The receiving antenna

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The basic theory of a receiving antenna is presented in sufficient detail to permit the calculation of the voltage across the load of an optimally designed TV antenna circuit. The rms load voltage is given by $V_L = \lambda H (30 R_r D)^{1/2}$, where λ is the wavelength of the incident radiation, H is the rms value of the magnetic field intensity, R_r is the radiation resistance of the antenna, and D is the directive gain. The result obtained from this formula for a typical monopole TV antenna is compared with that obtained from a simple, intuitive approximation. The reasons for the difference in the results are discussed.

I. THE PROBLEM

Although receiving antennas are widely used, textbooks on electricity and magnetism usually treat the receiving antenna cursorily when discussing electromagnetic radiation. Basically, the aim of this paper is to present the relevant theory in sufficient detail to enable one to calculate approximately the maximum voltage and current induced in the load of an optimally designed TV antenna circuit.

II. CIRCUIT REPRESENTATIONS

In view of the fact that an antenna is ordinarily coupled to an electronic circuit, it is useful to represent the entire system by means of a circuit diagram. Thus, if the antenna is used to radiate energy into space, the Thevenin equivalent circuit is the one shown in Fig. 1, where $\mathscr{E}$ is the impressed emf, Z the equivalent impedance of the source, and R_r the radiation resistance. If the same antenna is used to receive energy, the equivalent circuit as seen from the load terminals is the one shown in Fig. 2, where $\mathscr{E}$ is the equivalent of the emf induced in the antenna due to the *incident* electromagnetic waves, R_r' the "reradiation" resistance, and R_L the load resistance. The reradiation resistance of the receiving antenna arises from the fact that the incident waves not only induce an emf but are also reflected or scattered. Only a portion of the incident energy is absorbed by the load. Inherent in the circuits shown in Figs. 1 and 2 is the assumption that the lengths of the antennas are such that at the frequency of operation the impedance of the antennas is simply resistive. Also, we must remember that these circuits are only the Thevenin equivalents as seen from the load terminals. We shall now demonstrate that the reradiation resistance R_r' is equal to radiation resistance R_r.

III. THE RERADIATION RESISTANCE

To study the reradiation resistance R_r', we shall first consider a current element or Hertzian dipole of length dl. If the current in the Hertzian dipole is $I = I_0 \cos \omega t$, the radiated electric field intensity **E** and magnetic field intensity **H** are given[1] in SI units by

$$\mathbf{E} = -\mathbf{i}_\theta [60\pi I_0\, dl \sin\theta \sin(\omega t - 2\pi r/\lambda)]/\lambda r, \quad (1)$$

$$\mathbf{H} = -\mathbf{i}_\phi [I_0\, dl \sin\theta \sin(\omega t - 2\pi r/\lambda)]/2\lambda r, \quad (2)$$

where r is the distance from the center of the dipole to the field point (r,θ,ϕ) and $\mathbf{i}_\theta$ and $\mathbf{i}_\phi$ are unit vectors, respectively, in the directions of increasing θ and ϕ. The electric vector **E** is proportional to $\sin\theta$ and the field radiation pattern of the dipole consists simply of a plot of $\sin\theta$ vs θ.

If the dipole is used as a receiving antenna at a great distance from the transmitter, the incoming radiation consists of plane waves. Figure 3 shows the Poynting vector **P** of the incident radiation at an angle θ with the axis of the dipole and the vector **E** in the plane of incidence. The z component of **E** is $E \sin\theta$ and the voltage induced in the dipole is $E \sin\theta\, dl$. If **E** is normal to the plane of incidence, the induced voltage is zero. Thus, for **E** with arbitrary polarization, the induced voltage is proportional to $\sin\theta$. Clearly, the directional characteristics of the dipole when used as a receiver are the same as those when used as a transmitter. Since any antenna that is small compared to its distance from a given field point can be decomposed into a series of very short dipoles, we can assert that directional reciprocity of receiver and transmitter is valid in general.

Some of the radiation incident on a receiving antenna is reflected. Since the angle of incidence (which determines the receiving pattern) equals the angle of reflection (which determines the reradiation pattern), the directional characteristics of the outgoing radiation are the same as those of radiation emitted by the antenna when radiating. In fact, we may view the reflected radiation as having been reradiated. *The departing energy is broadcast by the antenna just as though the antenna were used as a transmitter.* Actually, insofar as reradiated energy is concerned, we may think of R_L in Fig. 2 as simply replacing Z in Fig. 1 so that Figs. 1 and 2 become identical provided the sources are

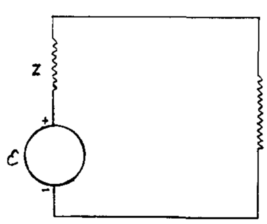

Fig. 1. The equivalent circuit for a transmitting antenna having a radiation resistance R_r.

