Understanding the penetration of electromagnetic velocity fields into conductors

Timothy H. Boyer
Department of Physics, City College of the City University of New York, New York, New York 10031-9198

(Received 11 January 1999; accepted 15 April 1999)

The electromagnetic fields due to a point charge can be broken into velocity fields and acceleration fields. The acceleration fields give rise to electromagnetic waves whose penetration into ohmic conductors is described by exponential damping with a characteristic skin depth. The velocity fields allow a steady-current limit where the magnetic field penetrates even a good conductor. Here we note the contrasts between velocity fields and wave fields in their interactions with conductors. We derive a new time-integral invariant of the magnetic field in an ohmic conductor. Finally we note the disparate analyses in the literature and suggest the following summary regarding the penetration of the electromagnetic fields of a point charge moving parallel to a conducting surface. (1) The falloff of the electromagnetic fields is algebraic, not exponential, and cannot be characterized by a skin depth. (2) In the limit of low velocity, the magnetic field penetration is independent of the conductivity of the material. (3) In general the penetration of the electromagnetic fields depends upon the velocity of the particle and becomes vanishingly small for a perfect conductor. (4) The time integral of the magnetic field at a fixed spatial point inside (or outside) an ohmic conductor is independent of the conductivity of the material; thus as the conductivity of the material becomes larger and the magnetic field inside becomes smaller, the time of penetration becomes longer. The penetration of time-dependent velocity fields into conductors has become of interest largely in connection with the Aharonov–Bohm effect. It is curious that the classical explanation for the Aharonov–Bohm effect depends upon the time integral of the magnetic field, which is independent of the conductivity of any ohmic shielding material. © *1999 American Association of Physics Teachers.*

I. INTRODUCTION

In ohmic conductors, changing magnetic fields cause eddy currents which tend to oppose the changes in the magnetic fields. Yet ohmic conductivity is irrelevant for time-independent magnetostatic fields, which penetrate even excellent conductors. This contrasting behavior between the penetration of time-dependent and time-independent magnetic fields suggests the possibility of significant differences between the penetration into conductors of electromagnetic wave fields, which do not have a time-independent limit, and of electromagnetic velocity fields, which do have a time-independent limit. In this analysis we provide a consistent understanding of the contrasting views in the literature regarding the penetration of electromagnetic velocity fields into conductors.

II. VELOCITY FIELDS AND ACCELERATION FIELDS OF A POINT CHARGE

The electromagnetic fields of a charged particle e at position $\mathbf{r}_e$ break up naturally into the form[1]

$$\mathbf{E}(\mathbf{r},t)=e\left[\frac{(\hat{n}-\boldsymbol{\beta})(1-\beta^2)}{(1-\hat{n}\cdot\boldsymbol{\beta})^3|\mathbf{r}-\mathbf{r}_e|^2}\right]_{t_{\rm ret}}+\frac{e}{c}\left[\frac{\hat{n}\times\{(\hat{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\}}{(1-\hat{n}\cdot\boldsymbol{\beta})^3|\mathbf{r}-\mathbf{r}_e|}\right]_{t_{\rm ret}}, \tag{1}$$

$$\mathbf{B}(\mathbf{r},t)=\hat{n}_{\rm ret}\times e\left[\frac{(\hat{n}-\boldsymbol{\beta})(1-\beta^2)}{(1-\hat{n}\cdot\boldsymbol{\beta})^3|\mathbf{r}-\mathbf{r}_e|^2}\right]_{t_{\rm ret}}+\hat{n}_{\rm ret}\times\frac{e}{c}\left[\frac{\hat{n}\times\{(\hat{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}}{(1-\hat{n}\cdot\boldsymbol{\beta})^3|\mathbf{r}-\mathbf{r}_e|}\right]_{t_{\rm ret}}, \tag{2}$$

