AMERICAN JOURNAL of PHYSICS

A Journal Devoted to the Instructional and Cultural Aspects of Physical Science

VOLUME 18, NUMBER 1 JANUARY, 1950

The Teaching of Electricity and Magnetism at the College Level

I. Logical Standards and Critical Issues

(Report of the Coulomb's Law Committee of the A. A. P. T.)

TABLE OF CONTENTS

Appendices

List of Principal Symbols

F_e, $\mathbf{F}_e$ = net mechanical force of electrostatic origin including contribution from electrostriction.
f_e = direct electrostatic force of one charge on another.
q = electric charge (in formulating Coulomb's law).
q_c = conduction charge.
ρ_c, σ_c = volume and surface densities, respectively, of conduction charge.
ρ_p, σ_p = volume and surface densities, respectively, of polarization charge.
$\rho_t = \rho_c + \rho_p$ = volume density of *net* charge.
V = electric potential.
$\mathbf{P}$ = electric polarization.
κ_e = relative dielectric constant.
ϵ_0 = dimensional constant for D/E in free space.
$\epsilon = \epsilon_0 \kappa_e$.
χ_e = electric susceptibility (dimensionless).
I = conduction current.
$\mathbf{J}_c$, $\mathbf{J}_m$ = current densities for conduction currents and uncanceled Amperian currents, respectively.
$\mathbf{F}_m$ = net mechanical force of magnetic origin including contribution from magnetostriction.
$\mathbf{L}_m$ = net mechanical torque of magnetic origin.
p = pole strength (in formulating Coulomb's law for poles).
p_0, p_t = pole strength due to hard polarization only and to total polarization, respectively.
ρ_{m0}, ρ_m = pole density due to hard polarization only and to total polarization, respectively.
$\mathbf{M}_0$, $\mathbf{M}$ = magnetic polarization, hard and total, respectively.
μ_0 = dimensional constant for B/H in free space.
κ_m = relative permeability.
$\mu = \mu_0 \kappa_m$.
χ_m = magnetic susceptibility (dimensionless).
χ_m' = differential susceptibility of hard magnet.
$\eta = 1/\mu$ = reluctivity.
$\mathbf{n}$ = outward normal to surface.

1. Introduction

1.1 Scope and Plan of the Report

THE Coulomb's Law Committee was appointed in the fall of 1944 to formulate recommendations to the American Association of Physics Teachers based on a paper by Professor Carleton C. Murdock[1] criticizing the handling of Coulomb's law in some current elementary textbooks. The committee was further requested by President Lloyd W. Taylor to examine and report on the whole problem of the presentation of the fundamental concepts of electricity and magnetism in elementary courses in physics. It was suggested that it call attention to commonly occurring textbook errors and indicate ways of correcting them without making the texts unduly difficult.

The members of the committee approached their task from widely different points of view and with decidedly different biases. We have debated the issues at length. Certain crucial points have been subjected to detailed independent analyses by various members of the committee. We are pleased to report that on the basis of these investigations it has been possible to reach complete agreement as to the facts at issue. Regarding questions of taste and judgment our differences have been substantially reduced by protracted discussions. Where differences remain, the report presents both sides. In its present form it has the united support of all members of the committee.[2]

In attempting to carry out its assignment the committee has undertaken three major tasks,

[1] "Coulomb's law and the dielectric constant," *Am. J. Physics* **12**, 201–3 (1944).

[2] The constitution of the committee signing this report is as follows:

W. F. Brown, Jr.,	W. H. Michener,
N. H. Frank,	C. C. Murdock,
E. C. Kemble, *Chairman*,	D. L. Webster.

This list differs from the original one due to the regrettable resignation of Professor Norman E. Gilbert in 1946 and to the subsequent addition to the committee of Professors Murdock and Webster.

viz., (a) a careful examination of the facts of the theoretical situation with respect to the basic concepts of the classical theory of electricity and magnetism, (b) an examination of the practices commonly used in developing the basic concepts in current elementary textbooks of physics, and (c) a study of various possible correct methods of presenting the essential points of the theory to elementary and intermediate classes.

The report itself is arranged in three parts. Of these, *Part 1* is primarily devoted to an introductory discussion of the logical problems involved in teaching elementary physics on the college level. Here the reader will find a formulation of our general philosophy and also a resumé of our impressions regarding current logical standards in the writing of elementary college textbooks of physics. We did not find it necessary or feasible to make a collection of horrible examples of textbook errors, but have preferred to stress positive recommendations of acceptable practices.

Part 2 contains a detailed discussion of various critical issues in the logical development of electromagnetic theory. These include the original question of the proper application of Coulomb's law in electrostatics and magnetostatics, the problem of defining the various electric and magnetic vectors at points inside ponderable media, and finally a discussion of the roles to be assigned to magnetic poles and to Amperian currents in developing the theory of magnetism. In order to justify our conclusions as they affect elementary instruction, it has been necessary in Secs. 2.1 and 2.2 to delve into questions that go well beyond the scope of the elementary course. We believe, however, that the reader who wishes to do so will be able to skim much of the material in these sections without missing the main points.

Part 3 contains our positive recommendations in the form of two acceptable outlines, or synopses, for teaching the fundamental principles of electricity and magnetism, one using the traditional approach to magnetism by way of magnetic pole concept and the other using the new approach in which all magnetic fields are referred directly to electric currents of one kind or another. These outlines show how, in practice, we think the problem of logical standards might be handled in the teaching of electricity and magnetism. We hope the suggestions in Part 3 will be helpful to many teachers, but realize fully that there is nothing unique about our proposals.

Appendix A contains a derivation of fundamental importance for the discussion of forces on an electrified body in a fluid dielectric. *Appendix B* deals with the problem of relating the concept of the permanent magnet to the properties of actual ferromagnetic materials and especially with the effect of immersion in a magnetic fluid on the pole strength of a permanent magnet.

1.2 General Discussion of Logical Standards

From the point of view of the committee, the situation revealed by the study of current textbooks is rather discouraging. Although many authors have made a commendable effort to maintain high logical standards, the attempt to bring a complex and difficult theory down to the level of the poorly trained freshmen who throng so many of our colleges has too often led to confusion and absurdities. A full treatment of the theory involves the use of mathematical tools far beyond the reach of the college freshman and frequently unfamiliar to the authors of the textbooks themselves. Moreover, many of the students are unaccustomed to the precise use of abstract words and are unprepared to appreciate exact thinking. The problem of presenting the subject matter to elementary classes is, therefore, one that of necessity calls for some compromises between the requirements of logical rigor and precision, on the one hand, and the need for simplification as an aid to intelligibility, interest, and brevity, on the other. It is, perhaps, not surprising that the tendency has been to develop the ideas by successive approximations without great regard for logical rigor, treating special cases as if they were general, and relying in effect on the imitative solution of certain standardized classes of problems to act as a corrective to inaccurate statements in the text.

The situation seems to have been further complicated by the recent introduction of mks units, which, valuable as they may ultimately prove when fully standardized and universally accepted, involve a redefinition of some physical quantities and wholesale revision of basic formulas. The normal procedure for the author of an elementary textbook to use in checking the accuracy of his definitions is to refer to an advanced treatise of established merit. This procedure has been made difficult by the abandonment of the conventions of the older books based on the Gaussian units. Two of the more prominent recent advanced texts[3] in electricity and magnetism differ not only from the older books as a result of adopting rationalized mks units, but differ from each other in fundamental formulas because they adopt different definitions of magnetic moment. Important as it is that the language of physics be expressed in terms of a wise choice of conventions, one cannot help wondering whether the difference between one choice and another is sufficient to justify an indefinite period in which a multiplicity of conventions operate in competition. We therefore urge that every effort be made to terminate the present transition period as soon as possible.

Recognition of the difficulties that beset the elementary teacher should not, however, lead to the easy acceptance of current practices and confusions. At least, such is the opinion of this committee. We believe that great improvement in the logical form of our elementary textbooks should be possible without making them unduly formidable. In our opinion the difficult points should be faced rather than side-stepped. At the top of the list of desiderata for a textbook should be the requirement that every statement be accurate and meaningful. Physics teachers betray their trust if they accept the logical standards which students bring with them to the classroom.

In this connection it may be permissible for us to emphasize the importance for physics teachers of the present widespread interest in semantics. The laws of physics are all expressed in a theoretical language involving concepts that we have created, or invented, in order to explain facts of experience. The laws are meaningless unless the conceptual schemes which lie behind them are properly understood. Moreover, these conceptual schemes have their origin and justification solely in their relation to experimental realities. A crucial test of good physics teaching is then the grasp which the student acquires of the relation between the word and letter symbols from which the language of physics is built and the experiments which they interpret. To give good instruction in this sense is the semantic problem of the physics teacher. If the student conquers the semantic difficulties of his subject, he can be expected to grow in self-confidence and in ability to do creative thinking in the field. If he fails on the semantic level, the subject matter becomes a mumbo-jumbo of words and verbal forms which he vainly attempts to memorize in the conviction that understanding is beyond him. Surely the last state of the student who is exposed to a course in physics, but fails to make the essential connections between symbols and experience, is worse than the first. Success with respect to some items, failure with respect to others, is, of course, the rule.

The discussion of the meanings of the electric vectors **E** and **D** at points inside solid or fluid materials given in Sec. 2.4 is an attempt to straighten out what is essentially a semantic problem.

While many statements in textbooks are undoubtedly wrong because the authors themselves have an incorrect understanding of the facts involved, others are at fault because the authors have chosen to introduce a general subject by the consideration of a special case and have not made clear to the student that the preliminary discussion is of limited scope. In some textbooks the general case is not mentioned. In others the more general case appears later without adequate reference to the previous treatment of a special case. For example, an author may define the permeability μ as a *constant* occurring in Coulomb's law and later disclose without explanation that in some materials μ is *not constant.*

The committee suggests that *authors should take pains to make clear at all times any limitations on the generality of the treatment of special topics.* One possible way to do this would be to supplement each equation by an italicized statement of the conditions under which it is applicable. This statement should preferably be separated from the rest of the text.

Carelessly worded definitions constitute a serious weakness of many elementary textbooks. In general, the definitions of physical quantities should not be wholly arbitrary. Ordinarily the definition of a new physical quantity is based on an experimental situation into which the quantity must fit. *Some reference to this experimental situation should accompany the definition.* It is confusing, for example, to define the magnetic flux through a coil of wire as that quantity whose time-rate of change is equal to the induced emf

[3] J. A. Stratton, *Electromagnetic theory* (McGraw-Hill, 1941) and G. P. Harnwell, *Principles of electricity and electromagnetism* (McGraw-Hill, 1938). The *Principles of electricity and electromagnetism* was issued in a second edition in 1949.

divided by the number of turns of wire, without making clear to the student that there is ample experimental evidence that the quantity so defined has an existence independent of the emf and can be computed, at least approximately, from the cross-sectional area of the coil, the nature and temperature of the material which constitutes the core, the distribution of magnetic intensity inside the core, and the previous magnetic history of the core. Unless the background physical situation is explained, the definition hangs in the air. One could arbitrarily define a physical quantity X to be associated with a body B as a quantity whose time-rate of change is equal to the volume of B, but such a definition would have no point, for X would not be associated with any significant physical law.

A satisfactory definition of a new physical quantity should indicate how the quantity is to be measured, or, if it is not directly measurable, how the term representing it is to be used.[4, 5] Otherwise there is a logical gap before the definition can be put to work. Definitions by analogy, such as "Potential is electric pressure," do not meet this requirement and are to be avoided.

Finally, the definition of the unit for a physical quantity is not a satisfactory substitute for a definition of the physical quantity itself. Simply to say, for example, that the ampere is the current which, when passed through a single-turn circular coil of 1-cm radius, produces a force of $2\pi/10$ dynes on a unit magnet pole at its center, is not to give adequate instructions for the measurement of an arbitrary current.

Unfortunately, the setting up of definitions against their experimental background with complete logical rigor frequently leads to wordy statements boring to the average elementary student. Consider the definition of charge in electrostatics and its relation to Coulomb's law. It is customary to define the unit of charge and then to state Coulomb's law for point charges in a vacuum with no more ado. The law then gives an implicit completion of the definition of charge so that the student learns to use the concept correctly in the solution of problems, but the failure of the instructor to give a full and explicit definition works against the development of the habit of clear thinking in the mind of the student.

