical symmetry of the field's motion, we would expect it to be an eigenfunction of L_z, the component of angular momentum about the axis of propagation. This angular momentum operator is

$$L_z = h/(2\pi i)\partial/\partial\varphi. \tag{32}$$

Identification of L_z (usually the operator for the z-component of *orbital* angular momentum) with the spin angular momentum is justified by the theoretical result that for light-speed particles (photons and neutrinos) the orbital angular momentum is indistinguishable from the spin angular momentum.[44]

Physically appropriate solutions arise in the following way. The form $\exp(+ik\varphi)$ is an eigenfunction of L_z with eigenvalue $+k\hbar$, and similarly, the eigenvalue of $\exp(-ik\varphi)$ is $-k\hbar$. Since the field is to represent a fundamental particle, the only angular momentum must be intrinsic angular momentum; i.e., spin, and since a particular particle is known to always have the same spin, its field must be of the form of (17) for a single value of k.

The general form of the L_z eigenfunctions is deduced as follows. Operation of L_z on the nonzero components of (17) produces

$$L_zE_x = k\hbar(\alpha f + \beta g) \times (AP - BQ) \times S(u) = \mu_0 c L_z H_y \tag{33a}$$

$$L_zE_y = k\hbar\ \exp(i\pi/2) \times (\alpha f - \beta g) \times (AP + BQ) \times S(u) = -\mu_0 c L_z H_x. \tag{33b}$$

It is apparent that if one of A and B is zero, then all four components are eigenfunctions of L_z with the same eigenvalue. If $B = 0$, the eigenvalue is $+k\hbar$, and if $A = 0$, it is $-k\hbar$. There is, however, a more general form for the eigenfunctions. The field **F** defined by

$$\mathbf{F}^{\pm} = [\mathbf{E} \pm (\mu_0 c/i)\mathbf{H}]/2 = (\mathbf{E} \mp i\mu_0 c\mathbf{H})/2 \tag{34}$$

has [in view of (17)] the simple form

$$\mathbf{F}^{+} = S(\alpha f BQ + \beta g AP)(\hat{i} - i\hat{j}) \tag{35a}$$

$$\mathbf{F}^{-} = S(\alpha f AP + \beta g BQ)(\hat{i} + i\hat{j}), \tag{35b}$$

where $\hat{i}$ and $\hat{j}$ are the Cartesian unit vectors and $i = \exp(i\pi/2) = \sqrt{-1}$.

This vector field is an eigenfunction of L_z if any one of the four constants α, β, A, and B is zero. Thus it can be an eigenfunction of L_z even if both A and B are not zero; in this case, the electric and magnetic field components themselves are *not* eigenfunctions of L_z. In the special cases where the radial dependence of the field is either monotonically increasing (i.e., $\beta = 0$) or decreasing (i.e., $\alpha = 0$) with increasing radius, the L_z eigenvalues of $\mathbf{F}^+$ and $\mathbf{F}^-$ are given by

	$\alpha = 0$	$\beta = 0$
$\mathbf{F}^+$	$+k\hbar$	$-k\hbar$
$\mathbf{F}^-$	$-k\hbar$	$+k\hbar$.

(36)

The vector field $\mathbf{F}^{\pm}$ is a particular combination of the six nonzero components that comprise the Lorentz invariant electromagnetic field tensor.[10] That it is an eigenfunction of L_z when **E** and **H** are not separately eigenfunctions (i.e., $A \neq 0$ and $B \neq 0$) as well as when they are eigenfunctions (i.e., $A = 0$ or $B = 0$) is a reflection of the fact that the electromagnetic field should be regarded as a second-rank tensor in four-dimensional space-time, rather than as two three-dimensional vectors.

The case of $A = \pm B$ and $k = 1$ corresponds to plane (i.e., linearly) polarized light. That the wavicle field (17) can be an eigenfunction of L_z in this case (if $\alpha = 0$ or $\beta = 0$) accords with the existence of plane-polarized photons as well as circularly polarized photons ($A = 0$ or

$B = 0$). More generally, it allows for elliptically polarized photons ($|A| \neq |B|$), since the photon field must always be an eigenfunction of L_z. A reason for choosing $\beta = 0$ is given in Sec. 2.8.

2.7 The Cylindrical Field and Spin-eigenfunctions

The radial and tangential components, E_r, H_r, E_φ, and H_φ are defined in terms of the Cartesian components E_x, H_x, E_y, and H_y by

$$E_r = E_x \cos\varphi + E_y \sin\varphi \quad H_r = H_x \cos\varphi + H_y \sin\varphi$$
$$E_\varphi = -E_x \sin\varphi + E_y \cos\varphi \quad H_\varphi = -H_x \sin\varphi + H_y \cos\varphi. \tag{37}$$

Substitution of the forms of the Cartesian components from (17) yields (after some simplification):

$$E_r = S(u)\{\alpha f[AP(k+1) + BQ(k+1)] + \beta g[AP(k-1) + BQ(k-1)]\} = \mu_0 c H_\varphi \tag{38a}$$

$$E_\varphi = \exp(-i\pi/2)S(u)\{-\alpha f[AP(k+1) + BQ(k+1)] + \beta g[AP(k-1) - BQ(k-1)]\} = -\mu_0 c H_r \tag{38b}$$

where $P(k \pm 1) = \exp[i(k \pm 1)\varphi]$, $Q(k \pm 1) = \exp[-i(k \pm 1)\varphi]$; the other components are defined immediately after Eq. (17).

A special case worthy of note is that if $\alpha = 0$ (radial dependence of the field $= r^{-k}$) and $k = 1$ (the photon), then all four cylindrical components are independent of the angle φ. This special case raises this question: How can the angular momentum operator $L_z = (\hbar/i)\partial/\partial\varphi$ operating on the cylindrical components reproduce the eigenvalues computed from the Cartesian components? The answer is that while the components themselves may (in this special case) be independent of φ, the curvilinear unit vectors $\hat{e}_r$ and $\hat{e}_\varphi$ are not fixed in space, and hence they are functions of φ. By expressing them in terms of space-fixed Cartesian unit vectors ($\hat{i}$ and $\hat{j}$) it is easily shown that

$$\partial\hat{e}_r/\partial\varphi = \hat{e}_\varphi \text{ and } \partial\hat{e}_\varphi/\partial\varphi = \hat{e}_r. \tag{39}$$

Hence, writing $\mathbf{E} = E_r\hat{e}_r + E_\varphi\hat{e}_\varphi$ and $\mathbf{H} = H_r\hat{e}_r + H_\varphi\hat{e}_\varphi$, it is easily proved that $\mathbf{E}$ and $\mathbf{H}$ are eigenfunctions of L_z (when $A = 0$ or $B = 0$) with the same eigenvalues derived in Sec. 2.6 from the Cartesian components of $\mathbf{E}$ and $\mathbf{H}$.

