COM 2030 Exercise sheet 1:
Finite Automata and Regular Languages

1. Consider the following regular grammar with start symbol A:

$$\begin{aligned} A &\rightarrow aB \\ A &\rightarrow a \\ B &\rightarrow bC \\ B &\rightarrow b \\ C &\rightarrow cB \\ C &\rightarrow c \end{aligned}$$

   (a) Derive a transition diagram for a nondeterministic finite automaton that accepts the language generated by this grammar using the technique suggested in the proof of Proposition 3.1 and formally define the associated automaton.

   (b) Formally define a deterministic finite automaton that will accept the same strings as this nondeterministic finite automaton using the technique of Proposition 2.4 and draw a transition diagram for the derived deterministic finite automaton.

   (c) Derive a regular expression representing the language generated by this grammar using the technique suggested in the proof of Proposition 4.1.

2. (a) Write a regular expression that represents the language of zero or more a's followed by one or more b's followed by zero or more a's.

   (b) Write a regular expression that represents the language consisting of one or more pairs ab where between any two instances of ab exactly one c may or may not occur.

   (c) Draw a transition diagram for a finite automaton that accepts

      i. the language in (a);

      ii. the language in (b);

      iii. the Kleene star of the union of the languages in (a) and (b) – this automaton should be constructed according to the principles for constructing automata corresponding to regular expressions demonstrated in Part 1 of the proof for Proposition 4.1.

   (d) Using the technique of Proposition 3.1 derive a regular grammar that generates the same language as that accepted by the automaton in (c) iii.

3. (a) Show that if L_1 and L_2 are regular languages then $L_1 \cap L_2$ is regular.

   (b) Show that if L_1 and L_2 are regular languages then $L_1 \setminus L_2$ is regular.

   (c) Show that if L is a regular language then the language obtained from writing the strings of L backwards is also regular.

Due This exercise should be completed by 9am on Monday, October 20th, since the solutions will be disclosed in that morning's tutorial.