where $\hat{n}_{\rm ret}=(\mathbf{r}-\mathbf{r}_e)/|\mathbf{r}-\mathbf{r}_e|$, $\boldsymbol{\beta}=\mathbf{v}_e/c$, and all quantities are evaluated at the retarded time. The velocity fields correspond to the first bracket on the right-hand sides of Eqs. (1) and (2); they fall off as the inverse distance squared $|\mathbf{r}-\mathbf{r}_e|^{-2}$ from the retarded position of the charge, and depend upon the position $\mathbf{r}_e$ and velocity $\mathbf{v}_e=c\boldsymbol{\beta}$ of the charge e at the retarded time. The second bracket on the right-hand sides of Eqs. (1) and (2) gives the electromagnetic wave fields which fall off as the inverse distance $|\mathbf{r}-\mathbf{r}_e|^{-1}$ from the retarded source point, depend linearly on the particle acceleration $\dot{\mathbf{v}}_e=c\dot{\boldsymbol{\beta}}$ at the retarded time, and are transverse to the displacement of the field point from the retarded source point. At large distances, the wave fields can be expressed as a superposition of spherical wave fields, each spherical wave having its own characteristic frequency derived from a time-spectral analysis of the source.[2] The contrast in the properties of electromagnetic velocity and wave fields leads to contrasting interactions with conductors.

III. DIFFERENTIAL EQUATIONS FOR THE FIELDS IN CONDUCTORS

Inside an ohmic conductor characterized by conductivity σ, and with $\mu=1$, $\epsilon=1$, the electromagnetic fields satisfy

Maxwell's equations with current $\mathbf{J}=\sigma\mathbf{E}$. Any free charge in the conducting material decays away exponentially;[3] thus no charge will appear inside the body of a conductor unless it is introduced by some nonelectromagnetic source, and so one may assume $\nabla\cdot\mathbf{E}=0$ inside a conductor. Maxwell's equations, $\nabla\cdot\mathbf{E}=0$, $\nabla\cdot\mathbf{B}=0$, $\nabla\times\mathbf{E}=-c^{-1}\partial\mathbf{B}/\partial t$, $\nabla\times\mathbf{B}=(4\pi/c)\sigma\mathbf{E}+c^{-1}\partial\mathbf{E}/\partial t$, then lead to the wave equation[4]

$$\left(\nabla^2-\frac{4\pi}{c^2}\sigma\frac{\partial}{\partial t}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\begin{Bmatrix}\mathbf{E}\\ \mathbf{B}\end{Bmatrix}=0. \tag{3}$$

An electromagnetic wave in vacuum which is incident upon a conducting plane is represented by a superposition of waves of different frequencies. Each of the waves $\mathbf{E}(\mathbf{r},t)=\mathbf{E}_0(\mathbf{r})e^{-i\omega_0 t}$, $\mathbf{B}(\mathbf{r},t)=\mathbf{B}_0(\mathbf{r})e^{-i\omega_0 t}$, of a given frequency ω_0 can be treated independently in its interaction with the conducting wall in Eq. (3), leading to an equation of the form

$$\left(\nabla^2+i\frac{4\pi\sigma\omega_0}{c^2}+\frac{\omega_0^2}{c^2}\right)\begin{Bmatrix}\mathbf{E}_0(\mathbf{r})\\ \mathbf{B}_0(\mathbf{r})\end{Bmatrix}=0. \tag{4}$$

This equation corresponds to exponential spatial damping characterized by a skin depth involving the length parameter $c/(\sigma\omega_0)^{1/2}$, which includes the characteristic frequency ω_0 of the wave.