Thus, while the individual cylindrical components E_r, E_φ, H_r, and H_φ are *not* eigenfunctions of L_z (as is obvious in the special case, because they are independent of φ), their vectorial combinations $\mathbf{E}$ and $\mathbf{H}$ are eigenfunctions of L_z when $A = 0$ or $B = 0$. More generally, when $A \neq 0$ and $B \neq 0$, neither the components (Cartesian or cylindrical) nor the vectorial combinations $\mathbf{E}$ and $\mathbf{H}$ are eigenfunctions of L_z; only the combined field $F^{\pm}$ defined by (34) is an eigenfunction of L_z – providing that at least one of the four constants α, β, A, and B is zero.

2.8 Wavicle Energy and Action

The energy U of the field is given by[28]

$$U = (1/8\pi) \int\int\int (\epsilon_0|\mathbf{E}|^2 + \mu_0|\mathbf{H}^2|) dz\,r\,dr\,d\varphi, \tag{40}$$

where $|\mathbf{E}|^2 = |E_x|^2 + |E_y|^2$ and $|\mathbf{H}|^2 = |H_x|^2 + |H_y|^2$ since $E_z = H_z = 0$. The energy is alternatively obtained by integrating the magnitude of Poynting's vector[45] over r, φ, and t for one period of the oscillation.[23] The integration of (40) is over the volume enclosed within the ellipsoid that is the wavicle's light-speed-limited domain, the integration limits being z: $0 \rightarrow \lambda$, r: $r_{\min} \rightarrow k\lambda/2\pi$, φ: $0 \rightarrow 2\pi$.

The components of the integrand are [from (17)]:

$$|E_x|^2 = S_0^2(\alpha^2 r^{2k} + \beta^2 r^{-2k} + 2\alpha\beta) \times [A^2 + B^2 + 2AB\cos(2k\varphi)] = (\mu_0 c)^2|H_y|^2 \tag{41a}$$

$$|E_y|^2 = S_0^2(\alpha^2 r^{2k} + \beta^2 r^{-2k} - 2\alpha\beta) \times [A^2 + B^2 + 2AB\cos(2k\varphi)] = (\mu_0 c)^2|H_x|^2. \tag{41b}$$

Integration of $\cos(2k\varphi)$ produces $\sin(2k\varphi)/2$ which vanishes at both integration limits $\varphi = 0$ and $\varphi = 2\pi$. Thus, in effect, the $\cos(2k\varphi)$ can be discarded in (41), and on addition of the x and y components the $2\alpha\beta$ terms cancel. Thus the integrand in (40) becomes (using $\epsilon_0\mu_0 = 1/c^2$)

$$[\epsilon_0|\mathbf{E}|^2 + \mu_0|\mathbf{H}|^2] = 4\epsilon_0 S_0^2(\alpha^2 r^{2k} + \beta^2 r^{-2k})(A^2 + B^2). \tag{42}$$

Integration over z produces a factor of λ, and over φ a factor of 2π. Thus (40) simplifies to

$$U = \lambda\epsilon_0(A^2 + B^2)S_0^2 \int (\alpha^2 r^{2k} + \beta^2 r^{-2k}) r\,dr. \tag{43}$$

The term $\beta^2 r^{-2k}$ produces a divergent integral at the formal lower limit $r_{\min} = 0$ for all half-integer values of $k > 1/2$. One way of avoiding this divergence is to set $\beta = 0$; i.e., presume that the radial dependence of the field is r^k so that it is zero at $r = 0$. An alternative way would be to presume that the physical lower limit on r is a very small distance such as the Planck length $= (hG/c^3)^{1/2} = 4.05 \times 10^{-35}$ m. This latter choice arises from the idea that the Planck length is the smallest physical length.[46],[47]

The choice $\beta = 0$ is attractive, because it resolves the problem of continuity of the field through the origin $r = 0$, and it also fulfills the angular momentum requirement that at least one of the four arbitrary constants be zero. Hence we proceed with the assumption that $\beta = 0$, in which case (40) becomes [with $r_{\max} = k\lambda/2\pi$ from (28)]

$$U = \lambda^{2k+3}\epsilon_0(A^2 + B^2)S_0^2\alpha^2/[(2k + 2)(2\pi)^{2k+2}]. \tag{44}$$

Thus this finite expression for the energy is obtained by cutting off the radial field at $r = r_{\max}$; this introduces a discontinuity that merits further study as discussed at the end of Sec. 2.5.

The action of the wavicle is obtained by integrating this expression for its energy over the time of one period of its oscillation. This simply introduces a factor of (λ/c):

$$\text{action} = \lambda^{2k+4}\epsilon_0(A^2 + B^2)S_0^2\alpha^2/[c(2k + 2)(2\pi)^{2k+2}]. \tag{45}$$

Two of the arbitrary constants S_0, A, B, and α are redundant, and we choose to set

$$\alpha = 1 \quad \text{and} \quad (A^2 + B^2) = 1. \tag{46}$$

Since the action is a Lorentz-invariant quantity, the wavicle's action must be a universal constant for a given value of k. Our basic premise, by analogy with the Bohr quantization postulate (in the old quantum theory), is that the wavicle's action is an integral multiple n of Planck's constant h. Hence [using (45)] the normalization constant S_0^2 has the value

$$S_0^2 = 2nhc(k + 1)(2\pi)^{2k+2}/(\epsilon_0\lambda^{2k+4}). \tag{47}$$

From (46) and (47) the energy expression (44) simplifies to:

$$U = nhc/\lambda = nh\nu. \tag{48}$$

Thus the Bohr postulate leads to this generalization of the Einstein photon-defining equation (1) – the photon energy can be an integral multiple n of $h\nu$. This is discussed further in Sec. 2.12.