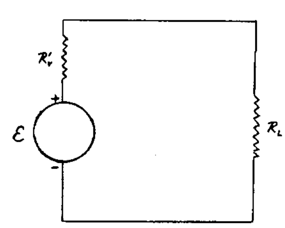

Fig. 2. The equivalent circuit for a receiving antenna having a reradiation resistance R_r' and a load resistance R_L.

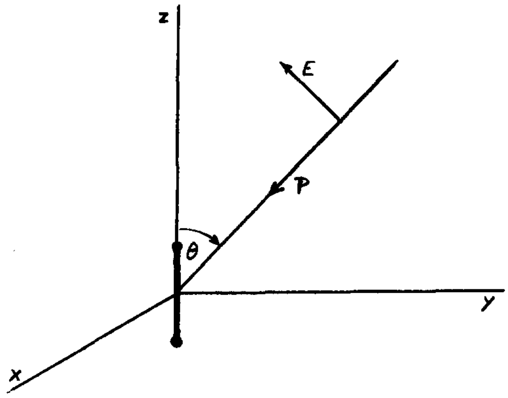

Fig. 3. The Poynting vector of plane waves incident on a Hertzian dipole makes an angle θ with the dipole axis.

equivalent. We may therefore conclude that the reradiation resistance R_r' is the same as the radiation resistance R_r.

IV. A HERTZIAN DIPOLE TRANSMITTER AND A HERTZIAN DIPOLE RECEIVER

To clarify further the concepts presented above, we shall consider two very short dipoles of lengths dl_1 and dl_2 parallel to the z axis and separated a large distance y as shown in Fig. 4. The first dipole acts as a transmitter and the second as a receiver. From Eq. (1) we can see that the peak value of the electric field intensity E_1 provided by the first dipole at the site of the second is

$$E_1 = 60\pi I_0 \, dl_1/\lambda y. \tag{3}$$

Consequently, the peak value of the induced emf $\mathscr{E}_2$ in the second dipole is

$$\mathscr{E}_2 = 60\pi I_0 \, dl_1 \, dl_2/\lambda y.$$

We now assume that the receiver operates under matched conditions. That is, the receiving antenna is connected by means of a lossless coaxial cable to the load, and the radiation resistance R_{r2} equals the load resistance R_{L2} which in turn equals the characteristic resistance of the coaxial line. Under these conditions there is no reflected wave in the coaxial cable, and the load receives maximum power. Since the radiation resistance of the receiving dipole is[2]

$$R_{r2} = 80\pi^2 (dl_2/\lambda)^2, \tag{4}$$

the total resistance of the Thevenin equivalent circuit is $160\pi^2 (dl_2/\lambda)^2$ and the peak value of the induced current in the receiver is

$$I_2 = \frac{3\lambda I_0}{8\pi y} \frac{dl_1}{dl_2}$$

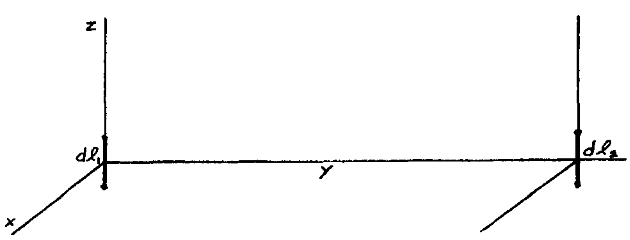

Fig. 4. Two Hertzian dipoles of lengths dl_1 and dl_2 parallel to the z axis are separated a distance y.

and the peak value of the voltage across the load is

$$V_{L2} = 30\pi I_0 \, dl_1 \, dl_2/\lambda y = \mathscr{E}_2/2.$$

The total average power absorbed by the receiver is

$$\tfrac{1}{2} I_2^2 R_{L2} = \tfrac{1}{2} I_2^2 R_{r2} = (45 I_0^2/8)(dl_1/y)^2.$$

From Eqs. (1) and (2) we can see that the time-average value of the incident Poynting vector $\langle \mathbf{P} \rangle = \langle \mathbf{E} \times \mathbf{H} \rangle$ at the site of the receving antenna is given by

$$|\langle \mathbf{P} \rangle| = 15\pi I_0^2 (dl_1)^2/\lambda^2 y^2. \tag{5}$$

The latter quantity is the average incident power per unit area. By definition, the absorption cross section A of the receiving antenna is the total average power absorbed divided by the average incident power per unit normal area. Thus

$$A = \frac{\tfrac{1}{2} I_2^2 R_{L2}}{|\langle \mathbf{P} \rangle|} = \frac{3\lambda^2}{8\pi}. \tag{6}$$

Clearly, if λ is large, the absorption cross section of a short dipole may be quite large in absolute terms although A may be small in comparison to the surface area over which the power of the transmitter is distributed.

