

**COM 2030 Exercise sheet 4:  
Recursive Functions**

1. (a) Show that the function  $f$  defined by

$$f(x, y, z) = \begin{cases} x & \text{if } z \text{ is even} \\ y & \text{if } z \text{ is odd} \end{cases}$$

is primitive recursive. You may assume that the functions *plus*, *eq*, *quo*, *monus* and *mult* have been shown to be primitive recursive, if you find it useful to do so.

- (b) Show that for any  $n \geq 1$  the function  $ADD_n : \mathbf{N}^n \rightarrow \mathbf{N}$  defined by

$$ADD_n(x_1, \dots, x_n) = x_1 + \dots + x_n$$

is primitive recursive.

2. Show that the function  $gcd : \mathbf{N}^2 \rightarrow \mathbf{N}$ , where  $gcd(x, y)$  is the greatest common divisor of  $x$  and  $y$ , is partial recursive.

3. Using the simple `while` language introduced in Lecture 16

- (a) write a program that computes the function  $f : \mathbf{N} \rightarrow \mathbf{N}$  defined by

$$f(x, y) = \begin{cases} 1 & \text{if } x > y \\ 0 & \text{otherwise} \end{cases}$$

- (b) show how to simulate the programming structure

`if x = 0 then S1 else S2`

where  $S_1$  and  $S_2$  represent program segments.