The following discussion may serve to represent the result of an attempt on the part of the ultraconscientious instructor to tell the story with complete logical rigor and clarity. Even to an experienced teacher of electrical theory it may be surprising to see how long the story is.

After exhibiting the elementary phenomena of frictional electricity and explaining the usual convention regarding the signs of charges, he confronts the class with the need for devising a workable scheme for the measurement of electric charges. He notes that any scheme of this kind within the framework of electrostatics must be based upon the forces which charges exert on one another. He observes that air, itself, under the influence of charged bodies, exerts on them forces which are quite small in comparison with the forces they exert on one another. Ideally the experiments to be considered should all be performed in a vacuum, but if they are actually performed in air and analyzed as if performed in a vacuum, the error will be small. On the basis of this introduction he proceeds to develop the experimental facts and conventions somewhat as follows:

(*a*) *Experimental fact.*—The force between two charged bodies is constant in time if the bodies are well insulated and are kept at a fixed distance apart.

(*b*) *Convention.*—The charges on a set of bodies are to be reckoned as constant so long as the forces of interaction depend only on the relative positions of the bodies.

(*c*) *Experimental fact.*—The force between two bodies carrying constant charges depends on their distance of separation r, on their shape, and on the way in which the charges are distributed over the bodies. However, if the linear dimensions of the bodies are small compared with r, the shape and the distribution are immaterial. In this case the force varies as $1/r^2$.[6]

(*d*) *Convention and corollaries.*—It is convenient to introduce the term "point charge" for an idealized charge concentrated at a geometric point. The force between point charges must then be supposed to vary as $1/r^2$ at all distances. A charge spread over a body of negligible dimensions approximates a point charge and can be treated like a point charge. Any distribu-

[4] See discussion in Sec. 2.4.

[5] Sometimes it is pedagogically helpful to supplement the definition of a new concept with information regarding interpretations not contained in the definition. Thus the definition of charge can be accompanied by the explanation that the quantity defined can ultimately be interpreted as a number of particles and the definition of magnetic moment may be supplemented by observing that a current loop has a moment which is equal to, or proportional to, the product of current and area.

[6] The instructor should note at this point that there are indirect methods of demonstrating the inverse-square law far more accurate than direct measurements of the forces on small charged bodies.

tion of charge can presumably be regarded as equivalent to a distribution of point charges.

(*e*) *Convention.*—The charges on two bodies shall be said to be *equal* in magnitude if they exert equal forces on a third charge equidistant from them, the distance being such that all three can be treated as point charges.

(*f*) *Experimental fact.*—If a charge X is subject to simultaneous electric forces from two or more charges A, B, C, $\cdots$, the net force which it experiences is the vector sum of the forces which it would experience if the several charges A, B, C, $\cdots$, were to act one at a time.

(*g*) *Corollary.*—If point charges A, B, $\cdots$, are brought together at a point P, the force which the combination of charges exerts on a point charge X is numerically equal to the algebraic sum of the force components along the line PX which they would separately exert if placed one after the other at P.

(*h*) *Corollary.*—If n equal point charges of the same sign are brought together at the point P, the force that the combination exerts on any other point charge X will be n times the force which any one of them would exert if acting by itself.

(*i*) *Definition.*—The ratio of the numerical values of any two point charges is to be identified with the ratio of the forces which they individually exert on a third point charge at the same distance r.

(*j*) *Definition.*—Let the unit point charge be defined (provisionally) as one which exerts a force of magnitude k_0/r^2 on an equal point charge at a distance of separation r. The numerical value to be assigned to k_0 varies from one system of units to another in a manner to be described hereafter.

The definitions of the unit point charge and of the ratio of the numerical values of any two point charges serve to define the numerical value of any point charge. The algebraic value q is obtained by multiplying the numerical value by ± 1 according to the usual sign convention. The value of an extended charge is defined by the same rule of measurement with the restriction that r must always be large compared with the linear dimensions of the bodies on which the charges are located.

(*k*) *Corollary.*—The total charge created by bringing together a number of separate charges is equal to the sum of their algebraic values expressed in terms of a common unit.

(*l*) *Corollary on* (*i*).—The force exerted by a point charge of magnitude $|q|$ on any other point charge X at a distance r is $|q|$ times the force exerted by a unit charge on X at the same distance.

(*m*) *Corollary on* (*c*) *and* (*l*).—The magnitude of the force exerted on each other by any two point charges of magnitude $|q|$, $|q'|$, respectively, whose distance apart is r, is

$$|F| = k_0 |q| \, |q'| / r^2. \qquad (1\text{–}1)$$

If the algebraic value of the force be defined as positive for repulsion and negative for attraction, this equation can be replaced by

$$F = k_0 q q' / r^2. \qquad (1\text{–}2)$$

This is Coulomb's law of force for point charges.

(*n*) *Corollary.*—The law (1–2) holds for extended charges provided that r is large compared with their maximum linear dimensions.

The instructor then proceeds with the specification of the value of k_0 in the system or systems of units which he chooses to employ.

In conclusion the instructor should point out that, although this outline gives a useful logical analysis of the way in which the theory is constructed from elementary observations, it does not describe the intuitive mental processes by which theories are ordinarily created; nor does it give an adequate conception of the wealth of evidence for the theory contained in the wide range of experimental tests that have been made to check the total consequences as well as the basic assumptions.

This method of presenting the first steps in electrostatics is admittedly involved and tedious. A teacher of elementary physics who desires to maintain a high logical level while holding the interest of his class is thereby confronted with a delicate pedagogical problem. Fortunately, his story can be made less formidable by introducing an operational definition of charge based on the Faraday ice-pail experiment and independent of direct reference to Coulomb's law. Or the presentation can be simplified by a quasi-deductive postulational approach. These permissible modifications of procedure are described in Sec. 3.21 of Part 3. Suffice it to say here that every teacher and textbook writer should realize that there are real hazards to clear thinking in a very radical cutting of the argument. Physics majors should at some time be exposed to the full story. Even in freshman courses complete failure to define the ratio of two charges is not to be recommended.

2. Critical Issues in the Logical Development of Electromagnetic Theory

2.1 Improper Use of Coulomb's Law in Electrostatics

The first issue confronting the committee relates to the use of Coulomb's law in the form[7, 8]

[7] To avoid misunderstanding, the equations in this report will ordinarily be written in both Gaussian and mksc units. The Gaussian system is the much-used mixed scheme in which all electrical quantities are expressed in unrationalized cgs electrostatic units, whereas the magnetic

(Gauss) "$F_e = qq'/\kappa_e r^2$," (2–1a)
or (Giorgi) "$F_e = qq'/4\pi\epsilon_0\kappa_e r^2$," (2–1b)

and the equivalent magnetic Eqs. (2–15) as the respective basic equations of the electrostatics of dielectric materials and the magnetostatics of magnetic materials. Although the corresponding equations for free space, Eqs. (2–2), can properly occupy a central position in the development of electrostatics and magnetostatics, it required little deliberation to disclose that the committee was from the beginning unanimously of the opinion that Eq. (2–1) and its magnetic analog Eq. (2–15) are not of general validity and are, therefore, not suitable for use as the basic equations of electrostatics and magnetostatics. In this the committee supports the primary thesis of the previously mentioned article by Murdock.[1] It seemed desirable, nevertheless, to make a critical examination of these two much-used, but dubious, equations. The conclusions to which we have been led are reviewed herewith, first for the electrostatic case and then, in Sec. 2.2, for the magnetostatic case.

Since the classical theories of the electrostatics and magnetostatics of material media have a much higher standing than any available experiments designed to give a direct check on the validity of the equations under consideration, our discussion must take a theoretical form.

quantities are expressed in the unrationalized cgs electromagnetic units.

The second system employed is the rationalized Giorgian meter-kilogram-second-coulomb system. Since there are two different sets of proposals regarding the units of magnetic pole-strength, magnetic moment, and magnetization within the mksc scheme, and since the merits of these two systems come up for discussion in the report (see Sec. 2.6), we indicate where necessary whether an equation follows the Sommerfeld or the Kennelly definitions.

In general, the different forms of the same relationship in different unit-schemes are labeled with the same equation number, but with different letters.

Vector quantities are indicated by bold-faced type.

Quotation marks around Eqs. (2–1) and (2–15) indicate that these equations are introduced not as statements of fact, or of accepted theory, but as assertions whose validity is in question.

The reader is warned that on account of an error in the preparation of the manuscript there are no equations for the numbers (2–30), (2–31), and (2–32).

[8] The choice of symbols for a report that uses both Gaussian and Giorgian units is a bit difficult. To avoid ambiguities we denote the relative dielectric constant and the relative permeability by κ_e and κ_m, respectively. The basic mksc dimensional constants ϵ_0 and μ_0 represent the ratios **D**/**E** and **B**/**H** for free space. Although ϵ and μ are now commonly used for the products $\epsilon_0\kappa_e$ and $\mu_0\kappa_m$, respectively, it has seemed best for our present purposes to write out these products explicitly.

The reader will recollect that the classical theory of electrostatics, in harmony with the Lorentz electron theory, postulates that Coulomb's law in the primitive form

(Gauss) $f_e = qq'/r^2$, (2–2a)
or (Giorgi) $f_e = qq'/4\pi\epsilon_0 r^2$, (2–2b)

applies in a ponderable medium as well as in a vacuum, provided we identify f_e with the *direct* force of one charge on another. The apparent influence of a ponderable medium on the observable force F_e exerted by one charge on another is assumed to be due to the electrical structure of matter and to the distribution of electric polarization created in the medium by an electric field. The electric polarization **P** gives rise to polarization charges in addition to the conduction charges. The potential V and the electric intensity **E** are to be computed as in a vacuum, except that the polarization charge densities are included with the conduction charge densities to give total charge densities which are the sources of the field.

The observable force $\mathbf{F}_e$ acting on an element of conduction charge in the presence of dielectrics is the vector sum of the direct forces $\mathbf{f}_e$ exerted on it by other conduction charges, the direct forces exerted on it by the polarization charges, and a mechanical force of electrostrictive origin.

The relation between the polarization vector **P** and the electric intensity **E** depends on the nature of the medium. If **P** is a homogeneous linear function of **E**, the medium is conveniently designated as *linear*. If it is nonconducting, we call it a *dielectric*. If it is homogeneous in the sense that its properties are independent of the positional coordinates, and if it is electrically isotropic, **P** is proportional to **E** and throughout the medium we have an electrical susceptibility and a dielectric constant with unique values. Most treatments of dielectric behavior apply only to such media. Clearly all these restrictions are necessary if Eq. (2–1) is to be meaningful. On the other hand, there is no fundamental difficulty in dealing with crystalline media which are electrically anisotropic, if a linear vector operator (dyadic) is used to relate **P** and **E**. There are also media to which the linearity postulate, as formulated above, does not apply, e.g., piezoelectric and pyroelectric crystals and materials which show saturation effects.

It follows from these considerations that *Eq. (2–1) is not applicable throughout the whole range of electrostatic problems and, on that account, cannot be considered fundamental.* The meaning and validity of the equation for problems involving only homogeneous, isotropic, and linear dielectrics—let us say *Class A dielectrics*—remains to be discussed.[9]

Since the point charges of Coulomb's law are ideal concepts, it is important to stipulate the physical interpretation of Eq. (2–1) in connection with a discussion of its validity. The simplest and most direct interpretation identifies F_e with the limit of the force of interaction between two small charged bodies immersed in a fluid dielectric when the linear dimensions of the bodies are made very small compared with their distance of separation. This may be designated the *point-charge interpretation.*

A second interpretation identifies F_e with the differential force between two charge elements in a continuous distribution of charge. The equation can then be written in vector form as[10]

$$\text{(Gauss)} \qquad d\mathbf{F}_e = \frac{dq_c dq_c'}{\kappa_e r^2}\mathbf{1}_r, \qquad (2\text{–}3\text{a})$$

$$\text{or (Giorgi)} \qquad d\mathbf{F}_e = \frac{dq_c dq_c'}{4\pi\epsilon_0\kappa_e r^2}\mathbf{1}_r, \qquad (2\text{–}3\text{b})$$

where $\mathbf{1}_r$ is a unit vector directed from the source-charge dq_c' to the charge dq_c which experiences the force. This may be designated the *continuous charge interpretation.* Finally we have the possibility, needing no explicit discussion, of the forces of interaction between a point charge and a continuous distribution of charge.