2.9 Physical Interpretation

The wavicles having $k = 1/2$ are the neutrinos, since these are zero-rest-mass particles with a spin of ½ while those having $k = 1$ are photons.[31] The spectrum of neutrinos having different wavelengths (energies) is analogous to the spectrum of photons; there is, from a relativistic viewpoint, only one (electron) neutrino and one photon. The perceived wavelength, frequency, and energy are simply functions of the observer's frame of reference. The two helicity states of the neutrino correspond to the two experimentally distinguished particles called the neutrino and the antineutrino. The photon helicity states correspond to light of different states of circular polarization.

That the neutrino is indeed a zero-rest-mass particle is supported by the Shelton supernova of 24 February 1987; neutrinos were observed to arrive at three earth observatories essentially coincident with the arrival of photons, and hence the neutrinos must have traveled *at* the speed of light.[48]-[51] The neutrinos were actually detected a few hours *before* the photons due to experimental exigencies. Even if they had arrived after the photons, an experimental error of a few hours within the 150 light-year distance of the supernova means that the speed of the neutrinos was within about one part in 10^6 of the velocity of light. From the measured neutrino energy it was inferred that the neutrino rest mass is at most a few electron volts.[48]-[51]

The solutions for $k = 1$ correspond to particles with a spin of $1 \times \hbar$, and hence these must be the photons.[31] These are discussed in greater detail in Sec. 2.10.

Wavicles having $k > 1$ may be regarded as neutrino-photon composites (for half-integral k) or as photon-photon composites (multiphotons) for integral values of k.[52] These composites are probably unstable and therefore experimentally difficult to observe. However, multiphotons are now commonly produced in focused laser beams (Secs. 2.12 to 3.1).

2.10 Photons

With $k = 1$ the radial factor of the field components (17) is either r^{+1} and/or r^{-1}, and since the formal range of r is $0 \leq r < \infty$, r^{+1} diverges as $r \to \infty$, and r^{-1} diverges at $r = 0$. When Thomson[17] was faced with these divergences he suggested variable coefficients for r^{+1} and r^{-1}. However, this is unacceptable, because the functions would no longer be solutions of the two-dimensional Laplace equation.[32] As indicated in Sec. 2.8, we choose $\beta = 0$ so that the radial dependence of the field components is r^{+1}.

The nonzero Cartesian components of the electromagnetic field are, from (17)

$$E_x = (X + Y)Sr \quad E_y = \exp(-i\pi/2)(X - Y)Sr$$
$$H_y = (X + Y)Sr\epsilon_0 c \quad H_x = \exp(i\pi/2)(X - Y)Sr\epsilon_0 c, \tag{49}$$

where $X = A\exp(i\varphi)$, $Y = B\exp(-i\varphi)$, $S = S_0\exp[2\pi i(z - ct)/\lambda]$, and λ is the wavelength of the light. The dimensionless normalized constants A and B satisfy $A^2 + B^2 = 1$ from (46).

The relative magnitudes of A and B determine the helicity state of the wavicle. The two special cases, $B = 0$ and $A = 0$, have positive and negative helicity and correspond to left and right circularly polarized photons, respectively.[53] When both A and B are unequal and nonzero, the photon is elliptically polarized.

When $A = B$ or $A = -B$ the complex field does not have the simple one-component form of a plane-polarized plane wave[28]; a plane wave is (17) for $k = 0$. Nevertheless, these states, containing equally weighted right and left circularly polarized functions, must be what are usually called "plane-polarized" photons. The discussion at the end of Sec. 2.6 shows that the whole electromagnetic field is an eigenfunction of L_z in this case even though its individual electric and magnetic components are not themselves eigenfunctions. However, since the discussion in Sec. 2.5 depended upon A or B being zero, we cannot infer that a plane-polarized photon is confined to the ellipsoidal domain (31). The difficulty of defining the spin of a plane-polarized photon has been discussed by Jauch and Rohrlich.[9]

2.11 Photon Generation

Thomson[17] and Honig's[18] photon models were mechanistic, being based upon generation of a pulse of radiation by an oscillating electric dipole.[8] This is probably quite a good model of a radiating atom, except that we believe that the photon's creation involves a rotating electron rather than an oscillating dipole. The atom is, in effect, a point source of the radiation, because it is typically 1000 times smaller than the wavelength of the emitted photon. The wave front of the emitted light will travel away from the point source at the velocity of light, so that after radiation for one photon period, it will be precisely one wavelength away from the source.[18],[54] Hence generation of a photon in one period of its oscillation predicts that its length will be precisely equal to its wavelength.

The change in the electron's orbital angular momentum is radiated as the photon's spin angular momentum. The photon's field has an angular velocity of $\omega = 2\pi c/\lambda$, and since angular momentum is moment of inertia times angular velocity, and since photons of all frequencies have the same angular momentum of $\hbar$, it follows that the moment of inertia of a photon must be proportional to its wavelength. Since moment of inertia has the form mr^2, and since the mass of a photon $= h/c\lambda$, it follows that the effective radius of a photon must be proportional to λ. This supports our inference in Sec. 2.5 that the photon's diameter is λ/π.

2.12 Energy Eigenstates: Multiphotons

Confinement of the field within the ellipsoid (31) together with continuity of the field at the surface of the ellipsoid would imply that the field must be zero outside the surface of the ellipsoid. In order to impose this condition the solution (17) of Maxwell's equations should be modified to conform with this boundary condition. This can be accomplished by transforming from the coordinates r and z to confocal elliptical coordinates.[55] While the mathematics is somewhat tedious, because it involves a pair of simultaneous eigenvalue problems,[56] its essential results are the allowed eigenvalues for the frequency c/λ and angular momentum of the confined field. We have thus far not completed this mathematical imposition of the ellipsoidal boundary conditions. However, the eigenvalues of the energy are obtained in a simpler way, as follows.