V. THE RECIPROCITY THEORM

To obtain similar results for antennas of finite size, we need to develop the reciprocity theorem. Since we are interested in antennas that transmit or receive radiation at a particular frequency, we shall express the electric field intensity as $\mathrm{Re}[\mathbf{E}(x,y,z)e^{j\omega t}]$ and the magnetic field intensity as $\mathrm{Re}[\mathbf{H}(x,y,z)e^{j\omega t}]$. Thus, for a field $(\mathbf{E}_1,\mathbf{H}_1)$, Maxwell's curl equations become

$$\nabla \times \mathbf{E}_1 = -j\omega\mu \mathbf{H}_1, \tag{7}$$

$$\nabla \times \mathbf{H}_1 = j\omega\epsilon \mathbf{E}_1 + \sigma \mathbf{E}_1 + \mathbf{J}_1, \tag{8}$$

where $\mathbf{J}_1$ is the *impressed* current density and serves as the source of the electromagnetic field. Similarly, for a field $(\mathbf{E}_2,\mathbf{H}_2)$, we have

$$\nabla \times \mathbf{E}_2 = -j\omega\mu \mathbf{H}_2, \tag{9}$$

$$\nabla \times \mathbf{H}_2 = j\omega\epsilon \mathbf{E}_2 + \sigma \mathbf{E}_2 + \mathbf{J}_2. \tag{10}$$

The sources $\mathbf{J}_1$ and $\mathbf{J}_2$ may be zero everywhere except in certain special regions. We now dot-multiply Eq. (7) by $\mathbf{H}_2$ and Eq. (10) by $\mathbf{E}_1$ and subtract. We obtain

$$\mathbf{H}_2 \cdot \nabla \times \mathbf{E}_1 - \mathbf{E}_1 \cdot \nabla \times \mathbf{H}_2 = -j\omega\mu \mathbf{H}_1 \cdot \mathbf{H}_2 - (\sigma + j\omega\epsilon) \mathbf{E}_1 \cdot \mathbf{E}_2 - \mathbf{E}_1 \cdot \mathbf{J}_2. \tag{11}$$

Similarly, dot-multiplying Eq. (8) by $\mathbf{E}_2$ and Eq. (9) by $\mathbf{H}_1$ and then subtracting, we get

$$\mathbf{E}_2 \cdot \nabla \times \mathbf{H}_1 - \mathbf{H}_1 \cdot \nabla \times \mathbf{E}_2 = (\sigma + j\omega\epsilon) \mathbf{E}_1 \cdot \mathbf{E}_2 + \mathbf{E}_2 \cdot J_1 + j\omega\mu \mathbf{H}_1 \cdot \mathbf{H}_2. \tag{12}$$

The left-hand side of Eq. (11) may be written $\nabla \cdot (\mathbf{E}_1 \times \mathbf{H}_2)$, and the left-hand side of Eq. (12) may be written $\nabla \cdot (\mathbf{H}_1 \times \mathbf{E}_2)$. We now add Eqs. (11) and (12) and obtain

$$\nabla \cdot (\mathbf{E}_1 \times \mathbf{H}_2) - \nabla \cdot (\mathbf{E}_2 \times \mathbf{H}_1) = \mathbf{E}_2 \cdot \mathbf{J}_1 - \mathbf{E}_1 \cdot \mathbf{J}_2. \tag{13}$$

We continue by integrating Eq. (13) throughout a volume V and apply the divergence theorem. The result is

$$\int_V (\mathbf{E}_2 \cdot \mathbf{J}_1 - \mathbf{E}_1 \cdot \mathbf{J}_2) dV = \oint_S (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot d\mathbf{S}, \tag{14}$$

where S represents the bounding surfaces of the volume V. *Equation* (14) *is the reciprocity theorem.*