Whether we have in mind point charges or continuous distributions it is convenient to resolve the problem of the validity of the equations in question into two subproblems by introducing the electric intensity $\mathbf{E}'$ due to q_c' as an intermediate factor. Equation (2–3) is then replaced by

$$\text{(Gauss)} \qquad d\mathbf{E}' = \frac{dq_c'}{\kappa_e r^2}\mathbf{1}_r, \qquad (2\text{–}4\text{a})$$

$$\text{or (Giorgi)} \qquad d\mathbf{E}' = \frac{dq_c'}{4\pi\epsilon_0\kappa_e r^2}\mathbf{1}_r, \qquad (2\text{–}4\text{b})$$

$$\text{and} \qquad d\mathbf{F}_e = dq_c\mathbf{E}'. \qquad (2\text{–}5)$$

Our first question is accordingly, "What is the standing of Eq. (2–4) for point charges q_c' and for continuous distributions of charge?" In answer, we first point out that, according to the basic definition of the theory,[11]

$$\text{(Gauss)} \qquad \mathbf{E} = \int \frac{\mathbf{1}_r dq_t}{r^2}, \qquad (2\text{–}6\text{a})$$

$$\text{or (Giorgi)} \qquad \mathbf{E} = \int \frac{\mathbf{1}_r dq_t}{4\pi\epsilon_0 r^2}. \qquad (2\text{–}6\text{b})$$

---

[9] In Maxwell's *Treatise on electricity and magnetism* (Oxford, 1904), 3rd ed., Coulomb's law is found only in the form (2–2) in passages written by Maxwell himself; moreover, both in the electrostatic statement of the law (Vol. 1, pp. 45–46) and in the magnetostatic (Vol. 2, p. 3), the original wording contains no mention of the medium. At both these points the editor, J. J. Thomson, felt it necessary to insert (in curly brackets; see Vol. 1, p. xvi) the qualification that the medium is supposed to be air; and on p. 122 of Vol. 1, at the conclusion of the discussion of systems of conductors in a liquid, he inserted the following sentence: "It follows from the preceding investigation that the force between two electrified bodies surrounded by a medium whose specific inductive capacity is K is $ee'/Kr^2\cdots$." It is clear that Maxwell himself did not consider Eq. (2–1) either basic or important and that "the force between two bodies" was to him an unambiguous concept, not requiring any mention of the intervening medium.

A reading of Maxwell's preface and introductory chapters makes it evident why this was so. Maxwell's object was the mathematical formulation of Faraday's concepts, which located all action in the medium, and the demonstration that when so formulated they were *mathematically equivalent* to the theory based on action at a distance. The two theories, however, are *conceptually distinct.* When one speaks of the force between two bodies, one is using the action-at-a-distance theory; this theory, in Maxwell's own words (Vol. 1, p. 155), "supposes the electric force to act directly between bodies at a distance, no attention being bestowed on the intervening medium." If one wishes to use the Faraday-Maxwell theory, one must—whether in a vacuum or in a material dielectric—speak not of the force between the bodies but of the force between adjacent portions of the stressed medium. The idea that the "force between the charges" can "depend on the medium" represents an incongruous mixture of the two theories, never used by Maxwell but introduced later by his followers. Equation (2–1) occurs in Heaviside's papers, as well as in the editorial insertion by Thomson mentioned above.

[10] From this point on we shall use subscripts c, p, and t, respectively, for conduction charges, polarization charges, and total charges.

[11] In Eqs. (2–4) and (2–5) the primes are used to distinguish between the external charges q_c' (producing the external field $\mathbf{E}'$) and the charges q_c on which the force $\mathbf{F}_e$ acts. In Eqs. (2–6)–(2–9) this distinction is dropped because the relations hold for any charges and corresponding field.

Here dq_t is an element of *total charge*, including polarization charge as well as conduction charge. The integral of Eq. (2–6) as written is intended to include contributions from volume densities, surface densities, line densities and, in the limit, point concentrations of charge.

Evidently the replacement of Eq. (2–6) by Eq. (2–4) is valid if, and only if, the charge elements dq_t are everywhere equal to the corresponding conduction charges dq_c divided by κ_e. But if, for simplicity, we confine our discussion for the moment to volume distributions of charge and stick to Gaussian units, the basic relation between the two kinds of charge[10] is

$$\text{(Gauss)}\quad \text{div}\mathbf{E}/4\pi=\rho_t=\rho_c+\rho_p=\rho_c-\text{div}\mathbf{P}. \quad (2\text{–}7)$$

To justify the replacement we must first assume that we have to do with linear isotropic dielectrics. This special assumption yields

$$\left.\begin{aligned}&\text{(Gauss)}\\ &\qquad \text{div}\mathbf{E}/4\pi=\rho_c-\text{div}(\chi_e\mathbf{E})\\ &\qquad\qquad =\rho_c-\chi_e\,\text{div}\mathbf{E}-\mathbf{E}\cdot\text{grad}\chi_e,\end{aligned}\right\}\quad (2\text{–}8)$$

and, since $1+4\pi\chi_e=\kappa_e$,

$$\text{(Gauss)}\qquad \rho_t\kappa_e=\rho_c-\mathbf{E}\cdot\text{grad}\kappa_e. \qquad (2\text{–}9)$$

Here χ_e denotes the electrical susceptibility. Thus the validity of Eq. (2–4) requires not only the restriction of the discussion to linear isotropic dielectrics, but also that $\mathbf{E}\cdot\text{grad}\kappa_e$ shall vanish throughout the field. If surfaces of discontinuity occur in the field the normal component of the intensity on these surfaces must vanish. *In practice this means that all parts of space in which there is a field must be filled with a single Class A dielectric if Eq. (2–4) is to be valid.* Conductors in electrostatic equilibrium may be present since the field inside them vanishes. Of course, Eq. (2–4) can be used to compute a useful approximation when the foregoing condition is not completely fulfilled provided that the field is sufficiently weak at the boundaries of the homogeneous dielectric and provided that the point at which the intensity is being computed is at a sufficient distance from such boundaries. The restrictions are very severe, however. *The equation is definitely not of general applicability if the dielectric is inhomogeneous, electrically anisotropic, or nonlinear.*

We turn now to Eq. (2–5) and consider first applications to extended charge distributions q_c on bodies immersed in Class A fluid dielectrics. The total force $\mathbf{F}_e$ acting on such a body X consists of two parts: the force due to *direct* electrical interaction with other charged bodies and with the fluid; and the resultant mechanical force acting on X due to the asymmetry of the distribution of mechanical pressure around it resulting from electrostriction in the fluid. Investigation of this force $\mathbf{F}_e$ may be carried out by energy methods or by direct examination of the force contributions from various sources. The usual procedure is to use the energy method to work out the body force and surface forces acting in an electrostatic field and responsible for electrostriction. These can then be integrated directly over the body or replaced by a stress system to compute the total force $\mathbf{F}_e$ to be balanced by mechanical supports. We shall concern ourselves here solely with this total force, ignoring the problem of its distribution over X.

If the vectors $\mathbf{D}$ and $\mathbf{E}$ have the same direction in the body X, so that we can use the relation $\mathbf{D}=\kappa_e\mathbf{E}$, and if we postulate the necessary transition layers to replace surface contributions by volume contributions, the total force $\mathbf{F}_e$ can be computed from the total electric intensity $\mathbf{E}$ by the equation[12]

$$\text{(Gauss)}\quad \mathbf{F}_e=\int_X\left[\rho_c\mathbf{E}-\frac{E^2}{8\pi}\,\text{grad}\kappa_e\right]d\tau; \qquad (2\text{–}10)$$

where $d\tau$ is an element of volume. It is immediately evident from this equation that Eq. (2–5) cannot be of general validity. In fact, as is well known, an uncharged dielectric body whose dielectric constant differs from that of the surrounding fluid will experience a net force different from zero when placed in a nonuniform external electric field.

An alternative and, in some respects, more general equation for the total force acting on the body X is

$$\mathbf{F}_e=(\kappa_e)_A\int_X\mathbf{E}\rho_t d\tau, \qquad (2\text{–}11)$$

where $\mathbf{E}$ is again the total electric intensity, ρ_t is

[12] Cf. J. A. Stratton, Ref. 3, Secs. 2.26, 2.29, and especially Eq. (13), p. 158.

the total charge density including the polarization charge, and $(\kappa_e)_A$ is the relative dielectric constant of the Class A fluid which surrounds X. The integration is to be extended over all charges in and adjacent to the surface of X including polarization charges.[13]

There are three important special cases in which the general equation (2–11) can be reduced to the special form envisaged in Eq. (2–5) for an extended distribution of charge, namely,

$$\mathbf{F}_e=\int_X \mathbf{E}'\rho_c d\tau. \quad (2\text{–}12)$$

The first case is the one already discussed in which X has the same dielectric constant as the surrounding fluid. One qualification only is needed for this case in view of the fact that in Eq. (2–12) $\mathbf{E}'$ is the intensity due to the *external* distribution of conduction charge q_c'. The total intensity can be resolved into the sum of three terms as follows:

$$\mathbf{E}=\mathbf{E}_s+\mathbf{E}_0'+\Delta\mathbf{E}', \quad (2\text{–}13)$$

where $\mathbf{E}_s$ is the self-field of the total charge distribution on X, $\mathbf{E}_0'$ is the external field before the body X was placed in position, and $\Delta\mathbf{E}'$ is the field due to the alteration in the net charge distribution on other bodies due to the introduction of X and its charge. Since the self-field can exert no resultant force on X it is correct to substitute $\mathbf{E}_0'+\Delta\mathbf{E}'$ for $\mathbf{E}$ in Eq. (2–10) after eliminating the term involving gradκ_e. Hence we must understand *either* that $\mathbf{E}'$ in Eq. (2–12) means the external field *as modified* by the presence of X and its charges (i.e., $\mathbf{E}_0'+\Delta\mathbf{E}'$), *or else* that the size of X, its dielectric constant, and its charge distribution are all so chosen that $\Delta\mathbf{E}'$ is negligible.

The second case is the one in which the body X is a conductor carrying a surface density of conduction charge σ_c. The corresponding surface density of total charge is $\sigma_c/(\kappa_e)_A$. In this case Eq. (2–11) reduces at once to Eq. (2–12) or to the equivalent surface integral. The same remarks about the interpretation of $\mathbf{E}'$ apply in this case as in the previous one.

The third case is that in which $\mathbf{E}_0'$ is constant and $\Delta\mathbf{E}'$ is negligible. Replacing $\mathbf{E}$ in Eq. (2–11) by $\mathbf{E}_0'$ we obtain

$$\mathbf{F}_e=(\kappa_e)_A\mathbf{E}_0'\int_X \rho_t d\tau=(\kappa_e)_A\mathbf{E}_0'\int_S E_n dS$$

$$=\mathbf{E}_0'\int_S D_n dS=\mathbf{E}_0' q_c,$$

where dS is an element of surface. This completes our examination of the applicability of Eqs. (2–5) and (2–12) to cases in which the charge q_c is distributed.

We turn finally to the point-charge case which most teachers of elementary courses envisage in connection with Coulomb's law. It is not difficult to see in the light of Eq. (2–11) that if we allow the body X to shrink to zero size about some fixed point P, the charge in each volume element being carried over at each stage into an equal charge in the corresponding reduced volume element, then the force $\mathbf{F}_e$ will approach the value $q_c\mathbf{E}'$, where $\mathbf{E}'$ is the external field at the point P in the presence of the point charge q_c. This is because the multipole moments of the distribution on X approach zero as the linear dimensions of the body are reduced.[14] We conclude that Eq. (2–5) is valid for a small charged body immersed in a Class A fluid dielectric in the sense that

$$\mathbf{E}'=\lim_{a\to 0}(\mathbf{F}_e/q_c),$$

where a is a linear dimension of the body and the limiting process indicated is to be carried out in the manner just described. If $\mathbf{E}_0'$ is desired, it will suffice to let q_c as well as a approach zero. Then $\mathbf{E}'$ passes over into $\mathbf{E}_0'$, and we can write

$$\mathbf{E}_0'=\lim_{\substack{a\to 0\\ q_c\to 0}}(\mathbf{F}_e/q_c). \quad (2\text{–}14)$$

Thus the theory tells us that $\mathbf{E}$ *should be equally measurable in a Class A dielectric fluid and in a*

[13] For a brief derivation of Eq. (2–11) and a proof of Eq. (2–10) based on Eq. (2–11), see Appendix A.