The energy eigenvalues arise from the time-independent Klein-Gordon equation.[57] This covariant eigenvalue equation is derived from d'Alembert's classical, covariant wave equation by differentiating the wave function with respect to time to eliminate the time, followed by use of the de Broglie relation, the energy-inertia equivalence relation

$$E = mc^2 \tag{50}$$

and the expression for the relativistic mass

$$m = m_0/[1 - (v/c)^2]^{1/2} \tag{51}$$

to eliminate the velocity v and the relativistic mass m. It has the form

$$-(\hbar^2/m_0)\nabla^2\psi = [E^2/(m_0c^2) - m_0c^2]\psi. \tag{52}$$

Since the rest mass m_0 of the photon is zero, Eq. (52) simplifies, using (50) and (51), to

$$-(\hbar^2/m)\nabla^2\psi = [E^2/mc^2)]\psi = E\psi. \tag{53}$$

This covariant eigenvalue equation is for a free photon; i.e., there is no potential energy in the Hamiltonian.

The energy eigenstates of the photon field are obtained by solving Eq. (53) for a particle confined within the ellipsoidal box that is the photon's domain. The energy eigenvalues of a particle in a linear or spherical box are[58]

$$E_n = (gnh/L)^2/m, \tag{54}$$

where n is the quantum number taking any positive integral value, L is the linear size of the box, m is the particle's mass, and g is a geometrical factor determined by the shape of the box.

Elimination of the relativistic mass m between (50) and (54) for a box of length $L = \lambda$ (the wavelength) produces, on taking the positive square root,

$$E_n = gnhc/\lambda = gnh\nu. \tag{55}$$

From the Einstein relation for photons (1) we deduce that the geometrical factor g is unity, and (55) thus states that the photon energy may be any integral multiple of the quantum $h\nu$

$$E_n = nh\nu \tag{56}$$

as deduced in (48) from the Bohr quantization condition.

This is a generalization of the Einstein photon relation (1). The energy $E_1 = h\nu$ is the ground state (a single photon), and for $n > 1$ the excited states of the electromagnetic field correspond to what are experimentally known as multiphotons. A multiphoton has the same frequency as the ground state, but it has n times as much energy. Note that the multiphoton quantum number n is independent of the angular momentum quantum number k.

For $n > 1$ there will be degeneracy; i.e., more than one multiphoton state with the same energy. These degenerate states differ in the disposition of the nodal surfaces of their fields. For example, for $n = 2$ one state will have a nodal plane φ = const, and another will have the plane $z = 0$. The former would be expected to decay into two $n = 1$ photons moving inphase along adjacent, parallel axes, whereas the latter would be expected to decay into two $n = 1$ photons moving along the same axis, one after the other separated in phase by a wavelength λ.

We believe that multiphotons are commonly produced by the process of stimulated emission. An ellipsoidal photon of energy $nh\nu$ interacts with an atom in an excited state with energy $h\nu$ above a lower stationary state. The photon emerges from the interaction with $(n + 1)h\nu$ of energy (in the same ellipsoidal volume) leaving the atom in the lower state. Multiphotons will also be produced whenever a large number of single photons are packed together; e.g., in a light beam focused to produce a high intensity. Experimentally, this is achieved in focused laser beams (see Sec. 3.1).

Like most excited states, multiphotons will spontaneously decay to the ground state, in this case producing n spatially separated but coherent single photons. Such a flock of photons is observed experimentally in the phenomenon of "photon bunching."[59],[60] The distinct phenomenon of "photon antibunching" is simply a way of saying that the energy of a light beam is localized (in the ellipsoidal volumes of our model) rather than being evenly distributed throughout the volume of the beam.

3. EXPERIMENTAL CONFIRMATION

3.1 Multiphoton Phenomena

Multiphoton absorption by atoms is considered to take place within "one optical period"; i.e., within $\tau = 1/\nu$.[61] This is the transit time of the photon wavicle. Mainfray and Manus[61] express the need for having a large number of photons concentrated within 10^{-15} s (i.e., one optical period). This need is satisfied by our multiphoton model, in which the n photons are congruent in space and time within the same ellipsoidal volume as that of the single photon.

Multiphoton absorption of visible light is observed to take place at intensities above about 1 MW/cm^2.[62],[63] Since our photon wavicle has a cross-sectional area of $\lambda^2/4\pi$ and an intrinsic power of $h\nu/\tau = h\nu^2$, its intrinsic intensity I_P (power/area) is given by

$$I_P = 4\pi hc^2/\lambda^4, \tag{57}$$

which is 1 MW/cm^2 for λ = 523 nm. At higher intensities than this (disregarding a geometrical packing factor of order unity), single photon-wavicles necessarily overlap to form multiphotons.

This accord of the wavicle's intrinsic intensity with the experimental threshold for multiphoton absorption is also confirmed for infrared photons (λ = 10.5 μm) from a carbon dioxide laser. The intrinsic intensity in this case is 6 W/cm^2, the experimental intensity required for absorption of several photons being about 30 W/cm^2 (experiments being conducted by Gokhan Baykut, Chemistry Dept., University of Florida, Gainesville). This experimental-theoretical agreement over nearly six orders of magnitude in intensity indicates that the fourth-power-dependence upon the wavelength predicted by (57) is correct.

A recent experiment by Verma and Chanda[64] has confirmed the multiphoton threshold predicted by (57) within about 25 percent. The desired experimental excitation of barium atoms requires the absorption of a multiphoton having just two quanta (i.e., $2h\nu$) from a laser beam of wavelength 650 nm. The experimental intensity has to be varied to produce a maximum in the desired signal; they estimate the optimum intensity in their 10 ns pulses to be 0.5 MW/cm^2. The threshold intensity for multiphoton production predicted by (57) is 0.42 MW/cm$_2$. This agreement between a sensitive (to intensity) experimental result and the theoretical prediction of (57) is quite remarkable, bearing in mind the nonuniformity of typical focused laser beams and the many orders of magnitude variation in intensity used in multiphoton absorption experiments.[61]

For randomly distributed photons, the probability of finding a multiphoton with energy $nh\nu$ is proportional to the ratio of the beam intensity I_B to the photon intrinsic intensity I_P raised to the n^{th} power. When $I_B < I_P$ this factor $(I_B/I_P)^n$ is much smaller than unity, and when $I_B > I_P$ it is much larger than unity. This explains both the threshold at about $I_B = I_P$ and the observed n^{th} power dependence of the ionization rate upon the intensity of the focused beam.[61]