Electromagnetic velocity fields are characterized not by any frequency but rather by the velocity of their source. For a particle moving parallel to a conducting wall with constant speed $\mathbf{v}$, the entire pattern of electromagnetic fields must move with constant speed $\mathbf{v}$. Thus if the particle is moving in the x direction, the electromagnetic fields must have the functional form $\mathbf{E}(x-vt,y,z),\mathbf{B}(x-vt,y,z)$. Inside the conductor, the fields satisfy Eq. (3) in the form

$$\left(\nabla^2+\frac{4\pi\sigma v}{c^2}\frac{\partial}{\partial x}-\frac{v^2}{c^2}\frac{\partial^2}{\partial x^2}\right)\begin{Bmatrix}\mathbf{E}(x-vt,y,z)\\ \mathbf{B}(x-vt,y,z)\end{Bmatrix}=0. \tag{5}$$

One should notice how different this equation is from Eq. (4) for wave fields. The only length parameter appearing here in Eq. (5) is c/σ, which makes no reference to the characteristics of the electromagnetic velocity field in free space. Indeed, the free-space motion of the particle provides no natural length parameter; any length in the problem must be introduced as a distance between the charge and some preferred spatial point, such as the distance from the particle to the conducting wall.

IV. EXISTENCE OF A TIME-INDEPENDENT LIMIT

The contrasting characteristics of velocity and wave fields lead to a crucial distinction involving the existence of a time-independent limit. This limit is essential for understanding interactions of the fields with conductors.

Since Maxwell's equations for ohmic materials are linear in the electromagnetic fields and the sources, the fields corresponding to the presence of several charges can be found simply by superimposing the fields due to the individual charges in the absence of the others. If a set of charged particles is arranged so as to allow the passage to the limit of a steady current, then the electromagnetic fields of the steady current (obtained by adding the electromagnetic fields of the individual charges) have nonvanishing contributions only for the velocity fields. The radiation fields of the particles give no contribution in the steady-current limit.[5] There is no such thing as a nonvanishing, time-independent superposition limit for electromagnetic wave fields, whereas there is indeed for electromagnetic velocity fields. We will use this limit as a touchstone in understanding the interaction of velocity fields and conductors.

V. THE TIME INTEGRAL OF THE MAGNETIC VELOCITY FIELD

When a charged particle moves with constant velocity past any ohmic conductor, its magnetic field must penetrate into the conductor. The requirement of penetration follows from the existence of a magnetic field inside the conductor in the steady-current limit. If the magnetic field of a single point charge did not penetrate, then there would be no field when the field of many charges were superimposed to form a constant line current; yet we know that the magnetic field of a line current indeed penetrates a good ohmic conductor as though the conductor were not present. Although the existence of a steady-current limit assures us that the magnetic velocity field must penetrate the conductor, it does not tell us that the field is not modified in some fashion in time or space while maintaining the superposition limit. The magnetic field corresponding to the steady-current superposition limit for a charge moving with constant velocity $\mathbf{v}=\hat{i}v$ corresponds to a line charge λ moving with velocity $\mathbf{v}$ along its length. Using Ampere's law and symmetry, the static magnetic field $\mathbf{B}(r)$ a distance $r=(y^2+z^2)^{1/2}$ from the moving line charge is found to be

$$\mathbf{B}_\lambda(r)=\hat{\varphi}\frac{2\lambda v}{cr}, \tag{6}$$

where $\hat{\varphi}$ is the unit vector in cylindrical coordinates.

Now in going from the magnetic field $\mathbf{B}_q(x,y,z,t)$ due to a point charge moving with constant velocity $\mathbf{v}=\hat{i}v$ over to the superposition limit, we consider a succession of charges q spaced at equal intervals along the path of the moving charge. We think of the line-charge magnetic field $\mathbf{B}_\lambda$ as a sum over the fields $\mathbf{B}_q$ arising from the individual charges. Since all the charges are equivalent and differ only in their positions in time, the magnetic field $\mathbf{B}_q$ at a fixed point (x,y,z), which one charge q produces at the instant t, is the same as the field $\mathbf{B}_q{}'$ another charge q' produced or will produce at the instant t' when it is located at the point where the particle q is at time t. Thus we can write

$$\mathbf{B}_\lambda(x,y,z)\cong\sum_{n=-\infty}^{\infty}\frac{\lambda\Delta x'}{q}\mathbf{B}_q\left(x,y,z,t-n\frac{\Delta x'}{v}\right). \tag{7}$$