[14] Introducing $\bar{\rho}_t$ for the mean value of ρ_t on X and defining $\delta\rho_t$ by the relation $\delta\rho_t=\rho_t-\bar{\rho}_t$, we can replace Eq. (2–11) by

$$\mathbf{F}_e=(\kappa_e)_A\left[\bar{\rho}_t\int_X \mathbf{E}'d\tau+\int_X \mathbf{E}'\delta\rho_t d\tau\right].$$

If the linear dimensions of X are reduced to zero the first term will approach $q_c\mathbf{E}(P)$ and the second term, which involves only the multipole forces, will approach zero.

vacuum by the traditional force-per-unit-test-charge method.[15]

This conclusion is, of course, in harmony with the interpretation of **E** as the force-per-unit-charge acting on the movable charges in the theory of current electricity. It can be simply checked by means of a mental experiment if one postulates that the dielectric model is to be in harmony with the principle of conservation of energy. Imagine a fluid dielectric in a container with insulating side walls and charged conducting plates A and B at top and bottom. Let the space around the container be evacuated. Then it becomes possible in principle to carry a small test-charge slowly from A to B either through the fluid or through the evacuated space. For the energy to be conserved, the work done against the electrostatic forces must be the same in the two cases. Hence, the work done in the fluid as well as in the vacuum must equal the product of the charge and the potential difference through which it is carried.[16]

As a corollary on our last result and the previous discussion of the validity of Eq. (2–4), we infer that Eq. (2–1) holds for "point charges" in an infinite, Class A dielectric fluid. It would *not* hold for point charges near the interface of two dielectrics or near a conducting surface.

In view of the limitations on the validity and usefulness of Eq. (2–1) and the likelihood of misunderstanding regarding it, we believe that it would be better not to introduce it into courses in general physics for beginners.

2.2 *Improper Use of Coulomb's Law in Magnetostatics*

We turn next to the magnetic form of Coulomb's law analogous to Eq. (2–1) and given by

(Gauss) $$\text{``}F_m=\frac{pp'}{\kappa_m r^2}\text{''} \qquad (2\text{–}15\text{a})$$

(Giorgi-Kennelly) $$\text{``}F_m=\frac{1}{4\pi\mu_0}\frac{pp'}{\kappa_m r^2}\text{''} \qquad (2\text{–}15\text{b})$$

(Giorgi-Sommerfeld) $$\text{``}F_m=\frac{\mu_0}{4\pi}\frac{pp'}{\kappa_m r^2}.\text{''} \qquad (2\text{–}15\text{c})$$

(Quotation marks indicate that the validity of this equation is in question. See footnote 7.)

The classical theory of magnetostatics starts with Coulomb's law in the form analogous to Eq. (2–2):

(Gauss) $$f_m=\frac{pp'}{r^2} \qquad (2\text{–}16\text{a})$$

(Giorgi-Kennelly) $$f_m=\frac{1}{4\pi\mu_0}\frac{pp'}{r^2} \qquad (2\text{–}16\text{b})$$

(Giorgi-Sommerfeld) $$f_m=\frac{\mu_0}{4\pi}\frac{pp'}{r^2}. \qquad (2\text{–}16\text{c})$$

Although this equation usually is stated, as here, for point poles and is illustrated by Coulomb's experiments, the theory does not depend on the possibility of approximating point poles experimentally; and in precise analyses of experimental situations, the poles are supposed distributed over surfaces and throughout volumes. It is postulated that Eq. (2–16) gives the direct force f_m of one pole on another, in a ponderable medium as well as in a vacuum; the apparent influence of a medium on the observable force $\mathbf{F}_m$ is attributed to magnetization (magnetic polarization) of the medium and to magnetostriction. The mathematical theory is formally analogous to that of electrostatics, with the magnetizing force, or magnetic intensity, **H** replacing the electric intensity **E**.[17] It follows at once that the primary conclusion reached in Sec. 2.1 for electrostatic theory carries over to magnetostatics. *The equation under investigation, Eq. (2–15), is not generally valid and is anything but fundamental.* In fact, as will appear, the standing of Eq. (2–15) in magnetostatics is much less respectable than the standing of the corresponding Eq. (2–1) in electrostatics.

[15] In practice it will, of course, suffice to make the dimensions of the test-body small enough to eliminate multipole forces and to make the test-charge small enough to eliminate image-forces. These restrictions apply equally to the measurement of electric intensity in a vacuum or in a fluid.

[16] It may be observed that when Eq. (2–5) is known to be correct, the resultant force of electrical origin due to the action of **E**′ on the polarization charges in, and immediately adjacent to, the body under consideration is canceled by the resultant mechanical force due to electrostriction in the fluid. See D. L. Webster, *Am. Physics Teacher* **2**, 149–151 (1934).

[17] It is, of course, possible to develop magnetic theory from the beginning in terms of electric currents—a procedure discussed in Sec. 2 and illustrated in detail in Outline II, Sec. 3.3. The two approaches lead to the same results.

Magnetostatics as traditionally developed in terms of the magnetic pole concept is a much clumsier subject than electrostatics. In the absence of a magnetic equivalent of conduction charge the roles of ρ_c and σ_c in electrostatics are taken over by the volume and surface densities of "permanent" magnetic pole strength, ρ_{m0} and σ_{m0}, respectively. These quantities are defined by the equations[17a]

$$\rho_{m0} = -\mathrm{div}\mathbf{M}_0, \quad \sigma_{m0} = \mathbf{n}\cdot\mathbf{M}_0, \qquad (2\text{–}17)$$

where $\mathbf{M}_0$ is that part of the magnetization $\mathbf{M}$ identified as "permanent." It follows that the important electrostatic case of conductors separated by a dielectric has no parallel in magnetostatics, and that the analog of Eq. (2–4) does not hold rigorously except in a physically unrealizable limit. If a magnet is immersed in a Class A magnetic fluid, that is, a homogeneous, isotropic, and magnetically linear fluid, the interior of the magnet is inaccessible to the fluid and is not field-free. Therefore the conditions for validity of the magnetic form of Eq. (2–4) are not satisfied. The case of small charged bodies also has no exact parallel; the closest approximation is realized by using slender rods of magnetically hard material subjected to suitable magnetic preparation. It is to such magnets, if to any, that the analogs of Eqs. (2–1), (2–4), and (2–5), *viz.*, Eq. (2–15), and

$$\left.\begin{aligned} &\text{(Gauss)} && \frac{dp'}{\kappa_m r^2}\mathbf{1}_r && (2\text{–}18\text{a})\\ &\text{(Giorgi-Kennelly)} && \frac{dp'}{4\pi\mu_0\kappa_m r^2}\mathbf{1}_r && (2\text{–}18\text{b})\\ &\text{(Giorgi-Sommerfeld)} && \frac{dp'}{4\pi\kappa_m r^2}\mathbf{1}_r && (2\text{–}18\text{c})\end{aligned}\right\} = d\mathbf{H}'$$

$$\left.\begin{aligned} &\text{and (Gauss or Giorgi-Kennelly)} && \mathbf{H}'dp && (2\text{–}19\text{ab})\\ &\text{(Giorgi-Sommerfeld)} && \mu_0\mathbf{H}'dp && (2\text{–}19\text{c})\end{aligned}\right\} = d\mathbf{F}_m$$

should apply. Actually the validity of these equations in the idealized limiting case of permanent, infinitesimally thin magnets immersed in a large volume of Class A magnetic fluid can be verified without difficulty. They do not hold for magnets of other shapes, however, and Eq. (2–18) fails even for a needle-magnet if inhomogeneities in the medium have to be taken into account. The valid uses of Eq. (2–18) are so unimportant and the possibility of misuse is so great that its introduction into an elementary course is pretty clearly inadvisable.

[17a] Explicit definitions of magnetization **M** and magnetic induction **B** are given in Sec. 3.23.

Equation (2–19) in a form analogous to that of Eq. (2–14) may be used to introduce the magnetic intensity concept in an elementary course, but if it is so used the limitation to very thin slender magnets should be clearly pointed out.

Particular attention is called to the fact that the shape of a magnet has an important effect on the change in its field that is caused by immersing it in a magnetic fluid and also on the change in the force and torque experienced when it and other sources of magnetic field are immersed in a magnetic fluid.

A good introductory description of the effect of immersion on the field of a permanent magnet is provided by the elementary theory of the magnetic circuit. The field of a magnet may be regarded as the result of the action of a magnetomotive force through a reluctance. If the external reluctance is decreased by immersing the magnet in a paramagnetic fluid, the total flux increases, but the potential drop across the external part of the circuit decreases; that is, the external **B** increases but the external **H** decreases. In the limiting case of long thin needle-magnets magnetized longitudinally, the internal reluctance far exceeds the external and therefore determines the flux; consequently, the external flux is independent of the reluctivity of the fluid and the potential drop is proportional to the reluctivity, that is, the fluid leaves **B** unaffected and decreases **H** in the ratio $1/\kappa_m$. Conversely, in the limiting case of a thin disk-magnet magnetized transversely (magnetic shell) the internal reluctance is negligibly small and, therefore, the magnetomotive force acts directly on the external part of the circuit; consequently, the external potential drop is independent of the reluctance of the fluid and the flux is proportional to the permeability; that is, the fluid leaves **H** unaffected and increases **B** in the ratio κ_m.

This approach to the problem yields a generally correct account of the behavior of actual magnets and indicates that the needle-magnet is an extreme case—interesting because it gives the closest magnetic analog of the point charge, *but no more representative of magnets in general than the opposite extreme, the disk-magnet.*

Elementary magnetic circuit theory also provides a rough description of the behavior when the change of external reluctance is produced by bringing ferromagnetic objects into the neighborhood—a case of much greater practical importance than that of immersion in a fluid. However, such a description must not be misinterpreted as a substitute for field theory. A magnet is not, in general, bounded by equipotential surfaces and lines of induction. Therefore, the internal and external reluctances are not quantities that can be defined precisely until the flux distribution has been found. Moreover, a change of external reluctance usually changes the internal flux distribution and therefore somewhat alters the internal reluctance. An ultimate analysis by field theory is therefore as necessary with this approach as with one based on poles.

The conclusions just indicated in terms of magnetic circuit theory can be restated in another form as follows: Let p_0 and p_t denote, respectively, the vacuum pole strength and the total, or net effective, pole strength of a magnet when immersed in a Class A paramagnetic fluid. (Here p_0 is assumed to be constant for a given magnet, whereas p_t, which includes the pole strength induced in the adjacent medium, depends on the medium. Appendix B contains a more detailed discussion of the ratio p_t/p_0 for real as well as ideal magnets.) Then our first conclusion is that in the case of an ideal needle-magnet $p_t = p_0/\kappa_m$. In the second place, since the external magnetic field of a magnetic shell is proportional to the product of the pole strength per unit area and the thickness, the induced pole density in the medium at the surface of an immersed magnetic shell is always negligible in comparison with the vacuum pole-density. Hence the pole strengths p_t and p_0 are equal for such a shell. The **H**-field of a magnetic shell is accordingly independent of the permeability of a surrounding fluid.[18]

The law of action and reaction is sufficient to establish the proposition that the differences between the effects of immersion on the fields of magnets of different shapes must be reflected in corresponding differences in the forces which the two classes of magnets experience when subjected to external fields and immersed.

To work out these differences we may take over from electrostatic theory[19] the force equation (2–11) whose magnetic equivalent is

(Gauss or Giorgi-Kennelly)

$$(\kappa_m)_A \int_X \mathbf{H}dp_t \quad (2\text{–}20\text{ab})$$

(Giorgi-Sommerfeld)

$$\mu_0(\kappa_m)_A \int_X \mathbf{H}dp_t \quad (2\text{–}20\text{c})$$

$$= \mathbf{F}_m.$$

Here, as in Eq. (2–11), the subscript A is used to denote a property of the Class A fluid; dp_t denotes an element of total pole strength due to both the permanent and variable components of the magnetization. The integration is to extend over the volume of the magnet X that experiences the force $\mathbf{F}_m$ and over the surface distribution of poles induced in the fluid adjacent to the surface of X.[20]

[18] Since the **H**-field of an electric current is unchanged by immersion in a magnetic fluid, the familiar equivalence between a linear current and a corresponding magnetic shell is undisturbed if shell and circuit are both "dunked" in magnetic fluid.