This finite photon overlapping explanation of the n^{th} power law of multiphoton absorption is simple compared with the complexity of n^{th}-order perturbation theory; the latter is regarded, even by its proponents, as conceptually unsatisfactory.[61] A recent attempt to develop an alternative, adiabatic theory[65] also seems to have foundered in mathematical complexity. Our model of multiphoton processes differs from the "effective-photon" model[66] and from the "quantum potential" theory.[67]

3.2 Microwave Generation and Transmission

The axial mode of a helical microwave antenna has a wavelength equal to the circumference of the cylindrical helix, with maximum radiation along the axis of the helix in a well-defined, circularly polarized beam.[68] The axis of propagation and circumference of this antenna correspond precisely with the axis and circumference of the finite photon model. The radiation produced by this antenna consists of wavicles all propagating along the axis of the antenna and having the same helicity as that of the antenna. In view of the low intrinsic intensity of the microwave wavicle, the radiation will consist of multiphoton wavicles containing many quanta ($nh\nu$, $n \gg 1$).

3.3 Measurement of the Photon's Diameter

The transmission of microwaves through a circular aperture is predicted to be strongly attenuated at diameters smaller than λ/π.[69] This is the cross-sectional diameter of our finite photon model. The prediction is based upon the classical electromagnetic theory of a continuous wave. The attenuation is consistent with the simple mechanical notion that the photon-wavicle cannot pass through an aperture that is smaller than its own diameter of λ/π.

The photon-wavicle is not, of course, a rigid body, and its induced currents in the screen may cause some radiation to appear on the far side of the screen; in fact, the classically derived curve of Andrejewski[70],[71] predicts a small finite transmission coefficient at aperture diameters smaller than λ/π. The available experimental evidence is confined to localized field intensity measurements, and while it is broadly in agreement with classical electromagnetic theory and with our photon model, it is inconclusive as evidence for or against the finite-photon model. This led us to undertake experiments designed to resolve the issue.

We have investigated the phenomenon of microwave transmission through apertures in a series of experiments designed to measure the transmitted (undiffracted) power as a function of aperture size; we used both circular apertures (size = diameter) and rectangular apertures (size = width). A linear plot of transmitted intensity vs aperture area has an intercept on the aperture axis close to the photon-wavicle's intrinsic diameter of λ/π; the transmitted power is proportional to the difference between the aperture and wavicle areas.

Complete details of these experiments will be reported elsewhere[72]; there were naturally several experimental factors that had to be taken into account in order to obtain an accurate measurement. Our final result is summarized in Fig. 1. The measured cut-off diameter is equal to the value λ/π predicted by our ellipsoidal model (31) within the experimental error of 0.5 percent.

In view of the experimental inaccuracies we cannot claim to have proven that the wavicle model is quantitatively better than the classical, continuous field theory,[70] since both theories accord with the available experimental evidence. However, the wavicle model does provide an intuitively simple explanation for the experimental result, whereas in the classical, continuous

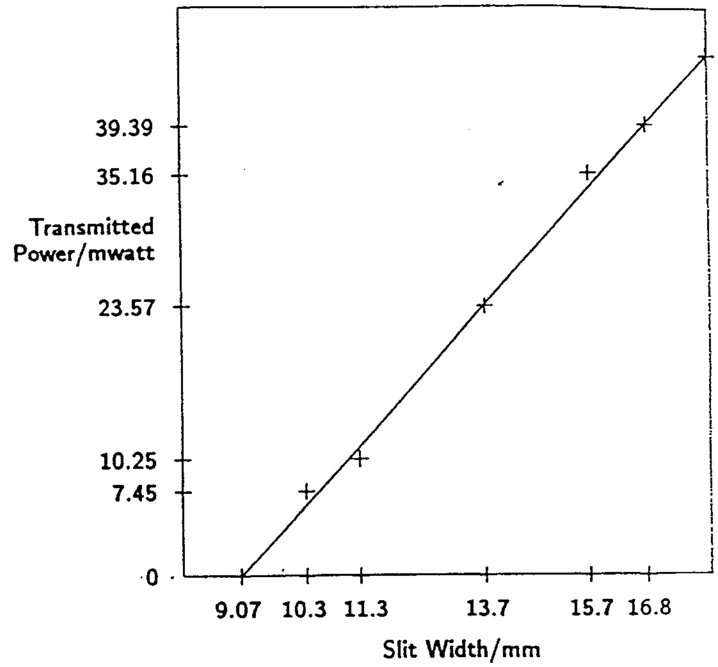

Figure 1. Experiment by Fritz Engler, 29 March 1986: X-band microwaves (wavelength = 28.5 mm), rectangular slit (constant length) in thin (0.5 mm) aluminum plates. The measured power was corrected by subtraction of the first harmonic content of the beam computed by the formula $H = 2.5 \times (SW - 4.54)/(8.1 - 4.54)$ based upon the signal at a slit width of 8.1 mm. The least-squares straight line (solid line in the graph) through the five experimental points (shown by +, SW = 10.3 − 16.8) yields an intercept on the slit-width axis of 9.07 ± 0.04 mm (0.5 percent error). The theoretically expected result is $28.5/\pi = 9.07$ mm.

wave theory the result only emerges as an approximate inference from a great deal of analysis, algebra, and calculation.[70],[71]

3.4 Diffraction and Interference at Low Light Intensities

Some experiments on the diffraction of light at low intensities have reopened the question of whether interference is a single-photon or a photon-photon phenomenon.[73],[74] The latter explanation accords with our model of the photon as an indivisible, physical entity. In our photon model, interference is thought to occur between the coherent photons of a photon bunch.

Light sources often exhibit the property of photon bunching.[59] Stimulated emission adds one quantum of energy ($h\nu$) to the incident photon; e.g., an incident single ($1h\nu$) photon stimulates the atom to give it an additional $h\nu$ of energy, emerging as a double photon having $2h\nu$ of energy. This subsequently decays into two spatially proximate, coherent photons. Repetition of this process thus produces bunches of coherent photons. For this physical concept of photon bunching the alternate term "flock" is very appropriate, for the photons of the bunch are traveling together just like a flock of migrating geese. Stimulated emission producing photon bunches is believed to be of common occurrence; it is, of course, dominant in lasers but is also believed to be important in thermal light sources.

