4. S. Pan, and W. Hsu, Single-feed dual-frequency microstrip antenna with two patches, Proc APS/URSI Symp, Orlando, FL, 1999, CD-Rom.
5. IE3D user's manual, Release 8, Zeland's Software, Inc.

THE FRACTAL HILBERT MONOPOLE: A TWO-DIMENSIONAL WIRE

Jaume Anguera, Carles Puente, Enrique Martínez, and Edouard Rozan
Technology Department
Fractus S.A. Avda.
Alcalde Barnils 64-68
Edificio Testa–Módulo
C. Sant Cugat, 08190 Barcelona, Spain

Received 11 July 2002

ABSTRACT: *A novel miniature antenna based on the fractal Hilbert geometry is presented. The antenna becomes electrically smaller as the fractal iteration increases at a higher rate than that of any other fractal geometry presented until now. Several fractal-shaped Hilbert monopoles have been constructed, measured, and compared with the classical λ/4 monopole.* © 2003 Wiley Periodicals, Inc. Microwave Opt Technol Lett 36: 102–104, 2003; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.10687

Key words: *fractal antennas; Hilbert monopole antenna; wire antenna*

1. INTRODUCTION

The invention of the first fractal-shaped antenna element was described in [1]. Fractal-shaped antennas are becoming a powerful tool to design advanced antennas such as multifrequency, small and high directivity antennas. Examples of multifrequency antennas using the Sierpinski monopole can be found in [2]. Small antennas such as the Koch monopole [3] and small microstrip patch antennas based on the Sierpinski bowtie [4] are also described in the literature. High directivity antennas have been recently achieved by using the Koch island [5] and Sierpinski bowtie microstrip patches [4]. In this paper, we present the Hilbert monopole as a small antenna. The Hilbert antenna invention was first described in [7]. In [6], an equation to predict resonant frequencies was presented. The advantage of this geometry, in comparison with the previous work on the Koch monopole [3], is that a higher frequency compression factor (CF) is achieved, that is, for the same antenna height, a larger frequency reduction is founded by shaping the monopole with the Hilbert geometry.

Rigorously, fractals are mathematical objects and in the real world only certain truncated versions of the ideal structure exists. So strictly speaking, an antenna with a fractal shaped cannot be physically constructed. Instead, other geometries, such as space-filling [7] and multilevel [8], can be used to approach ideal fractal shapes.

The Hilbert geometry is presented in the next section and the fractal dimension is introduced to relate this concept to factor compression. In section 3, experimental results are given and discussed.

2. SMALL ANTENNAS AND THE FRACTAL GEOMETRY

An antenna is said to be small when its larger dimension is less than twice the radius of the radian sphere, with radius $\lambda/2\pi$. Wheeler and Chu were the first who investigated on the fundamental limitations of such antennas [9, 10]. The goal of this paper is to analyze the miniature feature of the Hilbert monopole and compare it with the Koch monopole studied previously in [3, 11]. In this sense, when the total length of the Hilbert monopole increases as fractal iteration increases, the resonant frequency will decrease. Thus, the antenna becomes electrically smaller as fractal iteration increases. The frequency reduction of the Hilbert as well as the Koch monopole is related to the fractal iteration in the next section.