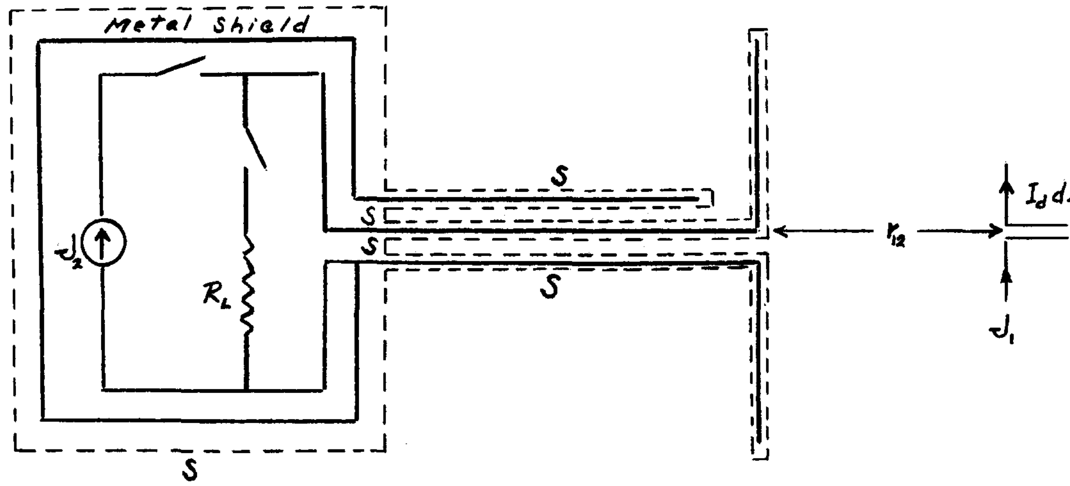

Fig. 5. An arbitrary antenna system that may either transmit energy to a Hertzian dipole or receive energy from the Hertzian dipole, which is oriented fc maximum reception.

VI. LOAD VOLTAGE OF A RECEIVING ANTENNA

Figure 5 shows an arbitrary antenna which may be connected by means of a lossless coaxial cable either to a current source of density $\mathbf{J}_2$ or to a load resistance R_L, both inside a metallic shield. The entire assembly is enclosed within a surface S that enters the coaxial cable as shown. At the section where S is normal to the cable axis, S cuts across the entire interconductor space. At a large distance r_{12} from the antenna assembly is a Hertzian dipole antenna system oriented for maximum reception. When the arbitrary antenna is radiating, the field is represented by $\mathbf{E}_2$ and $\mathbf{H}_2$, and when the dipole is radiating, the field is represented by $\mathbf{E}_1$ and $\mathbf{H}_1$.

We now use Eq. (14), the reciprocity theorem. We consider the volume V to be all of space except that enclosed by the surface S. Since $\mathbf{J}_1$ exists only within the Hertzian dipole antenna and $\mathbf{J}_2$ does not exist within the volume V, the left-hand side of Eq. (14) becomes $E_2(r_{12})I_d\,dl$, where $E_2(r_{12})$ is the peak value of $\mathbf{E}_2$ at the site of the dipole and $I_d\,dl$ is the integral of $J_1\,dV$, since for maximum reception the Hertzian dipole is oriented so that $\mathbf{E}_2\cdot\mathbf{J}_1 = E_2J_1$. If the medium in space is not entirely lossless, the fields will decrease more rapidly than $1/r$, and the integral over the surface at infinity will be zero. We assume this to be the case. Furthermore, since the surface S is everywhere parallel to conducting surfaces except at a certain section within the coaxial cable, $\mathbf{E}_1\times d\,\mathbf{S}$ and $\mathbf{E}_2\times d\,\mathbf{S}$ are zero everywhere except at this special section. Thus

$$\mathbf{E}_1\times\mathbf{H}_2\cdot d\,\mathbf{S} = -\mathbf{H}_2\cdot\mathbf{E}_1\times d\,\mathbf{S} = 0,$$

$$\mathbf{E}_2\times\mathbf{H}_1\cdot d\,\mathbf{S} = -\mathbf{H}_1\cdot\mathbf{E}_2\times d\,\mathbf{S} = 0,$$

and the surface integral vanishes except at one transverse section of the cable.