[19] A complete mathematical correspondence can be established between the computation of the resultant forces and torques on charged bodies in a Class A dielectric fluid as described in Appendix A and the computation of the corresponding forces and torques on magnets in a Class A magnetic fluid. In this correspondence **E** is replaced by **H**, the total electric charge density ρ_t is replaced by the total magnetic pole density $\rho_{mt} = -\text{div}\mathbf{M}$, and the conduction charge density ρ_c is replaced by the contribution of the "hard" polarization $\mathbf{M}_0$ to the total pole density. In Appendix A we replace the charged body X by an imaginary body X^* having the same total charge density but with a uniform relative dielectric constant equal to that of the surrounding Class A fluid. Similarly in the magnetic case we introduce an imaginary magnet having the same total pole density as X but provided with a "soft" polarization $\mathbf{M}_s = \mathbf{M} - \mathbf{M}_0$ which is proportional to **H**. By giving this substitute magnet a suitable susceptibility for soft polarization we can make its effective permeability the same as that of the surrounding Class A magnetic fluid. Thus the force equation (2–20) and a torque equation corresponding to Eq. (A–b) are readily established.

[20] There are a number of important equivalent expressions for the force and torque derivable from Eq. (2–20) and the magnetic analog of Eq. (A–b) of Appendix A by mathematical transformation. Assuming such transition

It is not permissible to include only one pole in the volume of integration, or in any way to fail to include the whole magnet and a thin layer of fluid around it. To see why, one may refer to Appendix A, where Eq. (2–11) is derived, and specifically to its use of the condition that the whole surface bounding the volume of integration is in a Class A fluid. As this condition is used there, the word *fluid* is as important as the uniform susceptibility implied in *Class A*.

On this account Eq. (2–20) and the corresponding torque equation do not establish the existence of forces concentrated at poles. Neither do the equations for force and torque in terms of Amperian currents (curl**M** of footnote 20) establish the existence of forces on the sides of the magnet, where the uncanceled Amperian currents are concentrated. On the contrary, we must assume that, aside from minor contributions due to the fluid, the bulk of the forces and torques acting on a magnet are spread throughout its volume along with the electrons whose spins account for the magnetization.

In problems having to do with the mechanical behavior of magnets either style of concentration may be assumed and either works as well as the other. But this is only because of the great rigidity of ferromagnetic materials; so it is appropriate that this rigidity, by contrast with the fluidity of the Class A fluid, is just what makes Eq. (2–20) inapplicable to anything less than a whole magnet.

In Eq. (2–20) we can substitute for the total intensity **H** the part **H**′ due to magnetization or currents outside X. Finally, we express **H**′ in terms of the magnetic induction **B**′ which would have existed at the point under consideration if the magnet X were taken out of the fluid. In this way we obtain the equivalent equation

$$\left.\begin{array}{ll}\text{(Gauss or Giorgi-Sommerfeld)} & \displaystyle\int_X \mathbf{B}' dp_t \\ \text{(Giorgi-Kennelly)} & \displaystyle\mu_0^{-1}\int_X \mathbf{B}' dp_t\end{array}\right\} = \mathbf{F}_m. \qquad \begin{array}{l}\text{(2–21ac)}\\ \text{(2–21b)}\end{array}$$

Similarly the torque $\mathbf{L}_m$ acting on X can be expressed in the form

$$\left.\begin{array}{ll}\text{(Gauss or Giorgi-Sommerfeld)} & \displaystyle\int_X \mathbf{r}\times\mathbf{B}' dp_t \\ \text{(Giorgi-Kennelly)} & \displaystyle\mu_0^{-1}\int_X \mathbf{r}\times\mathbf{B}' dp_t\end{array}\right\} = \mathbf{L}_m. \qquad \begin{array}{l}\text{(2–22ac)}\\ \text{(2–22b)}\end{array}$$

These apparently simple equations are deceptive and must be used with caution. They are valid only in a Class A fluid—a restriction not immediately evident as in the parallel form, Eq. (2–20). Moreover, it is *not* permissible to replace **B**′ in Eqs. (2–21) and (2–22) by the actual net magnetic induction **B**. This is because within the magnet the product $(\kappa_m)_A\mathbf{H}$, which occurs in Eq. (2–20), is very different from **B**, and there is no certainty that the difference would integrate to zero. Likewise, the **B** in the equations containing curl**M** in footnote 20 can be replaced by **B**′, and then $\mathbf{B}'/(\kappa_m)_A$ by **H**′; but it is *not* permissible for *this* **H**′ to be replaced by **H**.

Moreover, care must be used with the definitions of **H**′ and **B**′ as vectors which would have existed if the magnet were removed. Before this removal any magnetization induced by this magnet in other bodies would have to be *frozen into* them, so that it would contribute to **H**′ and **B**′.

Altogether, if Eqs. (2–21), (2–22) and their Amperian analogs are used, these hidden restrictions and those of Eq. (2–20) must all be kept in mind; but Eq. (2–20) and its Amperian analog are subject only to the more obvious restrictions, that the region of integration must include the whole magnet X and a surface film of a single surrounding Class A fluid. In any case one must

---

layers as are necessary to convert surface integrals into volume integrals and restricting ourselves to Gaussian units, we may write, for example:

$$\mathbf{F}_m = (\kappa_m)_A\int(-\mathrm{div}\mathbf{M})\mathbf{H}d\tau = [1/(\kappa_m)_A]\int[(\mathrm{curl}\mathbf{M})\times\mathbf{B}]d\tau,$$

$$\mathbf{L}_m = (\kappa_m)_A\int[\mathbf{r}\times(-\mathrm{div}\mathbf{M})\mathbf{H}]d\tau$$

$$= \frac{1}{(\kappa_m)_A}\int[\mathbf{r}\times\{(\mathrm{curl}\mathbf{M})\times\mathbf{B}\}]d\tau.$$

remember that the polarization of the surrounding medium alters the distribution of total pole strength and would, in fact, produce a net force and net torque on a completely nonmagnetic body immersed in a magnetic fluid and exposed to an external magnetic field.[21] For this reason one can draw simple conclusions from Eqs. (2–21) and (2–22) only in simple special cases.

Of prime interest is the case of an ideal needle-magnet subjected to an external field which is sensibly uniform in the small volume over which its effective pole strength is distributed. In this case it follows from the preceding equations that each pole can be considered subject to a concentrated force applied at the pole centroid whose value is expressible in terms of $\mathbf{B}'$ or $\mathbf{H}'$ as follows:

$$\left.\begin{array}{ll}\text{(Gauss)} & \\ \quad \mathbf{B}'p_t=\mathbf{B}'p_0/(\kappa_m)_A=\mathbf{H}'p_0 & \text{(2–23a)}\\ \text{(Giorgi-Kennelly)} & \\ \quad \mu_0^{-1}\mathbf{B}'p_t=\mathbf{B}'p_0/\mu_0(\kappa_m)_A=\mathbf{H}'p_0 & \text{(2–23b)}\\ \text{(Giorgi-Sommerfeld)} & \\ \quad \mathbf{B}'p_t=\mathbf{B}'p_0/(\kappa_m)_A=\mu_0\mathbf{H}'p_0 & \text{(2–23c)}\end{array}\right\}=\mathbf{F}_m.$$

Thus Eq. (2–19) holds as a satisfactory approximation for slender needle-magnets provided we identify dp with an element of the fixed vacuum pole-strength p_0. There is, consequently, no serious misrepresentation of the facts if we choose to identify the theoretically defined magnetic intensity $\mathbf{H}'$ in a Class A fluid (see Sec. 2.5) with the force-per-unit-pole-strength on a concentrated pole at the point in question.

Some authors[22] prefer to stress total pole-strength p_t rather than vacuum pole-strength p_0. If this is done the force equation is written as $\mathbf{F}_m=p_t\mathbf{B}'$ and the factor κ_m in the Coulomb's law equation (2–15) is thrown into the numerator. This practice may have advantages, but it introduces a variable where a corresponding invariant quantity could be used and forces a corresponding alteration in electrostatic theory if the usual analogy between electrostatics and magnetostatics is to be maintained. It is misleading indeed to indicate that $\mathbf{F}_m=p_t\mathbf{B}'$ is the *only* correct form.

[21] Moreover, a large external field, like that obtained when opposite poles of two different magnets are placed in contact, may be expected to bring about hysteresis effects not considered in elementary theory.

[22] For example, John A. Eldridge, *Am. J. Physics* **15**, 390 (1947), **16**, 327 (1948); E. Hallén, *Trans. Roy. Inst. Tech., Sweden*, No. 6 (1947).

In conclusion it should be pointed out that, if magnetostatics is developed from the law of interaction between current elements, there is the same objection to the premature introduction of κ_m as when we start from Coulomb's law for magnetic poles. The equation

$$\left.\begin{array}{lll}\text{(Gauss)} & \dfrac{I}{c}d\mathbf{s}\times\oint\left[\dfrac{I'}{c}d\mathbf{s}'\times\dfrac{\mathbf{1}_r}{r^2}\right] & \text{(2–24a)}\\ \text{(Giorgi)} & \dfrac{\mu_0}{4\pi}Id\mathbf{s}\times\oint\left[I'd\mathbf{s}'\times\dfrac{\mathbf{1}_r}{r^2}\right] & \text{(2–24b)}\end{array}\right\}=d\mathbf{F}_m$$

is analogous to Eq. (2–16) for this approach. It gives the force exerted on a current element $Id\mathbf{s}$ by a complete circuit composed of elements $I'd\mathbf{s}'$. The same equation with a factor κ_m inserted in the left-hand member is supposed to take into account immersion in a Class A magnetic fluid and is the analog of Eq. (2–15). This modified equation is not fundamental and holds only under restrictions that should be clearly specified if it is to be used at all.

The basically correct way to develop the theory is to begin with Eq. (2–24). The effect of a magnetic medium is then explained as due to the magnetization of the medium and the associated distribution of uncanceled Amperian currents, whose density is

$$\left.\begin{array}{lll}\text{(Gauss)} & c\,\mathrm{curl}\mathbf{M} & \text{(2–25a)}\\ \text{(Giorgi-Kennelly)} & (1/\mu_0)\,\mathrm{curl}\mathbf{M} & \text{(2–25b)}\\ \text{(Giorgi-Sommerfeld)} & \mathrm{curl}\mathbf{M} & \text{(2–25c)}\end{array}\right\}=\mathbf{J}_m.$$

The $\mathbf{B}$-field of the magnetization is then the field of the current distribution $\mathbf{J}_m$, while the $\mathbf{H}$-field is computed from the corresponding distribution of pole strength.

When a Class A magnetic medium fills all regions where there is a magnetic field, the magnetization produces no poles and the $\mathbf{H}$-field is the same as without the medium; $\mathbf{B}$ is therefore multiplied by the factor κ_m due to the medium. Under these conditions the vector potential differs from that without the medium only by a factor κ_m and the apparent forces between circuits are correctly given by Eq. (2–24) with κ_m inserted in the numerator. The situation in which a single Class A magnetic medium fills all space where there is a field, is not of great importance in practice, however.

In elementary courses Eq. (2–24) is not likely to be used; but the simpler equation for the force of attraction between parallel wires is practically always discussed. The introduction of a factor κ_m in this equation involves much the same dangers as its introduction in the more general equation.

To summarize: The magnetic equation (2–15) does not constitute a satisfactory basis for the theory of magnetostatics. Like its electrostatic analog, Eq. (2–1), it has only a limited validity and its applications are not of great practical importance. Since their use has led, and is likely to lead, to misconceptions on the part of the students, the committee recommends: *that Eqs. (2–1) and (2–15) be omitted altogether in elementary courses;*[23] *that Coulomb's and Ampere's laws be used only in the basic forms of Eqs. (2–2), (2–16), and (2–24); and that the effect of an intervening medium be treated in such a manner as to insure an understanding of the physical processes occurring and an appreciation of the fact that the problem is, in general, not simple.*

2.3 Polarization as a Basic Concept

A second important issue in connection with the teaching of electrostatics relates to the relative emphasis to be placed on the polarization and displacement vectors. The displacement vector **D** has traditionally had the major emphasis, and this vector is convenient for the formulation of the general partial differential equations of the electromagnetic field in linear media. On the other hand, from our present-day point of view, the polarization vector **P** is simpler and more fundamental as a physical concept. We are bound to explain the behavior of dielectrics in terms of the electrical structure of matter by means of the concept of polarization. In Maxwell's day **D** was fundamental because his interpretation of dielectric behavior was altogether different from ours, but from the physical point of view it must now be regarded as of secondary importance.