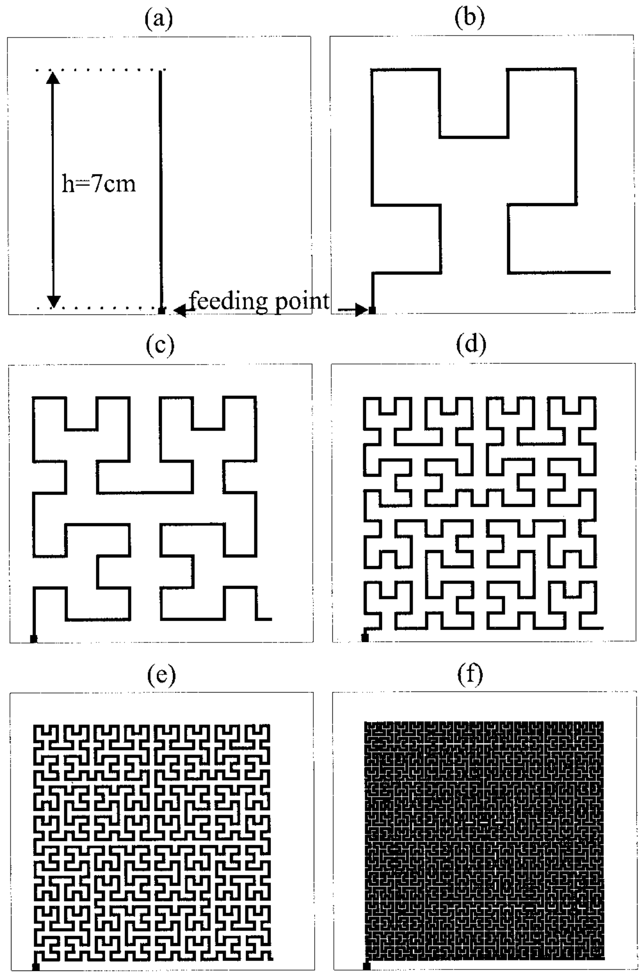

Figure 1 a) Vertical monopole; b) Hilbert 1; c) Hilbert 2; d) Hilbert 3; e) Hilbert 4; f) Hilbert 5. The antennas are etched on an FR4 substrate acting as a support

3. EXPERIMENTAL RESULTS

Figure 1 shows a λ/4 monopole and the first five fractal iterations of the Hilbert monopole where the total height for all antennas is $h = 7$ cm. The antennas are etched on a 1-mm FR4 substrate acting merely as an antenna support. The copper used is 0.4-mm wide.

The length L for the Hilbert monopole increases at each iteration n, given by

$$L(n) = \frac{4^{n+1} - 1}{2^{n+1} - 1} h. \tag{1}$$

For the 5th iteration, the length is 65 times h. One might think that the resonant frequency for a 5th iteration Hilbert monopole could be 65 times less than the resonant frequency of a linear monopole

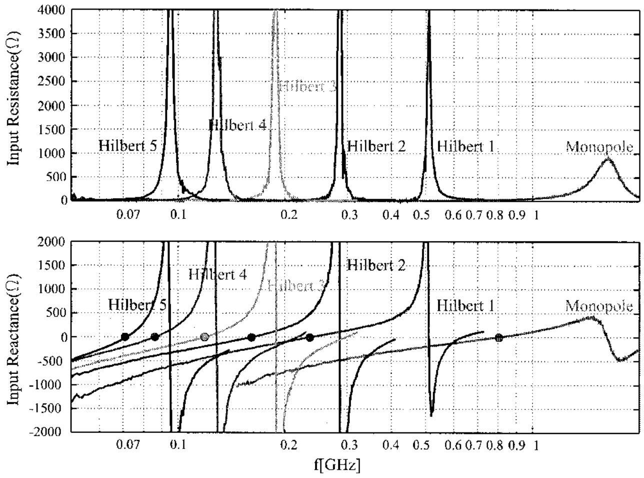

Figure 2 Measured input resistance (top) and input reactance (bottom)