In our photon model interference effects are typically produced by photon bunches. In a diffractometer the members of a bunch will be deflected at different angles depending upon the impact parameter of each photon with the edge of the slit; photons that miss the slit walls are transmitted undeflected. This model of diffraction explains why fringe visibility (relative intensity) is a maximum for slits of about $\lambda/3$ wide (the intrinsic width of a photon wavicle being λ/π). Interference is produced when a pair of coherent photons have different path lengths to the plane of observation, thereby interfering constructively or destructively.[1]

As the incident light intensity decreases, the number of photons in bunches may be expected to decrease. However, it should not be assumed that a reduction in intensity necessarily reduces the extent of photon bunching; the actual physical mechanism used to absorb or deflect some of the photons may, or may not, reduce the proportion of photons in bunches.

In a beam of statistically independent photons, diffraction still takes place; it results from a single photon's interaction with the walls of the diffracting aperture. However, the interference fringes should disappear in a low intensity beam of statistically independent photons. This effect has been observed in a series of experiments by Panarella[74] and in an earlier experiment by Dontsov and Baz.[73] However, similar experiments by Reynolds, Spartalian, and Scarl[75] and Jeffers[76] do not show the effect; the visibility of the side lobes of the diffraction pattern is independent of the beam intensity. In a 1978 review of five nonphotographic detection experiments,[77] the only one showing a decrease in fringe intensity with decreasing incident beam intensity was the Dontsov and Baz experiment.

It is now known that the original (1909) low-intensity interference experiments of Taylor[78] did not display single-photon interference, because photographic detectors (as used by Taylor and in three subsequent experiments reviewed by Pipkin)[77] are now known to be essentially photon-coincidence detectors; i.e., a silver halide grain must absorb at least four coincident (i.e., within about one optical period) photons to become developable.[79]

Despite the low incident intensity used by Taylor, his interference patterns must have been produced by photon bunches of at least four photons. These early experiments, the results of which were presumed to display single photon interference, apparently led Dirac into making his well-known pronouncement[80] that interference is exclusively a single-photon phenomenon. In the light of modern photographic knowledge, the correct inference from Taylor's (and the other photographic) experiments is that photon bunching is commonplace, even in low-intensity beams originating from noncoherent sources such as gas discharge lamps.

Hence, the key to resolving the experimental issue is to produce a light source of statistically independent photons; i.e., one with hardly any bunched photons. One way of detecting statistical independence would be to use photographic and nonphotographic detectors alternately in the same experiment. If the diffraction pattern is not observed with the photographic detector but is observed with the photomultiplier or other nonphotographic detector, then the inference would be that the observed pattern was produced by a beam with fewer than four photons in any bunches in it. On the other hand, if the diffraction pattern is also not observed with the nonphotographic detector, then the inference would be that the interference is a photon-photon phenomenon.

A recent photon-counting experiment[81] was designed to determine whether diffractive interference is a single-photon or a photon-photon phenomenon. The experimental results support the single-photon hypothesis. The conventional rationalization of single-photon interference is the dichotomy of the wave-particle duality; the single photon's wave traverses both paths of the interferometer simultaneously, interfering with itself and then manifesting its particlelike character at the detector. Explanations for single-photon interference based upon passage of each photon along only one of the two paths are under consideration by Buonomano[82] and Surdin.[83]

Experiments with two independent light sources have shown that a photon from one source can interfere with a photon from the other source.[84],[85] Thus photon-photon interference does occur contrary to Dirac's pronouncement.[80] The Grangier experiment indicates that a single photon can also interfere with itself. However, the interpretation of this experiment is currently the subject of controversy,[82],[86] and hence the question of whether a photon can interfere with itself as well as with other photons remains unresolved.

4. INTERPRETATION

4.1 The Mechanics of Photon Wavicles

An intimate relationship between the wavicle's linear motion and its rotational motion is revealed by its elementary mechanical properties. For frequency ν and wavelength λ ($\lambda\nu = c$) it has a relativistic mass m:

$$m = h\nu/c^2 = h/c\lambda. \tag{58}$$

Since it is moving at the speed of light c, its linear momentum p_z is

$$p_z = mc = h\nu/c = h/\lambda \text{ (the de Broglie relation).} \tag{59}$$

Supposing that the mass m is effectively at a distance r_{max} [given by (28)] from the axis of rotation/propagation, the wavicle's moment of inertia I is

$$I = mr_{max}^2 = m(k\lambda/2\pi)^2. \tag{60}$$

The tangential velocity at this maximum radius is the velocity of light c, and hence the angular velocity ω of the rotating field is

$$\omega = c/r_{max} = 2\pi c/k\lambda = 2\pi\nu/k. \tag{61}$$

Thus the magnitude L_z of the angular momentum about the z-axis is

$$L_z = I\omega = mc(k\lambda/2\pi) = kh/2\pi = k\hbar. \tag{62}$$

The rectilinear kinetic energy is mc^2 [the Newtonian expression $\frac{1}{2}mv^2$ becomes mv^2 for a relativistic particle; cf. (53)], and this is seen to be equal to its rotational kinetic energy $I\omega^2$ (likewise $I\omega^2$ rather than the Newtonian expression $\frac{1}{2}I\omega^2$)

$$E = I\omega^2 = (kh/2\pi)(2\pi c/k\lambda) = hc/\lambda = h\nu = p_z c = mc^2. \tag{63}$$

The action S of the wavicle is the angular momentum per complete cycle $\Delta\varphi = 2\pi/k$ or the product of its energy E and its period of oscillation $\tau = 1/\nu$:

$$S = k\hbar \times 2\pi/k = h = E\tau = E/\nu$$
$$= mc^2/\nu = mc\lambda = p_z\lambda = h. \tag{64}$$

The rectilinear kinetic energy and the rotational kinetic energy are *not* separate components of the wavicle's total energy (as would be the case for, say, a rifle bullet) but rather different manifestations of the same inertial energy.