Here we have introduced a charge per unit length λ. In the limit $\Delta x'\to 0$, this becomes the integral

$$\mathbf{B}_\lambda(r)=\frac{\lambda}{q}\int_{-\infty}^{\infty}dx'\,\mathbf{B}_q\left(x,y,z,t-\frac{x'}{v}\right)=\hat{\varphi}\frac{2\lambda v}{cr}, \tag{8}$$

where $r=(y^2+z^2)^{1/2}$ and the second equality follows from Eq. (6). However, the integral can be converted to a time integral because of the functional form $t-x'/v$. Canceling factors of v and λ on both sides, we have finally

$$\int_{-\infty}^{\infty}dt\,\mathbf{B}_q(x,y,z,t)=\hat{\varphi}\frac{2q}{cr}, \tag{9}$$

where $r=(y^2+z^2)^{1/2}$. Thus in order to maintain the superposition limit, we find that the time integral of the magnetic field, either inside or outside the conductor, must satisfy Eq. (9) and is independent of the conductivity of the material. It is also independent of the velocity of the charge. The argument goes through even if there are variations in the shape of the ohmic conductors near the path. We note that Eq. (9) can easily be seen to hold for the field of a constant-velocity point charge in vacuum where the magnetic field is given by

$$\mathbf{B}_{q(x,r,t)}^{\text{vacuum}}=\hat{\varphi}\frac{q\gamma v r}{c[\gamma^2(x-vt)^2+r^2]^{3/2}}. \tag{10}$$

with $\gamma=(1-v^2/c^2)^{-1/2}$.

In some calculations it is convenient to consider not a point charge moving parallel to a conducting surface, but rather a line charge moving perpendicular to its length. In this case the superposition limit corresponds to a sheet of charge Σ per unit area moving with velocity $\mathbf{v}=\hat{\imath}v$ and so giving a surface current $\mathbf{K}=\Sigma\mathbf{v}$. The time integral of the magnetic field $\mathbf{B}_{\lambda\perp}$ of the line charge oriented parallel to the y axis moving perpendicular to its length along $\mathbf{v}=\hat{\imath}v$ can be found in a fashion analogous to that for a point charge,

$$\int_{-\infty}^{\infty}dt\,\mathbf{B}_{\lambda\perp}=\frac{1}{v}\int_{-\infty}^{\infty}dx'\,\mathbf{B}_{\lambda\perp}\left(x,z,t-\frac{x'}{v}\right)$$
$$=\frac{\lambda\mathbf{B}_{\Sigma}}{\Sigma v}=\pm\hat{\jmath}\frac{2\pi\lambda}{c}. \tag{11}$$

This equation can be seen to hold for the magnetic field of a line charge oriented parallel to the y axis and moving perpendicular to its length $\mathbf{v}=\hat{\imath}v$ in free space where the magnetic field is

$$\mathbf{B}_{\lambda\perp}(x,z,t)=-\hat{\jmath}\frac{2\lambda v\gamma z}{c[\gamma^2(x-vt)^2+z^2]}. \tag{12}$$

Clearly, the idea of the invariance of the time integral of the magnetic field at a fixed spatial point due to the motion of a charged particle moving with constant velocity can be extended to periodic motions of charges which move around a circuit with constant speed. Perhaps the simplest example involves a point charge in uniform circular motion at a constant distance from a conducting half-space. Then the electromagnetic fields are periodic in time and so can be expanded in a time Fourier series involving multiples of the fundamental frequency. The time integral of the magnetic field over one period at a fixed spatial point must depend upon the charge and spatial coordinates and is independent of the conductivity of any materials; it corresponds to the existence of a magnetostatic limiting configuration of uniformly spaced charges. A single charge in uniform circular motion will emit radiation which is exponentially damped in the conductor. However, this radiation with its oscillating electric and magnetic fields will not contribute to the time integral of the magnetic field taken over one period. The radiation emission completely disappears in the limit of a large number of uniformly spaced particles moving around the circuit with constant speed,[5] while the time-averaged magnetic field remains. Although this simplest situation would involve a plane conducting half-space, the circuit might be near an arbitrary distribution of ohmic conductors. The time Fourier series arising from the periodic motion of the charge around the circuit would still be possible and one would still find the independence of conductivity for the time integral of the magnetic field at a fixed point in space. Since any current induced in the ohmic material according to Faraday's law is oscillating and hence produces magnetic fields of opposite sign at different points in the period, any such current will not contribute to the time integral of B over a complete period.

