THE UNIVERSITY OF SHEFFIELD

DEPARTMENT OF COMPUTER SCIENCE

AUTUMN SEMESTER 1999-00 2 HOURS

THEORY OF MACHINES

Answer THREE questions.

All questions carry equal weight. Figures in square brackets indicate the percentage of available marks allocated to each part of a question.

1. a) Compare finite automata and Turing machines by:

   (i) explaining informally what each is and how they differ; [10%]

   (ii) giving a formal definition of each and making it clear how transitions are defined for each. [10%]

   (iii) explaining the difference between them in terms of the classes of language in the Chomsky hierarchy that each accepts. [10%]

   b) Consider the set of all languages over the alphabet $\{0, 1\}$:

   (i) Write a regular expression that represents the language of strings containing at least two 0's and draw a transition diagram for a finite automaton that accepts this language. [15%]

   (ii) Write a regular expression that represents the language of strings in which the number of 0s is even and draw a transition diagram for a finite automaton that accepts this language. [15%]

   (iii) Draw a transition diagram for a finite automaton that accepts the union of the languages in (i) and (ii). [10%]

   c) Consider the grammar G with rewrite rules:

$$\begin{aligned} S &\rightarrow AcB \\ A &\rightarrow aBcAa \\ A &\rightarrow a \\ B &\rightarrow bAcBb \\ B &\rightarrow b \end{aligned}$$

   Construct a transition diagram for a pushdown automaton which accepts strings in the language generated by G. [30%]

2. a) Using any of the building block single tape Turing machines:

   1. L,R, and x;
   2. the searching machines R_x, $R_{\neg x}$, L_x, and $L_{\neg x}$; or
   3. the shifting machines S_R and S_L,

   construct a composite single tape Turing machine that **decides** the language

$$L = \{w \in \{a, b, *\}^* \mid w \text{ has the form } v * v, v \in \{a, b\}^*\}$$

   Explain the strategy your machine uses. [50%]

   b) What is a Universal Turing machine and how would you design one ? [30%]

   c) What is the halting problem and why is it significant ? [20%]

3. a) The class of primitive recursive functions is defined in terms of a set of initial functions and a set of construction operations for building new functions from existing functions.

   (i) Specify each initial function in terms of its input and output. [15%]

   (ii) Specify each construction operation by showing how in each case the operation generates a new function from more basic given functions. [15%]

   (iii) Show that the function $f : \mathbf{N} \times \mathbf{N} \to \mathbf{N}$ defined by

$$f(x, y) = min(x, y) \quad \text{(the minimum of } x \text{ and } y\text{)}$$

   is primitive recursive. You may assume that the functions $plus$, $\neg$, eq, quo, $monus$ and $mult$ have been shown to be primitive recursive, if you find it useful to do so. [20%]

   (iv) Show how the multiplication function $mult : \mathbf{N} \times \mathbf{N} \to \mathbf{N}$ can be defined using primitive recursion with the constant functions $K^n_m : \mathbf{N^n} \to \mathbf{N}$ and the plus function $plus : \mathbf{N} \times \mathbf{N} \to \mathbf{N}$.
   Use your definition to compute the value of $2 * 3$. [20%]

   b) The class of partial recursive functions is a larger class of functions than the primitive recursive functions because it increases the set of function construction operations to include minimalization.

   (i) Define minimalization. [10%]

   (ii) What is the value of $f(4)$ if f is defined by

$$f(x) = \mu y[monus(x, pred(y)) = 0] \ \ ?$$

   Show how you have derived your answer. [20%]

4. a) Define the classes of languages P and NP, making clear what the difference is. [10%]

   b) (i) What is a polynomial reduction from a language L_1 over an alphabet Σ_1 to a language L_2 over an alphabet Σ_2 ? [10%]

      (ii) What is the problem of propositional satisfiability ? [10%]

      (iii) State Cook's Theorem and explain its significance. [20%]

   c) Show that the language consisting of all palindromes in $\{x, y\}^*$ is in P. (A palindrome is a string that is the same written backwards as forwards.) [50%]

END OF QUESTION PAPER