This intimate relationship between linear and rotational motion may eventually lead to an explanation for the universal equivalence of inertial and gravitational masses, for rectilinear motion is related to inertial mass, and accelerated, rotational motion is equivalent to a gravitational field.[87]

4.2 The Minimum Quantum of Action

The wavicle's energy is $h\nu = pc$ and its transit time is $1/\nu$; alternatively, its momentum is $p = h\nu/c = h/\lambda$ and its length is λ. Hence its action, = energy $\times$ time = momentum $\times$ length = h, is Planck's constant. Thus the wavicle model of the photon is consistent with the idea that upon absorption, emission, or scattering, the action involved in the process is h; the action involved has this minimum value because of the photon's finite transit time (or finite length).

The finite wavicle model of the photon provides an explanation for the wave-particle duality of light. The wavicle is neither an indefinitely extended wave nor a pointlike particle; it is a wave with a well-defined finite extent. Its oscillating/rotating electromagnetic field is consistent with its wavelike properties, and its finite extent explains its particlelike properties (especially line spectra and the photoelectric effect). The photon-wavicle is, as Einstein anticipated,[3] a localized, indivisible physical entity that moves without dividing and is only produced or absorbed as a complete unit. It is the atom of light. The wavicle model unifies the wave and particle properties of light in a single physical entity. Thus the dichotomy of light exhibiting both wavelike and particlelike properties (the wave-particle duality paradox) is resolved in the unity of the finite wavicle.

The wavicle model of the photon also provides a resolution of the philosophical dispute between proponents of the Copenhagen interpretation of quantum mechanics (Bohr, Heisenberg, Dirac, etc.) and the persistent determinists (Einstein, de Broglie, Schrödinger, etc.).[88] The former believed that the uncertainty principle was intrinsic to nature, while the latter believed that nature was deterministic despite the *experimental* limitation of the uncertainty principle.

Internally, the photon-wavicle is a classical electromagnetic field whose domain is defined by the relativistic requirement for causality. However, the finite size of the photon limits the accuracy of measurements for it is an indivisible entity; for a momentum $p = h/\lambda$ its position can only be measured to within the length of its ellipsoidal domain = λ. Hence in interactions (measurements) the product of momentum and positional uncertainty is at least the minimum action h in accordance with the Heisenberg uncertainty principle. The wavicle's internal coordinates may be regarded as hidden variables.[2]

The wavicle model suggests an intimate relationship between quantum mechanics and the theory of relativity, since the length of the wavicle is the distance that it travels (moving at the velocity of light) in one period of its oscillation, and this finite length leads to the uncertainty principle, as explained above. This suggests that the finite value of Planck's constant h is related to the finite value of the velocity of light.[89] Bohm has alluded to the idea that the limitation imposed upon causality by the finite velocity of light is related to the uncertainty principle.[90] If this conjecture is correct, it should lead to a quantitative relationship between h and c.

Einstein regarded the relation $E = h\nu$ (1) as an enigma. This enigma is partially resolved by the wavicle model of the photon for there is a simple explanation for the fact that all photons carry the same action h and that all physical processes (in which photons are absorbed or emitted) involve integral multiples of this quantum of action; the action of a photon-wavicle is a Lorentz-invariant property. Energy and frequency transform in the same way under a Lorentz transformation,[91] and hence the ratio of energy to frequency (i.e., action) is invariant under a Lorentz transformation. In this sense all photons, from gamma rays to radio waves, are essentially the same particle, just as all electrons are regarded as identical even though they may have different energies (and hence different de Broglie wavelengths) in different experiments.

A corollary of the Lorentz invariance of action is that Planck's constant h must have a finite value, because if h were zero then the amplitude of the photon-wavicle's electromagnetic field would have to be zero in order to make its action zero, since its action is the product of its momentum and its length, and its length is finite. Its finite length is deduced from Maxwell's equations and the relativistic requirement for causality, regardless of the field's amplitude. Hence, if the quantum of action h were zero, light would not exist. The photon would be a null particle; i.e., its field amplitude would be zero. While this explains why the *de facto* existence of light implies that Planck's constant is not zero, it does not account for the specific, observed value of h.

4.3 Comparison with Quantum Field Theory

The essence of the quantum field theory of radiation is the association of a quantum mechanical harmonic oscillator with each mode (i.e., frequency) of the electromagnetic field.[59],[92] In this quantum theory, the photon is nothing more (and nothing less) than a quantum of energy $h\nu$ for the mode having frequency ν. This second-quantization theory is framed within the interaction picture, all processes being described in terms of absorption/emission of real/virtual photons. It avoids entirely any concept of the photon as a physical entity.[93] This avoidance of the issue, which stems from the Copenhagen philosophy of quantum mechanics, raises conceptual difficulties in the minds of many physicists. To quote a recent review by Strnad "the concept of the photon may be one of the main didactic issues of modern physics."[94]

Our Maxwellian model of the photon has some parallels in quantum mechanics. The model is the solution of a wave equation (3). The solutions are field amplitudes which in interactions must be added and squared to produce observable intensities, and hence the nonclassical interference phenomena that are characteristic of quantum mechanics[93],[95] arise also in the wavicle model. That it is valid to operate on electromagnetic field components with wave mechanical operators has been shown by Green and Wolf.[30]

A weakness of the conventional theory is the ubiquitous formulation in terms of plane waves.[96] A plane wave for a single mode [$k = 0$ in (17)] has the same z and t dependence as the wavicle's field (17), but it has no dependence upon r and φ, and hence it has the same amplitude throughout any plane perpendicular to the axis of propagation. For this reason, it is obvious that a plane wave cannot represent a narrow beam of light, much less a single photon. In the plane wave theory, the polarization property has to be added as an ad hoc supplement to the plane wave functions,[?] whereas this property is intrinsic and essential in our wavicle wave functions.

A distinct advantage of our theory is that it is couched entirely in terms of the electromagnetic field components, whereas in the conventional Lagrangian theory the use of the scalar and vector potentials leads to a physically insignificant ambiguity of gauge. To quote Jauch and Rohrlich: "the electromagnetic field components describe directly measurable quantities ... while the vector potential must be considered as a mathematical and auxiliary field which can be determined ... only to the extent expressed by the gauge transformation."[9]

Another disadvantage of quantum electrodynamics is that boson (photon) quantization is implicitly assumed. In our semiclassical wavicle theory quantization of the angular momentum à la Bohr leads to the prediction of fermion (neutrino) states of the electromagnetic field as well as boson (photon) states as explained in Sec. 2.4.