The treatment of dielectrics in introductory textbooks of general physics usually does not extend much beyond the application to parallel-plate condensers. It is easy to give a physical explanation of dielectric action for this special case in terms of the concept of polarization. The displacement vector is not required for this purpose and need not be introduced at all in such a text unless the author wishes to use it as a preparation for the magnetic induction **B**.

In more advanced textbooks the usefulness of the vector **D** is unquestionable, but in the opinion of the majority of the committee there is a great advantage even in such books in stressing the primary physical importance of **P**.

2.4 Definitions of the Electric Vectors at Points Inside Ponderable Media

The problem of defining **E**, **D**, **B**, and **H** at points inside ponderable media has long been a thorn in the side of the teacher of elementary physics and to a large extent the issue has been evaded in ways that are unsatisfactory.

Some of the definitions do not define; others are given as if they were general when actually they are applicable only to special cases. We see no objection in the construction of an elementary textbook to the use of definitions of limited scope in the place of general definitions, but in such cases the reader should be very clearly warned of the limitation.

We believe that confusion frequently arises in the minds of teachers and students alike as a result of the common failure to distinguish between two different ways in which we use the basic terms of our science. For example, we use the term "electric intensity" and the symbol **E** to denote what we conceive to be an objective physical reality susceptible of measurement in fact, or in principle, by the force-per-unit-test-charge technique. We use the same phrase and symbol to denote the corresponding theoretical concept, namely, the mathematical entity **E** as it appears in theoretical computations such as the evaluation of the capacitance of a condenser.

These two functions of the term are, of course, intimately related. The rules by which we use the theoretical concept in our calculations are formulated on the basis of experimental measurements in such fashion as to make the drawing of a distinction between the two meanings appear at first glance to be artificial and unnecessary. Nevertheless, the committee believes that a frank

[23] In making this recommendation the Committee follows the lead of Professor Murdock, whose paper (Ref. 1) led to the present inquiry.

recognition of a distinction between what we may describe as the experimental meaning of a physical quantity and the meaning of the corresponding theoretical concept can be very helpful.[24] One meaning is defined by a set of laboratory operations, while the other is defined by the mathematical rules governing the paper-and-pencil operations of problem-solving. The distinction is similar to that between physical dots and lines on paper and the ideal points and lines of the geometer.

In the past physicists have tended to ignore this difference in meaning because they have adopted a naive common-sense philosophy which identified the quantity under consideration in each case with a single objective property of the external world. Such a view does not adequately recognize the essentially tentative character of physical theories. Moreover, as we endeavor to show in the following paragraphs, at certain points this common-sense point of view muddies the business of formulating theories.

Consider the electric intensity vector. Direct measurements of electric intensity as force-per-unit-test-charge are infrequent and crude. On the other hand, the theoretical tool **E** is sharply defined as one element of a conceptual scheme containing many others, such as the polarization vector, electric charges, space coordinates, conducting surfaces, etc. It is a vector point-function used with other functions in the scheme to give solutions of Maxwell's equations. The elements of uncertainty which characterize experimental measurements have no place in the theory because the theory is a mental construction.

We are inclined to postulate a structural relation between elements of physical reality corresponding exactly to the structural relations of a useful theory, like the classical theory of electromagnetism. The need for a quantum theory of electrodynamics clearly indicates, however, that the postulated exact correspondence does not exist, at least for the classical theory of electricity and magnetism.

[24] The distinction has a close similarity to that drawn by F. S. C. Northrop between "concepts by intuition" and "concepts by postulation," although the experimental meaning of such a concept as the electric intensity does not make it a concept by intuition. See Northrop's *Logic of the sciences and the humanities* (The Macmillan Company, New York, 1947), esp. pp. 60, 82, 83.

There is, of course, a correspondence between the elements of the conceptual scheme used in any domain of physics and certain measurable physical quantities that must be observed in setting up a specific theoretical problem from data for a specific experimental situation, as well as in comparing the theoretical result with the outcome of the experiment. Rarely, however, are all the elements of the theoretical scheme susceptible of direct comparison with their objective physical correlates. We may and do use theoretical concepts which have no direct physical correlates at all, such as the wave functions of quantum mechanics.

The distinction we are emphasizing becomes important for the beginner when the discussion relates to the interior of a solid medium, where the electromagnetic vectors are never measured.[25] It is true that the Kelvin-cavity definitions provide us with mental experiments which give the semblance of physical reality to the theoretical concepts **E**, **D**, **H**, and **B**. These definitions are never converted into concrete physical experiments, however, and would involve insuperable difficulties in asymmetric crystals, in ferromagnetic materials exhibiting hysteresis, and in cases in which the vectors change rapidly in time, as in the passage of light waves through matter. From the point of view of the Lorentz electron theory, the electric and magnetic vectors **E** and **B** in material media are space averages of violently fluctuating "microscopic" field vectors and atomic magnitudes that are never susceptible of direct measurement. Hence, *the macroscopic vectors* **E**, **D**, **H**, **B**, *and the electric and magnetic polarizations should be defined for points in solid bodies as theoretical tools and not as directly measurable physical quantities. The definitions should serve to show the student how the various vectors are to be used in calculations and it should be made clear to the student that this is the purpose of the definitions.*

With these considerations in mind the committee recommends for the definition of the electric vectors a procedure somewhat as follows:

First discuss the electrostatics of free space, defining **E**

[25] Of course, we *do* make measurements of the average values of these vectors; for example, the measurement of the magnetic flux through a solenoid with an iron core determines the average value of B in the core.

by the usual force-per-unit-test-charge method. Introduce dielectrics by an account of the experimental behavior of condensers whose plates are separated by such media. Mention some other experiments of interest in connection with dielectrics, such as the observed change in an electric field due to the insertion of a block of dielectric into the field, the measurement of the force of attraction of a point charge for an uncharged block of dielectric, or the force between two charged bodies immersed in a fluid dielectric. Then confront the class with the problem of developing a theory which, if possible, shall give a quantitative prediction of many experimental results and which shall relate the observations in a reasonable way to other information regarding the probable nature of matter. Such a theory can reasonably consist of a *model* of the dielectric based on familiar concepts and forming a basis for the computation of quantities which are to be compared with experimental results.

As a first step toward the construction of a theory, introduce the idea that matter contains distributions of positive and negative charge which are very dense, but which ordinarily cancel one another, and which are bound together in most cases by strong "elastic" (linear) forces. Note that from the atomic point of view these distributions are undoubtedly discontinuous, but for the purpose in hand can be treated as continuous.[26] Under the influence of an electric field a small motion of positive charge density, ρ^+, relative to negative charge density, ρ^-, may be expected. Such a motion would create a net surface-density of bound charge (polarization charge) at the surface of the medium and under certain circumstances could create a non-vanishing net volume-density of polarization charge. *Postulate* that the electric intensity **E** outside the dielectric, where it is directly measurable, is to be computed from the total charge, including both free and bound parts, by the methods used in the absence of dielectrics. Note that although we can not directly explore the electric field inside a solid dielectric, we can still *define* the electric intensity **E** for points where it can not be measured as a vector computed from the charge distribution by the same rules that apply outside the dielectric. The emphasis to be laid on the distinction between the two meanings of the term "electric intensity" will depend on the level of the course and the bias of the teacher.

Next *define* the polarization vector **P** in terms of the model of the dielectric as electric moment per unit volume due to the relative displacement of positive and negative bound charges. Its value at any point A is equal to the product of the positive charge density and the vector displacement of positive charge relative to negative charge, both evaluated at A. From this definition show that the polarization of the medium must cause the flow of bound charge across every surface element dS in the direction of the positive normal in amount $P_n dS$, where P_n is the component of **P** in the direction of the positive normal.

[26] At the pleasure of the instructor the charges may be treated as discontinuous during the early stages of the discussion, the transition to continuous charge densities being deferred as long as possible.

It follows that at the exterior surface of a dielectric there should be a net bound surface-charge-density equal to the component of **P** at the surface in the direction of the exterior normal. If the mathematical level of the course permits, show that the negative divergence of **P** (if any) is the net volume-density of bound charge. Thus the vector **P** determines the complete distribution of net bound charge, or polarization charge.

At this point the conceptual scheme devised for the interpretation of the behavior of dielectrics consists of four essential elements, *viz.*, the density of conduction charge ρ_c, the polarization vector **P**, the net density of the polarization charge, and the electric intensity **E**. The first two quantities determine the second two, but none of the quantities is directly measurable in the interior of the dielectric. The four concepts *belong to the model.*

In applying the theory to special cases an additional assumption is needed in order to create a mathematical problem which has a definite solution. Consider the case of a parallel-plate condenser large enough so that edge effects can be neglected. Let the space between the plates be filled with an ideal homogeneous isotropic dielectric. In this case the density of conduction charge can be assumed equal to zero except on the surfaces of the plates, where it must have a uniform value σ_c (positive plate), or $-\sigma_c$ (negative plate). In conformity with the experimental proportionality of charge and potential difference the polarization **P** of the model is assumed to be proportional to **E** and in the same direction. This assumption completes the essential features of the model. Show that under these circumstances the model predicts that the dielectric will increase the capacitance of the condenser by the factor $1+4\pi\chi_e$ (Gauss) or $1+\chi_e$ (Giorgi), where the electric susceptibility χ_e is defined as the dimensionless constant of proportionality in the relation

(Gauss) $$\mathbf{P}=\chi_e\mathbf{E} \qquad (2\text{–}26a)$$
or (Giorgi) $$\mathbf{P}=\epsilon_0\chi_e\mathbf{E} \qquad (2\text{–}26b)$$

Define the relative dielectric constant κ_e by the equation

(Gauss) $$\kappa_e=1+4\pi\chi_e, \qquad (2\text{–}27a)$$
or (Giorgi) $$\kappa_e=1+\chi_e. \qquad (2\text{–}27b)$$

Then take one of the other special problems of the elementary theory of dielectrics and show that if the dielectric is homogeneous, isotropic, and linear, the model and the information contained in the specification of the experimental conditions suffice to yield a definite scheme for computing the result of the observations.

Attention should be called to the fact that the assumption that **P** is proportional to **E** does not give a satisfactory model for many dielectrics.

In conclusion, note the fact that a small charged body immersed in a fluid dielectric should experience three kinds of forces, *viz.*, electric forces due to its direct interaction with other conduction charges in the neighborhood, electric forces due to interaction with the electric dipoles which the dielectric contains, and a mechanical force due to the asymmetric pressure built up in the fluid by electrostrictive action. Note that the resultant of these forces is $q_c\mathbf{E}$ so that in principle **E** can be measured in a fluid dielectric by the same technique as in a vacuum.[27]

At this point the vector **D** can be introduced and defined in terms of **E** and **P** by the equation

(Gauss) $\mathbf{D}=\mathbf{E}+4\pi\mathbf{P}$, (2–28a)

or (Giorgi) $\mathbf{D}=\epsilon_0\mathbf{E}+\mathbf{P}$. (2–28b)

In a Class A dielectric in which **P** is proportional to **E** this definition leads at once to the simple relation $\mathbf{D}=\kappa_\epsilon\mathbf{P}$, or $\mathbf{D}=\epsilon_0\kappa_\epsilon\mathbf{P}$, according to the unit scheme adopted.

Call attention to the fact that unlike **E** the vector **D** is not derivable from a potential except in special cases where it has no component parallel to any dielectric interface in the field. When the mathematical properties of **D** have been sufficiently developed, the Kelvin-cavity interpretations of **E** and **D** may be given to show how far one can go in the way of giving a sense of physical reality to the vector concepts of the mathematical theory.