VI. THE LITERATURE OF THE PENETRATION PROBLEM

Although the exponential damping for electromagnetic wave fields in conductors is mentioned in all the textbooks and is familiar to physicists, the penetration problem for velocity fields seems to have been noticed only relatively recently, mainly in connection with the Aharonov–Bohm effect.[6] Here we wish to survey the literature of the velocity-field penetration problem.

In 1965 Liebowitz[7] noted that certain energy-conservation problems involving a charged particle passing a solenoid had not been discussed in connection with the Aharonov–Bohm effect. He suggested that the effect might be due to a relative lag between charged particles passing on opposite sides of the solenoid. In order to counter Liebowitz's suggestions, Kasper[8] undertook to show that the magnetic fields of a passing charge could not penetrate into the interior of a conducting solenoid. Kasper followed the reaction of many physicists by analyzing the problem as a skin-depth situation with a characteristic frequency parameter $\omega\sim v/d$, where v is the particle velocity and d the distance of the passing charge from the conductor. In 1975 Kasper again tried to analyze the velocity fields penetrating into a conductor assuming exponential damping.[9]

Shier[10] in 1968 was apparently the first to realize that the velocity penetration problem involved new aspects compared to wave fields. He thought in terms of the usual exponential damping familiar from wave fields, but noted that when the particle velocity was small and the penetration depth was very large, then the velocity field penetration at distances less than the assumed skin depth took a very different form which he proceeded to derive. Shier's expressions gave algebraic dependence for the electric and magnetic fields inside the conductor, but these fields were understood to be suppressed at large distances by the exponential damping of the skin-depth behavior.

In 1974, Boyer[11] and Furry[12] independently suggested that the usual skin-depth analysis for wave fields played no role whatsoever in the penetration of electromagnetic velocity fields. Both authors used as the crucial criterion the fact that velocity fields could be superimposed so as to give the fields of a steady line current, and the magnetic field of a line current penetrates even a good conductor. Using a first-order analysis in the particle velocity, they found expressions closely related to those which had been given by Shier six years earlier. Both Boyer and Furry were initially unaware of the other's work and also of Shier's results. In contrast to Shier, Boyer emphasized that the asymptotic behavior of the electromagnetic fields in the conductor was algebraic and not exponential damping; he showed that the superposition of the field expressions agreed with the line-current and current-sheet limits. In 1996 Boyer extended his analysis to the case of a conducting wall of finite thickness.[13] In the low-velocity limit obtained by Boyer and by Furry, the magnetic field penetration into the conductor was independent of the conductivity of the wall. Both authors erroneously suggested that the low-velocity expressions also held in the limit of perfect conductivity. Kasper[9] in 1975 correctly objected that the large-conductivity limit was not possible in the low-velocity perturbation expansion obtained by Boyer and by Furry. The low-velocity expansion analysis has also been followed by Aguirregabiria, Hernandez, and Rivas.[14] A different approach to the penetration of electric fields in the low-velocity limit has been given by Tomassone and Widom,[15] who find the same retarding force on a passing charge as given by Shier and by Boyer.

It was Jones[16] in 1975 who undertook the most extensive investigation of the penetration problem for velocity fields. Jones analyzed the penetration of the magnetic velocity fields of a line charge moving perpendicular to its length and parallel to a conducting wall without restriction on the speed of the line charge or the conductivity of the wall. He confirmed Boyer's claim that the asymptotic falloff of the velocity fields was algebraic, not exponential, and he obtained the same low-velocity limit as given by Boyer. However, Jones contradicted some of Boyer's claims. He noted that his calculations showed that the penetrating magnetic field in general depended upon the conductivity of the wall, and indeed gave vanishing penetration into a perfect conductor.