5. CONCLUSIONS

The photon model presented here is based upon classical electromagnetism (Maxwell's equations)[97] and the relativistic principle of causality. The theory predicts that the action carried by the photon-wavicle is a finite, universal constant for all photons, and relating this to experiment we recognize this constant as Planck's constant h. Causality confines the photon's oscillating field within a circular ellipsoid of length λ and diameter λ/π. The field is an eigenfunction of the wave mechanical operators for angular and linear momentum, the eigenvalues being those that are experimentally observed. It also has helicity states that correspond to right and left circularly polarized light, combinations of which correspond to an elliptically or linearly polarized field that is nevertheless an eigenfunction of L_z (Sec. 2.6). The predicted energy eigenvalues of the confined field are $E = nh\nu$, this being an extension of the Einstein relation $E = h\nu$; we identify the energy states for $n > 1$ as what are experimentally called "multiphotons."

The wavicle model of the photon conforms with many of the experimental properties of monochromatic light, the most direct supporting evidence being the almost total attenuation of transmission through slits whose width is less than the wavicle's diameter of λ/π.

This photon diameter also accords with a microscope's maximum resolving power using monochromatic light: "The resolving power ... giving the best resolution obtainable ... is in general a little less than a third of the wavelength of the light used."[98] This result from classical diffraction theory concurs with our concept that the resolving power is equal to the photon's diameter since $1/\pi = 0.3183$ is "a little less than" $1/3 = 0.3333$.

Our model predicts the threshold intensity for multiphoton phenomena to be the wavicle's intrinsic intensity of $4\pi hc^2/\lambda^4$; the prediction is confirmed (within experimental error) for both visible ($\lambda = 650$ nm, $I_P = 0.42$ MW/cm^2) and infrared ($\lambda = 10.5$ μm, $I_P = 6$ W/cm^2) multiphoton thresholds.

The theory also predicts the existence of a light-speed (zero-rest-mass) fermion (spin = intrinsic angular momentum = $\pm\hbar/2$), which we identify as the electron-neutrino, the two helicity states (spin = $+\hbar/2$ and $-\hbar/2$) corresponding to the neutrino and the antineutrino. Further correlation with experiment is needed in this case, although the latest experimental evidence (the Shelton supernova of February 1987) indicates that the rest mass of the neutrino is indeed zero.[48]-[51] The "multi-photon" states ($E = nh\nu$) of the neutrino may be the muon-neutrino, the tau-neutrino, etc. The neutrino emerges from the theory as the fermion particle that travels at the speed of light, being the natural counterpart of the boson-photon. Since both the photon and the neutrino emerge from the same equations of motion (Maxwell's equations), the theory implicitly provides a basis for the unification of the electromagnetic and weak interactions.

The photon-wavicle provides a physical basis for the Heisenberg uncertainty principle. The product of the wavicle's momentum and length is its Lorentz-invariant action h, and hence in measurement processes in which a photon is absorbed or emitted, the minimum product of momentum and length measurements is the photon's action h. The interpretation that the photon-wavicle *is* the physical quantum of action supports the philosophical view that the uncertainty principle is not intrinsic to nature (the Copenhagen philosophy), but rather it is simply a limitation upon experimental measurements. It *is* possible to know what cannot be measured directly,[4] and furthermore such knowledge does have predictive value as exemplified in Sec. 3.

This theory of electromagnetic wavicles is capable of several extensions: characterization of multiphoton states, mathematical description of the processes of spontaneous and stimulated emission, multiphoton formation in a focused laser beam, and spontaneous decay of a multiphoton. In addition, we speculate that stationary wavicles (say with spherical symmetry) may prove to be pure field models of such particles as the electron, the muon, and the tauon. Mackinnon[24],[99] has promoted the idea that a particle's mass m arises from an internal vibration of frequency mc^2/h; i.e., $E = mc^2 = h\nu$. A pure field model of elementary particles is being developed by Jennison.[11]

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Résumé

Les photons et les neutrinos sont représentés comme des états oscillatoires du champ électromagnétique confiné à un domaine local, le mouvement étant régi par les équations de Maxwell. Les dimensions et la forme du domaine sont limitées par le principe relativiste de causalité, en d'autres termes, l'intervalle entre des événements se produisant à l'intérieur du domaine est tel qu'un intervalle de temps. Ces ondes de soliton localisées sont appelées "ondicules."

Les solutions des équations de Maxwell sont des états propres du moment angulaire intrinsèque (spin) avec une valeur propre $\mathrm{k}\hbar$, k *étant un entier ou un demi-entier. Le domaine causal limité est un ellipsoïde circulaire ayant pour longueur* λ *(la longueur d'onde) et de circonférence* $\mathrm{k}\lambda$*. Les solutions comprennent une certaine hélicité, qui correspond à une polarisation gauche ou droite de la lumière pour* $\mathrm{k} = 1$*, et au neutrino et à l'antineutrino pour* $\mathrm{k} = \frac{1}{2}$*.*

Ce modèle de l'ondicule est en étroite corrélation avec de nombreuses propriétés de la lumière observées expérimentalement. Il prédit comment la lumière est transmise à travers une ouverture: en effet, nous avons pu procéder à une mesure confirmant le diamètre du photon dans le domaine des micro-ondes. Nous prédisons l'occurence de phénomènes multiphotoniques au-dessus des seuils observables d'intensité. La production d'ondicules multiphotoniques en cours d'émission stimulée explique l'apparition de "paquets" de photons. La notion d'ondicule explique aussi les propriétés directionnelles et de polarisation d'une antenne hélicoïdale à micro-ondes.

L'ondicule photonique est la base physique du principe d'incertitude de Heisenberg; en d'autres termes, l'ondicule est le quantum d'action. Le produit de la longueur d'ondicule et du moment est $\mathrm{k}\hbar$*, action relativiste invariante. Cette action est nécessairement impliquée dans tout processus observable dans lequel une ondicule est totalement absorbée ou émise.*

Endnote

[1] The electric permittivity ϵ_0 and the magnetic permeability μ_0 of free space are related by $\epsilon_0\mu_0 c^2 = 1$.

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