Some authors have preferred to introduce **D** at an early stage as a vector whose divergence is proportional to the conduction charge density and which is related to **E** by the rule, $\mathbf{D}=\epsilon\mathbf{E}$. Since curl**E** vanishes identically, curl**D** reduces to the vector product $\nabla\epsilon\times\mathbf{E}$. Thus curl**D** vanishes in a homogeneous isotropic medium, and in a transition layer between one dielectric and another if **E** happens to be normal to the interface. It also vanishes automatically at the interface of a dielectric and a conductor in which there are no currents. There are, accordingly, a number of problems in which **D** is irrotational and therefore correctly given by the simple formula

(Gauss) $\mathbf{D}=\Sigma q\mathbf{1}_r/r^2$, (2–29)

where $\mathbf{1}_r$ denotes a unit vector along the radius from the source-point to the field-point. Although this procedure is open to no logical objection if the severe restrictions on the formula are made clear, there is great danger that the use of Eq. (2–29) in the early stages of an elementary or intermediate course will lead to a serious misconception in the minds of many students. Hence *the committee regards such use of Eq. (2–29) as undesirable.*

2.5 Concerning the Definitions of the Magnetic Vectors and Some Related Matters

The business of defining the magnetic vectors **B**, **M**, and **H** in solid magnetic media involves the same need for distinguishing between theoretical and experimental meanings that we have discussed in connection with the electric vectors **E**, **P**, and **D**. The definitions of **B**, **M**, and **H** in terms of laboratory operations have significance only for media in which the operations can actually be performed. The theoretical meanings, valid for all media, are based on mathematical relations, or paper-and-pencil operations, that show how these vectors are to be used in calculations based on the classical theory. In this respect, however, the magnetic vectors present no new pedagogical problems.[27a]

A different problem connected with the definition of **M** has arisen, however, as the result of the possibility of interpreting the elementary magnets of the "molecular" theory either as pairs of equal and opposite poles, or as Amperian current loops. The ambiguity is purely formal, since nearly everyone agrees that the magnetic poles are mathematical fictions and that the Amperian currents correspond closely to present conceptions of the physical "reality."[28] Each interpretation is allied to a method of calculation and the two methods are mathematically equivalent. Nevertheless, considerable divergences in teaching practice and a recently developed disagreement as to the best choice of unit and dimensions for magnetic polarization in the Giorgi scheme are consequences of this double interpretation.

There are, in fact, three different, closely related issues which should be considered in this connection, namely, (a) the relative emphasis to be placed on magnetic poles and Amperian currents as initial concepts in the theory of magnetism, (b) the relative emphasis to be placed on the correlation of **H** with **E** (parallel correlation) and on the correlation of **B** with **E** (antiparallel correlation), and (c) the definition of **M** (choice of dimensions and unit in the Giorgi system). As will appear in Secs. 2.6 and 2.7, a decision to

[27] See Sec. 2.1, Eqs. (2–5) and (2–14).

[27a] The explicit working out of these definitions with their appropriate background is carried through in Sec. 3.25 of Part III.

[28] Yet too much emphasis should not be placed on the supposed reality of Amperian currents. Ferromagnetic polarization is primarily due to electron-spin magnetic moments. Few of those familiar with the rules of behavior of electron spin would consider them a very satisfactory justification for calling Amperian currents "real." Still more serious doubts regarding a naive acceptance of the reality of Amperian currents might be raised by a consideration of neutron magnetic moments.

introduce the subject of magnetism by way of the magnetic-pole concept leads almost inevitably to emphasis on **H** and its correlation with **E** and, if Giorgi units are to be used, to preference for the Kennelly definition of **M**. On the contrary, a decision to introduce magnetism by the Amperian current concept leads almost inevitably to emphasis on **B**, to the correlation of **B** with **E**, and to preference for the Sommerfeld definition of **M**. Thus, the three issues reduce to a single choice between what we may refer to as the traditional, or magnetic pole, approach to the study of magnetism and the Amperian current approach.

In discussing this broad problem the Coulomb's Law Committee has found itself confronted with matters of taste and personal judgment as well as with matters of logical rigor. Although we have no mandate to render opinions on such issues, the confusion created by the two definitions and the highly partisan arguments on one side and the other call for clarification. Under these circumstances we here undertake to present the chief arguments on both sides, not so much to assist readers to the best possible final choice of approach as to give them, if possible, a fuller realization of the implications of the choice and a clearer conception of the possibilities of improved instruction by either technique.

2.6 Amperian Currents versus Magnetic Poles

The traditional approach to magnetism (see Outline I of Part 3) is by way of point poles, distributed poles, and distributed dipoles. After the magnetic fields of permanent magnets have been described on the basis of these concepts it is explained that current loops produce magnetic fields that are indistinguishable at large distances from the fields of small magnets. Amperian currents are then introduced as the equivalent of static dipoles. The magnetic fields of conduction-current circuits are introduced by means of the somewhat artificial concept of the magnetic shell.[29] By this device one can ultimately derive Ampere's law [Eq. (3–12)] and make both scalar and vector potentials available for the computation of magnetic fields.

This familiar procedure starts magnetic theory along lines to which the student has been introduced by the previous study of electrostatics and the polarization of dielectrics. It makes a maximum use of familiar mathematical tools and postpones vector products as long as possible.

The alternative Amperian approach (see Outline II of Part 3) is not so well standardized. It deals with the magnetic fields of currents in free space without the assistance of magnetic poles or permanent magnets. It starts from the observed forces between parallel currents rather than from Coulomb's law for magnetic poles, and defines the magnetic induction **B** in free space in terms of the torque on an exploring coil, or in terms of the force exerted on a straight current-carrying wire. Emphasis is laid from the beginning on important contrasts between magnetic and electric fields such as the fact that like currents attract one another although like charges repel.

When magnetic materials are taken up, the process of polarization by an external field is interpreted from the beginning as the rotation of Amperian-current loops due to the attraction of parallel currents and the repulsion of those that are antiparallel. The field of a magnetized bar of iron is referred, not to poles which accompany magnetization, but to net uncanceled Amperian currents on its lateral surface. In this way the discussion of the fields of currents in a vacuum is unified with the treatment of magnetic materials.

The arguments in favor of these two different strategies are various. Our discussion takes up first the simpler and more obvious considerations, then those that are not so obvious. Since "the proof of the pudding is in the eating" the final section of the discussion is contained in the two outlines for teaching given in Part 3. The remainder of this section expounds more fully the contrasts between the two approaches. The next section, 2.7, takes up the argument from the practical point of view of the classroom.

The traditional method of introducing the subject of magnetism describes the forces on poles in terms of **H** [see Eq. (2–19)]; the Amperian method describes forces on currents in

[29] Since transversely magnetized sheets of metal can now be manufactured, these shells are not so artificial as they once seemed to be.

terms of **B**. Hence the method of calculation used in the traditional approach makes **H** the analog of the electrostatic force **E**, but the Amperian method makes **B** the analog of **E**. These correlations are initially dictated by the fact that in a Class A magnetic medium the observable force on a concentrated magnetic pole is most conveniently expressed as $\mathbf{H}'p_0$, where $\mathbf{H}'$ is the external field and p_0 is the vacuum pole-strength [see Eq. (2–23)], whereas the force on a current-carrying wire of length L is $L\mathbf{I}\times\mathbf{B}'$ [see Eq. (3–11), where the prime is omitted as unnecessary in view of the context]. The correlations are reinforced by the satisfactory way in which they can be developed. Therefore, an essential part of the problem of choosing between the two pedagogical approaches is to see how the corresponding correlations work out.

The two schemes have many corresponding features. In the older one emphasis is laid on the identical form of the Coulomb law for charges and poles in a vacuum. In the newer one, as indicated above, a similar emphasis is laid on the contrast between the repulsion of like charges and the attraction of like currents. This contrast is conveniently formulated in terms of the equations for the forces on parallel wires of length L carrying, in the one case, charges Q and Q' per unit length and, in the other, currents I and I'. Thus

Electrostatic

(Gauss) $$F_e=+(2QQ'L/r);\qquad (2\text{–}33a)$$
or (Giorgi) $$F_e=+(QQ'L/2\pi\epsilon_0 r);\qquad (2\text{–}33b)$$

Magnetic

(Gauss) $$F_m=-(2II'L/c^2r);\qquad (2\text{–}34a)$$
or (Giorgi) $$F_m=-(II'L/2\pi\eta_0 r).\qquad (2\text{–}34b)$$

Here η_0 is a symbol introduced for the reciprocal of the usual μ_0 in order to remove an apparent contrast that is not of an essential character. The $+$ and $-$ signs, however, represent a fundamental contrast which we designate as *essential antiparallelism*, because the signs cannot be made alike without spoiling the symmetry of the definitions of F_e, F_m, I, I', or Q, Q'. Antiparallelism of this sort is characteristic of the Amperian correlation just as similarity, or parallelism, is characteristic of the traditional correlation.

Consider the lines of force. In the traditional approach lines of **H** are introduced by means of permanent magnets and in the absence of currents shown to have properties similar to those of the lines of **E**. In the Amperian approach, using electric currents and lines of **B**, similarity is largely replaced by contrast.[30] Thus electrostatic lines terminate on charges, but the lines of **B** form closed loops linked with currents (conduction, or Amperian, or both). In the teacher's mind, assuring him of the permanent value of these contrasts are the advanced equations:

Electrostatic

$$\text{curl}\mathbf{E}=0,\qquad (2\text{–}35)$$
(Gauss) $$\text{div}\mathbf{E}=4\pi(\rho_c+\rho_p),\qquad (2\text{–}36a)$$
or (Giorgi) $$\epsilon_0\,\text{div}\mathbf{E}=\rho_c+\rho_p,\qquad (2\text{–}36b)$$

Amperian Magnetostatic

$$\text{div}\mathbf{B}=0,\qquad (2\text{–}37)$$
(Gauss) $$\text{curl}\mathbf{B}=4\pi(\mathbf{J}_c+\mathbf{J}_m),\qquad (2\text{–}38a)$$
or (Giorgi) $$\eta_0\,\text{curl}\mathbf{B}=\mathbf{J}_c+\mathbf{J}_m.\qquad (2\text{–}38b)$$

Here, evidently, the antiparallelism takes the form of an exchange of the curl and divergence operators.

The corresponding equations exhibiting the parallelism of the traditional approach are:

Traditional Magnetostatic

$$\text{curl}\mathbf{H}=0\qquad (2\text{–}39)$$
(Gauss) $$\text{div}\mathbf{H}=4\pi(\rho_{m0}+\rho_{ms}),\qquad (2\text{–}40a)$$
(Giorgi-Kennelly)
$$\mu_0\,\text{div}\mathbf{H}=\rho_{m0}+\rho_{ms},\qquad (2\text{–}40b)$$
(Giorgi-Sommerfeld)
$$\text{div}\mathbf{H}=\rho_{m0}+\rho_{ms}.\qquad (2\text{–}40c)$$

Here ρ_{m0} and ρ_{ms} denote, respectively, the volume density of magnetic pole-strength due to perma-

[30] There are many ways of locating and defining the lines of **B** in lectures. Some of them will be discussed in more detail in Part 3, Outline II. Here we may note one definition chosen for contrast with almost any definition of the lines of **E**, namely: "A magnetic line of force in free space is a line along which a wire may lie and feel *no* force, regardless of the current it carries." The paradoxical aspect of this definition makes it take hold of the student's mind and helps to keep the class alert.

nent polarization and to "soft" polarization (see definition of ρ_{mt} in footnote 19).

The teacher must, of course, bear in mind the limitations of these equations and of the analogies which they describe. The curl equations are limited to cases in which the fields are not time-dependent. Equation (2–39) is restricted to cases in which there are no conduction currents and the division of ρ_m into ρ_{m0} and ρ_{ms} in Eq. (2–40) to cases in which the relation between **M** and **H** conforms to the linear equation

(Gauss or Giorgi-Sommerfeld)

$$\mathbf{M}=\mathbf{M}_0+\chi_m'(\mathbf{H}-\mathbf{H}_0) \qquad (2\text{–}41\text{a, c})$$

(Giorgi-Kennelly)

$$\mathbf{M}=\mathbf{M}_0+\mu_0\chi_m'(\mathbf{H}-\mathbf{H}_0). \qquad (2\text{–}41\text{b})$$

(See footnote 19 and Eq. (B-8) of Appendix B). Although the range of the parallel correlation of the traditional approach can be extended by the introduction of magnetic shells as the equivalent of linear currents, it will be seen that the parallel correlation is basically much more restricted in scope than the antiparallel one.

In describing polarizable media the traditional approach continues with parallelism, the Amperian with antiparallelism. Both explain the electrostatic effect of a glass plate inserted in an air condenser as a partial counterbalancing of the effects of conduction charges by unlike polarization charges attracted toward them. The traditional approach can use a parallel interpretation of the effect of a soft iron sheet placed between the poles of a permanent horseshoe magnet in reducing its external field. The Amperian approach, on the other hand, can introduce soft iron or other ferromagnetic material as a plunger in a solenoid, showing how it reinforces the external field and explaining the effect as the result of the attraction of Amperian currents of like sign by the conduction currents which surround the plunger.