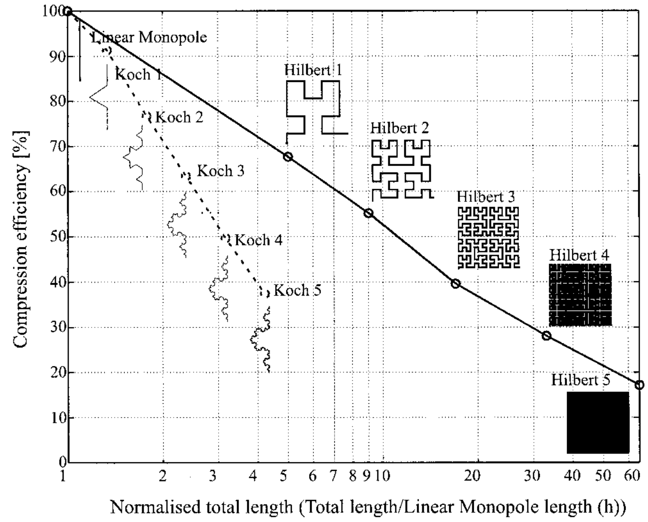

Figure 3 Compression efficiency for the Hilbert monopoles

with height h, which will be an extraordinary frequency reduction. Obviously, this is not true since coupling between turns provides a shorter path for currents flowing from one tip to the other. However, even with the coupling effect, the resonant frequency reduction or compression factor (CF) can achieve values up to 10, which is a very significant size reduction. A large CF is of interest for the design of miniature antennas, where space in a constraint factor, such as integrated low-frequency antennas in MMIC circuits.

To study how the resonant frequency is reduced at each iteration, the input impedance has been measured in an anechoic chamber. The antennas are fed through a SMA connector (see Fig. 1) and the antennas are placed on a square ground plane of 80 × 80 cm^2. Figure 2 shows the input resistance and input reactance where the first resonant frequency for each antenna has been marked with a dot. This first resonant frequency is associated to a low input resistance around 50Ω, therefore, the antennas could be easily matched. Table 1 summarizes the resonant frequencies—the electrical size as well as the CF. The electrical size for the linear monopole does not match the theoretic λ/4 because the dielectric support shifts the frequency to lower values. The CF increases as the fractal iteration increases, achieving CF = 11 for the 5th Hilbert iteration. This means that for the same frequency, using the Hilbert 5 monopole antenna one can achieve a size 11 times less than the size of the classical λ/4 monopole. It should be noted that the maximum CF achieved for the Koch monopole is CF = 1.57, which is a very poor value compared with the Hilbert monopole [3]. The large CF obtained with the Hilbert monopole can be related to the fractal dimension. The fractal dimension for the Hilbert and Koch monopoles are $D = 2$ and $D = 1.26$, respectively. This larger fractal dimension indicates that the Hilbert curve fills a surface more efficiently than the Koch curve, which, although it has not been theoretically proved yet, seems to be related to the miniaturization capabilities of fractal-shaped antennas.

TABLE 1 Performance Evolution as a Function of the Fractal Iteration

	Hilbert			Koch	
Antenna	Resonant Frequency (MHz)	Electrical Size (λ)	CF Hilbert	Electrical Size (λ)	CF Koch
Monopole	799.6	0.186		—	—
It 1	236.4	0.055	3.38	0.144	1.21
It 2	161.1	0.037	4.96	0.129	1.36
It 3	118.7	0.027	6.73	0.117	1.50
It 4	86.3	0.020	9.26	0.112	1.56
It 5	71.7	0.016	11.1	0.111	1.57

Finally, a new concept is introduced called compression efficiency (CE). CE is defined as the ratio between the first resonant frequency of the equivalent vertical monopole with a height equal to the total length of a Hilbert (Koch) monopole and the first resonant frequency of the Hilbert (Koch) antenna. CE is an important parameter for comparing different space-filling geometries in order to determine which structure can decrease the resonant frequency with less length.

Figure 3 shows the CE as a function of the fractal iteration showing that CE decreases as the fractal iteration increases. In the same figure, the CE for the Koch monopole is depicted [3]. It is clearly shown that the CE decreases more rapidly for the Koch monopole. For example, for the same height h of the vertical monopole, the Hilbert 1 and the Koch 5 monopole have approximately the same length, however CE is 70% for the Hilbert monopole and only 40% for the Koch monopole, which shows that the Hilbert monopole achieves a larger frequency reduction within the same wire length.