If the outer radius b of the coaxial cable is small relative to the wavelength of the radiation, only a transverse electromagnetic (TEM) wave can exist within the cable, and the peak values of the field intensities are

$$H_{2\phi} = I_2/2\pi r, \quad E_{2r} = (\mu/\epsilon)^{1/2}(I_2/2\pi r),$$

$$H_{1\phi} = -I_1/2\pi r, \quad E_{1r} = (\mu/\epsilon)^{1/2}(I_1/2\pi r).$$

The negative sign for $H_{1\phi}$ results from the fact that the TEM wave for the field $(\mathbf{E}_1,\mathbf{H}_1)$ travels to the left. The surface integral thus becomes

$$\int_0^{2\pi}\int_a^b 2\left(\frac{\mu}{\epsilon}\right)^{1/2}\left(\frac{I_1I_2}{4\pi^2r^2}\right) r\,dr\,d\phi$$

$$= \left(\frac{\mu}{\epsilon}\right)^{1/2}\left(\frac{I_1I_2}{\pi}\right)\int_a^b \frac{dr}{r} = \left(\frac{\mu}{\epsilon}\right)^{1/2}\left(\frac{I_1I_2}{\pi}\right)\ln\frac{b}{a}$$

$$= 120 I_1I_2 \ln\frac{b}{a} = 2Z_cI_1I_2, \tag{15}$$

where Z_c is the characteristic impedance of the lossless coaxial cable. We assume that the arbitrary antenna operates under matched conditions; namely

$$Z_c = R_L = R_r, \tag{16}$$

where we have also assumed that the impedance of the antenna is purely resistive. The reciprocity theorem thus yields the following result:

$$E_2(r_{12})I_d\,dl = 2Z_cI_1I_2 = 2V_LI_2, \tag{17}$$

where V_L is the peak value of the voltage across R_L when the arbitrary antenna is receiving.

We can rewrite Eq. (17) in more convenient terms. First, when the dipole radiates, the peak value of the electric field intensity at the arbitrary antenna, according to Eq. (3), is

$$E = 60\pi I_d\,dl/\lambda r_{12}. \tag{18}$$

Second, when the arbitrary antenna radiates, the average total radiated power is

$$\mathscr{P} = \tfrac{1}{2}I_2^2R_r$$

and the magnitude of the time-average value of the Poynting vector is given by

$$|\langle\mathbf{E}_2\times\mathbf{H}_2\rangle| = \frac{1}{2}\frac{[E_2(r_{12})]^2}{120\pi} = \frac{\mathscr{P}D}{4\pi r_{12}^2},$$

where, by definition, D is the directive gain of the arbitrary antenna in the direction of the Hertzian dipole. The maximum value of the directive gain is simply called the directivity. From the last equality, we have

$$[E_2(r_{12})]^2 = \frac{60\mathscr{P}D}{r_{12}^2} = \frac{30I_2^2R_rD}{r_{12}^2},$$

$$E_2(r_{12}) = \frac{I_2}{r_{12}}(30R_rD)^{1/2}. \tag{19}$$

Substituting the value of $I_d\ dl$ from Eq. (18) and the value of $E_2(r_{12})$ from Eq. (19) into Eq. (17), we find

$$V_L = (\lambda E/120\pi)(30R_r D)^{1/2}. \tag{20}$$

In free space,

$$H = E/120\pi,$$

where H is the peak value of the magnetic field intensity at the arbitrary antenna. Therefore

$$V_L = \lambda H(30R_r D)^{1/2}. \tag{21}$$

Equation (21) relates the peak voltage across the load of a receiving antenna to the peak value of the magnetic field intensity at the site of the antenna. This result is perfectly general except for the assumption of matched conditions. Equation (21) gives the rms value of V_L if H is the rms value of the magnetic field intensity.

We can check Eq. (21) by applying it to the situation discussed in Sec. IV. For the case of two parallel Hertzian dipoles separated a distance y as in Fig. 4, we have for the dipole on the right (acting as a receiver) from Eqs. (2) and (4)

$$H_1(y) = H = \frac{I_0\, dl_1}{2\lambda y}, \quad R_r = 80\pi^2\left(\frac{dl_2}{\lambda}\right)^2.$$

To obtain the value of D, we consider that the dipole on the right radiates energy. From Eqs. (1) and (19) we obtain

$$E_2(y) = \frac{60\pi I_2\, dl_2}{\lambda y} = \frac{I_2}{y}\left[30(80)\pi^2\left(\frac{dl_2}{\lambda}\right)^2 D\right]^{1/2}$$

$$D = 1.5.$$

Thus, substituting for H, R_r, and D in Eq. (21), we get

$$V_{L2} = 30\pi I_0\, dl_1\, dl_2/\lambda y,$$

which agrees with the value found previously for the peak value of the voltage across the load connected to the dipole on the right.