Jones' Fourier analysis does not lead naturally to the steady-current limit. Indeed, Jones never discussed this limit which played such a large role in Boyer and Furry's thinking, and Boyer largely ignored Jones' work in his finite-thickness wall calculation of 1996.[17] In order to extend Jones' analysis to a full understanding of the penetration problem, it is of interest to take the steady-current limit of his calculations. Jones' results for the magnetic field both inside and outside a conducting wall due to a line charge moving perpendicular to its length are given on the top of page 746 of Ref. 16. However, these contain a transcription error involving the omission of the term u_0. Once this is corrected, it is easily possible to integrate Jones' expressions numerically. One finds that the magnetic field at a fixed spatial point inside or outside the conductor increases and decreases smoothly in time as the line charge passes; in general, the magnetic field is delayed compared to the magnetic velocity field for a line charge in empty space, and also the field is both smaller and lasts for a longer time. Evaluation of the time integral of the magnetic field at a fixed spatial point as given in Eq. (11) of the present paper indeed shows the invariance of the integral with respect to conductivity, and velocity of the charge. The time integrals of the magnetic field do not correspond directly to the single current sheet situation of Eq. (11) of the present article, but rather indicate the presence of currents at spatial infinity, as noted by Boyer when calculating the low-velocity limit for this situation; the time integrals of Jones' expressions do correspond exactly to the expected current-sheet limit given by Boyer in Eqs. (89) and (90) of Ref. 11.

VII. CONCLUSION

When trying to reconcile the various calculations in the literature, it seems helpful to separate the generally correct calculations of the authors from their sometimes erroneous comments and extrapolations. Thus Kasper and Shier's references to skin depth damping are inaccurate. Boyer's extrapolation of his low-velocity field result to a perfect conductor is not allowed by his approximations, and his claim for independence of the magnetic field penetration from the conductivity holds only in the low-velocity limit which he actually calculated. Furry's assertion that he is working with a perfectly conducting wall is inaccurate. Jones' work (when corrected for the transcription error) seems accurate and in the low velocity limit fits with the actual calculations of Shier, of Boyer, and of Furry. His analysis when supplemented by the time-integral calculation of the present paper gives a consistent picture of the penetration of electromagnetic velocity fields into conductors.

Thus we suggest the following. (1) The electric and magnetic velocity fields in a conductor fall off algebraically with distance; all the calculations and suggestions involving a penetration skin-depth seem misdirected. (2) In the low-velocity limit, one recovers the expressions obtained by Shier, by Boyer, and by Furry which give a penetrating magnetic field independent of the conductivity of the material. (3) For a general velocity, the penetrating magnetic velocity field depends upon the conductivity of the material and vanishes in the limit of perfect conductivity. (4) The time of penetration of the magnetic velocity field becomes longer as the conductivity becomes larger, becoming infinite in the limit of perfect conductivity, so that the time-integrated magnetic field is independent of the conductivity.

It is curious that this last time-integral condition is precisely what is needed to maintain the possibility of a classical explanation[7,18] of the Aharonov–Bohm effect despite the presence of intervening conductors.

ACKNOWLEDGMENT

I wish to thank Professor Joel Gersten for a discussion in which he noted the possibility of time modification of the penetrating magnetic velocity fields.

[1]J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), 2nd ed., p. 657, Eqs. (14.13) and (14.14).

[2]Reference 1, Sec. 14.5 on pp. 668–672.

[3]See, for example, D. S. Jones, *The Theory of Electromagnetism* (Pergamon, New York, 1964), pp. 11–12. However, see also Hans C. Ohanian, ''On the approach to electro- and magneto-static equilibrium,'' Am. J. Phys. **51**, 1020–1022.