4. CONCLUSION

A new set of space-filling antennas based on the Hilbert fractal geometry has been presented. Measurement shows how the electrical size of a classical λ/4 monopole can be reduced up to factors of 11, which is a significant result in terms of miniaturization.

A novel concept called compression efficiency has been introduced as a parameter useful to compare different space-filling geometries. In this case, the Hilbert monopole has been compared with the Koch monopole, with the result that the Hilbert monopole achieves a larger CE and CF values. This appears to be related to the fractal dimension.

ACKNOWLEDGMENTS

This work has been supported by Fractus, S.A. Space-filling antennas are patent pending. Fractal antennas,® space-filling antennas,® and multilevel antennas® are registered trademarks.

REFERENCES

1. C. Puente, Fractal and multifractal antennas, Invention Patent OEPM 9501019.

2. C. Puente, J. Romeu, R. Pous, and A. Cardama, Fractal multiband antenna based on the Sierpinski gasket, IEE Electron Lett 32 (1996), 1–2.
3. C. Puente, J. Romeu, R. Pous, J. Ramis, and A. Hijazo, Small but long Koch fractal monopole, IEE Electron Lett 34 (1998), 9–10.
4. J. Anguera, C. Puente, C. Borja, R. Montero, J. Soler, Small and High Directivity Bowtie Patch Antenna based on the Sierpinski Fractal, Microwave Opt Technology Lett 31 (2001), 239–241.
5. C. Borja, G. Font, S. Blanch, J. Romeu, High Directivity fractal boundary microstrip patch antenna, IEE Electron Lett 36 (2000), 778–779.
6. K.J. Vinoy, K.A. Jose, V.K. Varadan, V.V. Varadan, Resonant Frequency of Hilbert Curve Fractal Antennas, IEEE Antennas Prop Inter Symp Digest 3 (2001), 648–651.
7. C. Puente, E. Rozan, J. Anguera, Space filling miniature antennas, Invention Patent WO0154225.
8. C. Puente, J. Romeu, M. Navarro, C. Borja, J. Anguera, Multilevel antennas, Invention WO0122528.
9. H.A. Wheeler, Fundamental Limitations on Small Antennas, Proc IRE, (1947), 1479–1488.
10. L.J. Chu, Physical Limitations on omni-directional antennas, J Appl Phys (1948), 1163–1175.
11. C. Puente, Fractal Antennas, Ph.D. Dissertation, Universitat Politècnica de Catalunya, 1997.

ANALYSIS OF TWO-CONCENTRIC ANNULAR RING MICROSTRIP ANTENNA

Binod K. Kanaujia and Babau R. Vishvakarma
Department of Electronics Engineering
Institute of Technology, Banaras Hindu University
Varanasi-221005, India

Received 8 July 2002

ABSTRACT: *The analysis of a two-concentric annular ring patch antenna using planar waveguide model is presented. Consequently, various parameters such as input impedance (Z_{in}), VSWR, return loss, resonance frequency, and radiation patterns have been theoretically investigated as a function of gap length and feed location. The various parameters of the two-concentric annular ring patch antenna are found to depend heavily on gap length and feed locations. The radiated power and 3-dB beamwidth improve significantly, as compared to a single radiator.*
 Microwave Opt Technol Lett 36: 104–108, 2003; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.10688

Key words: *microstrip antenna; concentric annular ring antenna; ring antenna; gap-coupled microstrip antenna; patch antenna*

1. INTRODUCTION

A number of papers have appeared in the literature on bandwidth enhancement of microstrip antennas. Electrically thick elements [1], stacked multi-patch/multilayer antennas [2], and the impedance matching network [3–4] are some of the methods that modify the resonant circuit by (i) adding parasitic patches on the top of the driven patch [5–6] or (ii) adding gap-coupled patches to the radiating or non-radiating edges [7–9] of the patch.