VII. APPLICATION TO A TV ANTENNA

We shall now apply Eq. (21) to a typical situation. A station transmits at a carrier frequency of 150 MHz, and the average power flux at the site of a TV receiving set is 10 mW/m^2. The TV set has a monopole antenna; that is, the lower end of the antenna is connected to the load, and the load is connected to ground. The length of the monopole is somewhat less than one-quarter of a wavelength so that its radiation impedance insofar as the equivalent circuit is concerned will be a pure resistance. The radiation impedance of a half-wave dipole antenna is[3] $73 + j42.5\ \Omega$, and its directivity[4] is 1.642. Because the monopole has an image in the ground, it is similar to a half-wave dipole except that only the monopole itself and not its image receives energy.[5,6] If exactly one-quarter of a wavelength long, the monopole would have a radiation impedance one-half that of a half-wave dipole antenna, namely, $36.5 + j\ 21.25\ \Omega$. Its length is usually reduced 4% so that its radiation impedance becomes purely resistive with a value[7] of approximately 33.5 Ω. The problem is to find the rms value of the voltage across a matched load connected to the monopole antenna which is oriented for maximum reception.

To calculate the rms value of V_L, we use Eq. (21). We know R_r and D, and, since the frequency is 1.5×10^8 Hz, the wavelength λ is 2 m. We can find the rms value of the magnetic field intensity as follows:

$$|\langle \mathbf{P}\rangle| = |\langle \mathbf{E}\times\mathbf{H}\rangle|, \quad E = 120\pi H,$$

where E and H now represent rms values.

$$|\langle \mathbf{E}\times\mathbf{H}\rangle| = 120\pi H^2 = 10^{-2}\ \text{W/m}^2,$$

$$H = 5.15\times10^{-3}\ \text{A/m}.$$

Substituting the above data in Eq. (21), we obtain

$$V_L = 0.418\ \text{V}.$$

The monopole may be considered an extension of the central conductor of a coaxial cable, thus forming the end section of an open-circuited transmission line. The radiation field causes a standing wave to be set up in the monopole such that the current is zero at the open end and a maximum at the other end. As an approximation, however, we may consider the monopole to be the equivalent of a concentrated voltage source (similar to a Hertzian dipole) with an internal resistance equal to the radiation resistance as shown in Fig. 2. Since the monopole is 0.48 m long and the rms value of the electric field intensity at the site of the monopole is

$$E = 120\pi H = 1.94\ \text{V/m},$$

the rms value of the lumped circuit induced emf is

$$\mathscr{E} = (1.94\ \text{V/m})(0.48\ \text{m}) = 0.932\ \text{V}.$$

If $R_L = R_r$, then the rms value of the induced voltage across the load is 0.466 V. Evidently the result of this very approximate calculation is reasonably close to the more accurate result provided by Eq. (21). The difference is 11%.

VIII. SUMMARY

The main result of this paper is expressed in Eq. (21). This equation shows that the matched load voltage of a purely resistive receiving antenna is proportional to the square root of the product of the radiation resistance and the directive gain. These two characteristics are determined by the induced current distribution along the antenna. In the sinusoidal steady state a standing current wave is set up in accordance with the boundary conditions.

In contrast, in the above approximate calculation the incoming electromagnetic waves are assumed to induce an emf in the antenna that is simply proportional to its length. The entire antenna acts as a concentrated voltage source having an internal resistance equal to the radiation resistance. Insofar as the load is concerned, the antenna acts as a lumped circuit element. This approximation is very good for antennas that are short compared to the wavelength of the incident radiation since the variation of the antenna current with position is unimportant for all practical purposes. The approximation becomes progressively worse as the length of the antenna increases relative to the wavelength.

[1]N. N. Rao, *Elements of Engineering Electromagnetics* (Prentice-Hall, Englewood Cliffs, NJ, 1977), p. 302.

[2]Reference 1, p. 304.

[3]C. R. Paul and S. A. Nasar, *Introduction to Electromagnetic Fields* (McGraw-Hill, New York, 1982), pp. 402–404.

[4]Reference 1, p. 311.

[5]R. E. Collin and F. J. Zucker, *Antenna Theory* (McGraw-Hill, New York, 1969).

[6]R. W. P. King and C. W. Harrison, Jr., *Antennas and Waves* (MIT, Cambridge, MA, 1969).

[7]W. L. Stutzman and G. A. Thiele, *Antenna Theory and Design* (Wiley, New York, 1981), pp. 86 and 198–203.