[4]See, for example, D. J. Griffiths, *Introduction to Electrodynamics* (Prentice–Hall, Englewood Cliffs, NJ, 1981), pp. 369–370.

[5]See, for example, Ref. 1, p. 697, problems #14.12 and 14.13.

[6]Y. Aharonov and D. Bohm, ''Significance of Electromagnetic Potentials in Quantum Theory,'' Phys. Rev. **115**, 485–491 (1959).

[7]B. Liebowitz, ''Significance of the Aharonov–Bohm Effect,'' Nuovo Cimento **38**, 932–950 (1965).

[8]Erwin Kasper, ''Zur Deutung der Streifenverschiebung in Elektronen-Interferometern durch einem magnetischen Fluss,'' Z. Phys. **196**, 415–423 (1966).

[9]Erwin Kasper, ''On attempts to interpret the Electron Optical Phase Shift by the Bohm–Aharonov Effect on a purely classical basis,'' Optik (Stuttgart) **42**, 367–376 (1975).

[10]John S. Shier, ''Fields Due to a Slow Charged Particle Moving Parallel to a Plane-Metal Surface,'' Am. J. Phys. **36**, 245–249 (1968).

[11]T. H. Boyer, ''Penetration of the electric and magnetic velocity fields of a nonrelativistic point charge into a conducting plane,'' Phys. Rev. A **9**, 68–82 (1974). I became interested in the penetration problem for velocity fields because of its connection with the Aharonov–Bohm effect and specially because of Liebowitz's work which was introduced to me by Baldwin Robertson. The existence of a steady-current limit for velocity fields suggested to me that Kasper's work must be in error.

[12]W. H. Furry, ''Shielding of the Magnetic Field of a Slowly Moving point Charge by a Conducting Surface,'' Am. J. Phys. **42**, 649–667 (1974).

[13]T. H. Boyer, ''Penetration of electromagnetic velocity fields through a conducting wall of finite thickness,'' Phys. Rev. E **53**, 6450–6459 (1996).

[14]J. M. Aguirregabiria, A. Hernandez, and M. Rivas, ''Velocity fields inside a conducting sphere near a slowly moving charge,'' Am. J. Phys. **62**, 462–466 (1994).
[15]S. Tomassone and A. Widom, ''Friction forces on charges moving outside of a conductor due to Ohms law heating inside a conductor,'' Am. J. Phys. **65**, 1181–1183 (1997).
[16]D. S. Jones, ''The penetration into conductors of magnetic fields from moving charges,'' J. Phys. A **8**, 742–750 (1975).
[17]Since Jones never confirmed that his results gave the known steady-current limit, which fitted so well with Furry's and with my results, I had been inclined to ignore his calculations, which involved considerable mathematical sophistication; because the low-velocity penetration results fitted with my own ideas about the Aharonov–Bohm effect, I restricted my attention to the low-velocity case and did not pursue the investigation further. However, recently, anonymous referees have criticized my calculations and pointed to Shier's skin-depth-limited results as representing the accurate point of view.
[18]T. H. Boyer, ''The Aharonov–Bohm Effect as a Classical Electromagnetic-Lag Effect; An Electrostatic Analogue and Possible Experimental Test,'' Nuovo Cimento **100B**, 685–701 (1987).

''IT IS WELL KNOWN THAT...''

A few years before [Fermi's] death, in a conversation in which I complained about the many subjects that are supposedly ''well known'' but in fact are just the opposite, Fermi suggested I make a note of any such questions I came across—such as validity conditions for Born's approximation, subtle questions of B and H in magnetism, signs in the energy expressions in thermodynamics, innumerable questions related to phases in quantum mechanics, and so on— and that when he retired, he would write a book giving all the explanations. Unfortunately, this did not come to pass. It would have been the best-seller in physics. Of course, there are many other physicists who could write such a book. I hope one of them will oblige, and write it with Fermi's clarity and simplicity.

Emilio Segrè, *A Mind Always in Motion—The Autobiography of Emilio Segrè* (University of California Press, Berkeley, 1993), p. 231.