In this paper, an attempt has been made to theoretically analyze the gap-coupled annular ring microstrip antenna (ARMSA) using a planar waveguide model. The investigations concentrate on the effect of gap length as well as feed-point location on the radiation characteristics of the two-concentric ring microstrip radiators.

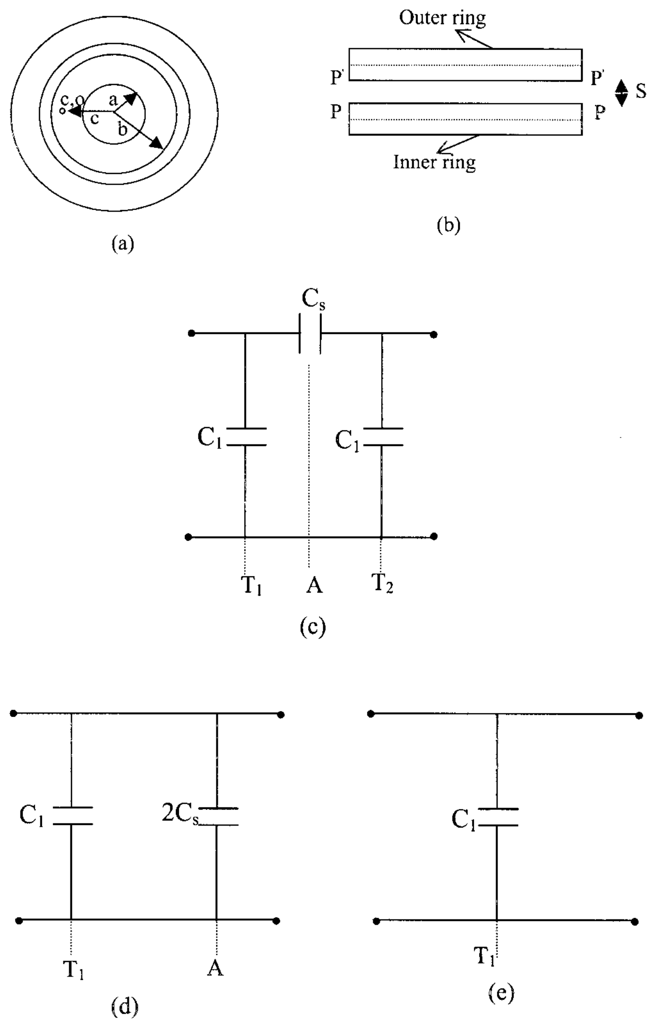

Figure 1 (a) Concentric annular ring microstrip antenna; (b) parallel gap-coupled microstrip lines; (c) equivalent circuit; (d) equivalent circuit for odd mode (C_{odd}); (e) equivalent circuit for even mode (C_{even})

2. EQUIVALENT CIRCUIT PARAMETERS OF THE GAP STRUCTURE

The physical gap structure in the concentric annular ring antenna is shown in Figure 1(a). The inner ring patch is excited by coaxial feed and the outer patch is considered as a parasitic element. The arbitrary discontinuity at the junction of the two ring patches can generally be represented by either the equivalent T or the π circuit [10]. The parallel gap-coupled radiator using a planar waveguide model [11] is shown in Figure 1(b). It is preferable to represent the gap structure by the equivalent π circuit as shown in Figure 1(c). The equivalent capacitances in even and odd modes for parallel symmetrical microstrip lines are given as [12–14]:

$$C_{odd\,mode} = 2C_s + C_1$$

$$C_{even\,mode} = C_1,$$

with C_1 plate capacitance and C_s gap coupling capacitance.

3. THEORETICAL ANALYSIS

A transverse gap in the strip conductor of a microstrip is usually in the form a symmetrical arrangement with conductors of equal width on either side of the gap (Fig. 1(b)). The equivalent gap circuit (Fig. 1(c)) consists of the gap-coupling capacitance (C_s)