On the basis of these explanations both approaches may define **D** by the equation

(Gauss)

$$\mathbf{D}=\mathbf{E}+4\pi\mathbf{P}, \qquad (2\text{–}42\text{a})$$

but the traditional approach now correlates **H** with **E**, whereas the Amperian correlates **B** with **E**. Thus

Traditional

(Gauss)

$$\mathbf{B}=\mathbf{H}+4\pi\mathbf{M}, \qquad (2\text{–}43\text{a})$$

Amperian

(Gauss)

$$\mathbf{H}=\mathbf{B}-4\pi\mathbf{M}, \qquad (2\text{–}44\text{a})$$

where the sign reversal of **M** in Eq. (2–44a) is the characteristic feature of antiparallelism.

To write either of these magnetic equations in Giorgi units, one must choose between the two conflicting definitions of **M** mentioned in footnote 7 and in Sec. 2.5. One of these, proposed by Kennelly and adopted by Harnwell and others,[31] corresponds to a definition of the magnetic moment of a plane current loop of area S as $\mu_0 IS$; the other, proposed by Sommerfeld and adopted by Stratton and others,[32] corresponds to the definition IS. The traditional equation (2–43) takes its simplest form in Giorgi-Kennelly units; the Amperian equation (2–44) in Giorgi-Sommerfeld units.[33] We accordingly write the Giorgi forms as follows:

(Giorgi)

$$\mathbf{D}=\epsilon_0\mathbf{E}+\mathbf{P}, \qquad (2\text{–}42\text{b})$$

Traditional

(Giorgi-Kennelly)

$$\mathbf{B}=\mu_0\mathbf{H}+\mathbf{M}, \qquad (2\text{–}43\text{b})$$

Amperian

(Giorgi-Sommerfeld)

$$\mathbf{H}=\eta_0\mathbf{B}-\mathbf{M}. \qquad (2\text{–}44\text{c})$$

The introduction of η_0 for $1/\mu_0$ is needed here again to give the antiparallel equations maximum symmetry.

For mechanical forces in polarizable media the traditional approach compares poles with charges as usual, while the Amperian approach contrasts

[31] A. E. Kennelly, Comité International des Poids et Mesures, *Procès-Verbaux des Séances* **17**, 1935; G. P. Harnwell, *loc. cit.*

[32] A. Sommerfeld, "Dimensionen der elektromagnetischen Grössen," *Zeits. f. Tech. Physik*, **16**, 420 (1935); J. A. Stratton, *loc. cit.*

[33] In *both* schemes the relations between magnetic moment, magnetization, magnetic pole-strength, and magnetic pole-density are the same. To convert any of these quantities from Sommerfeld units to Kennelly units we can start with pole-strength. For this the Gaussian unit is the (abampere×cm), the Sommerfeld unit is the (ampere ×meter), and the Kennelly unit is the (volt×second). The ratio of the Kennelly unit to the Sommerfeld unit is μ_0, or $4\pi\times10^{-7}$ henry/meter. The defining equation for susceptibility in the Sommerfeld scheme is $\mathbf{M}=\chi_m\mathbf{H}$, but in the Kennelly scheme it is $\mathbf{M}=\mu_0\chi_m\mathbf{H}$, which means that the values of χ_m and κ_m are the same for both.

currents with charges; so we can expect parallelism of signs in the one case and antiparallelism in the other. Equation (2–10) of Sec. 2.2 gives the force density resulting from electrostatic polarization as (Gauss) $-(E^2/8\pi)\,\text{grad}\kappa_e$, or (Giorgi) $-\frac{1}{2}\epsilon_0 E^2\,\text{grad}\kappa_e$. Corresponding to this, the traditional approach writes $-(H^2/8\pi)\,\text{grad}\kappa_m$, or $-\frac{1}{2}\mu_0 H^2\,\text{grad}\kappa_m$. In the Amperian approach, using B, η_0, and a variable relative reluctivity $\gamma_m = 1/\kappa_m$, the same term takes the form $+(B^2/8\pi)\,\text{grad}\gamma_m$ or $+\frac{1}{2}\eta_0 B^2\,\text{grad}\gamma_m$. That is, each approach leads to the sign which one would expect on the basis of its physical interpretation of magnetization.

Questions are often asked about what the field "really is" within a polarizable medium. As noted in Sec. 2.4 such questions have a preliminary answer in the conclusions of the Lorentz electron theory which make **E** the average of the "microscopic" electric field and **B** the average of the corresponding magnetic field. The Lorentz interpretations must be taken with a grain of salt since they antedate electron spin and the Heisenberg uncertainty principle. They harmonize very well with the Kelvin-cavity definitions, however, and the latter seem to have a certain permanent significance. So it may be well to point out that in the mental operation of cutting a drill-hole for the measurement of **E** one introduces no appreciable polarization charges. Just so, the cutting of a slot for the measurement of **B** introduces no appreciable uncanceled Amperian currents. So it is reasonable to identify these vectors with the average values of microscopic fields before making the cuts. It may be observed at the same time that if at this point one wishes to stick to poles, rather than currents, as the cause of magnetization, one should properly identify the **H** measured in a longitudinal drill-hole with the average initial microscopic field.

Finally, it may be well to note briefly what becomes of the correlations when we go beyond the limitations noted in connection with Eqs. (2–35) to (2–40). Going far beyond, into relativity, the Lorentz transformations deal with **E** and **B** in combination as one entity, and with **D** and **H** as another. Moreover, the four-dimensional vector analysis appropriate to relativity gives us Maxwell's equations for curl**E** and div**B** as parts of one vector equation, and those for curl**H** and div**D** as parts of another. In short, relativity confirms the correlation of **B** with **E**, and also the correlations of curls and divergences noted for static fields in the antiparallel system in connection with Eqs. (2–35) to (2–38).

In bringing this section to a close we briefly summarize the arguments on the two sides of the Sommerfeld-Kennelly unit question as we see them.

The Kennelly units are congenial to those who wish to use the traditional approach because (a) they are needed to give Eq. (2–43b) a form parallel to the electrostatic equation (2–42b); and because (b) the definition of **H** by $\mathbf{F}_m = \mathbf{H}' p_0$ [Eq. (2–23b)] would be spoiled if we introduced Sommerfeld units by substituting $\mu_0 p_0$ for p_0. (Our reasons for preferring $\mathbf{H}' p_0$ to $\mathbf{B}' p_t$ are indicated in Sec. 2.2.)

The Sommerfeld units, on the other hand, are congenial to those who use the antiparallel approach because (a) they are needed to give Eq. (2–44c) a form antiparallel to Eq. (2–42b); and because (b) they simplify the relation between magnetic moment and current. Under (b) we include the relation $\text{curl}\mathbf{M} = \mathbf{J}_m$ for the volume-density of Amperian currents and the corresponding equation for the surface-density, as well as the equation for the magnetic moment of a linear circuit.

Possibly other considerations of importance have escaped us. In any case we hope that we have clarified the unit question by pointing out its intimate relation to teaching methods. Since our discussions lead to no clear mandate on the best pedagogical approach (see Sec. 2.7) we make no recommendation regarding magnetic units.

2.7 Magnetic Fields in the Classroom

Agreeing on the laws and correlations of Sec. 2.6, teachers will still disagree as to the best way of presenting them. Some will prefer to stick to the traditional approach, while others will go "all out" for the newer Amperian procedure. Both ways are right. Each has many variations, still right, and much room for the individual teacher's favorite demonstration experiments and other grace notes. The old principle, *de gustibus non disputandum*, comes near enough to this case to be a warning. Arguments here should avoid dogmatic extremes, and they will be helpful only as they illuminate the problems.

For the traditional approach the chief positive argument has already been stated: It starts magnetic theory along familiar lines in conformity with the historical development of the subject and postpones the introduction of vector products as long as possible.

There are, however, a number of objections to this method that should be considered. Some teachers object to the emphasis that it places on a "fictional" concept (the magnetic pole) and to the fact that elementary textbooks using this approach are apt to give a false conception of the facts by creating the impression that every bar magnet has at its ends two point poles of fixed strength from which the magnetic field can be accurately computed according to Coulomb's law. Because the point pole is an idealization it may be argued that it is an illusion. In fact, in the minds of some the discrepancy between the point-pole approximation and the actual experimental situation seems to becloud the whole subject.[34]

In reply to these criticisms one can make two constructive suggestions for those who adhere to the traditional approach. The first suggestion is that the instructor make full use of the distinction between measured physical quantities and the idealized tools of physical theory[35] to point out that the crudity of the experiments which suggest a conceptual scheme in no way limits the precision with which the scheme itself is formulated. The theory of magnetic polarization, whether developed step-by-step from the pole concept, or from the notion of Amperian currents, is a well defined theory of great power and well able to account for the failure of the point-poles approximation at short distances. Its value is independent of the accuracy and range of the approximate formulas most helpful in enabling the beginner to grasp the initial concepts of the subject.

In the second place, it is clear that we need renewed emphasis on the necessity for treating the polar regions at the ends of a magnet as *distributions* of magnetic charge or magnetic pole-strength, which can be replaced by point poles *only* in a first crude approximation in order to simplify the computation of the field at large distances. The success of the concept of a distribution of magnetic pole-strength in explaining various experimental results, such as the properties of ellipsoidal magnets, long bar magnets, electromagnet pole pieces, etc., should be stressed. It would be helpful if tables of numerical data indicating the value and the limitations of the point pole approximation could find their way into elementary textbooks.[36]

For the Amperian approach the broadest argument is that it proceeds as directly as is practicable for a freshman toward the physical principles adequate for a satisfactory comprehension of the whole range of magnetic phenomena including induced electromotive forces. It unifies the treatment of the magnetic fields of bar magnets with those of electrical circuits and avoids the not-too-easy transition from one to the other that is inevitable in the magnetic-pole approach. It simplifies matters because it defines the magnetic field in free space at the beginning in terms of operations more suited to accurate measurement than the operation of measuring the force on a concentrated magnetic pole.[37]

Of course, the difficult spot in teaching by the Amperian approach lies in the initial step of defining the field by means of the transverse

[34] As one experienced teacher remarks, "No matter how carefully the statements about magnetic poles are qualified, the students who work formal problems involving magnetic poles almost invariably come to wrong conclusions. If later confronted with experimental tests of the departures from Coulomb's law at short distances, or with the fact that the attraction between unlike poles of a pair of similar magnets in contact is much greater than the repulsion of like poles in contact, they are at a complete loss to account for the observations and are likely to feel that the whole foundation of the subject of magnetism has evaporated."

[35] See Sec. 2.3.

[36] The committee feels the need of some new data—at least data more recent than the eighteenth century observations of Robison and Coulomb. A service would be rendered by any reader of this report who would undertake an accurate series of observations with carefully prepared magnets using the best modern materials. Incidentally, we should be interested to know just how effective the Robison design (with small iron spheres at the ends) actually is, in fixing the pole positions.

[37] An incidental practical advantage appears in the treatment of motors and generators. Every teacher has seen the confusion about the directions of the forces on the armatures of motors with the thumb and finger rules that have twelve geometric possibilities, of which six are right and six wrong. But because of the likeness of Amperian currents to the neighboring conduction currents, and because of the designs of modern motors (a.c. as well as d.c.) which place the field coils near the moving conductors, the directions of the forces can be found simply from the rule of attraction of like currents.

force on a suitably oriented current-carrying wire. A new physical concept, the magnetic field, and a new mathematical concept, the vector product, have to be introduced together. The average elementary student can well find himself baffled at this point even though the new vector **B** is defined in two steps, first fixing the direction in terms of lines of force revealed by iron filings and afterward fixing the magnitude by means of the mechanical force on a current element oriented at right angles to the line of force.

Advocates of the Amperian approach agree that the difficulty is a real one that can be met only by careful planning. Enough attention must be given to the lines, as lines, to make the student feel at home among them before any problem is raised about magnitudes. Iron filings should be used liberally. Also exploring coils, free to turn; also fairly straight, loose wires. There is plenty of work to do here before measuring any forces (see Sec. 3.34 of Outline II).

Only when he has this foundation is a freshman ready for the magnitude of **B**; but he can see it at this point in terms of either the force on a straight wire or the torque on an exploring coil. The latter view prepares him for the more usual method of measurement with the coil by electromotive impulse.

The arguments outlined here are not by any means all there are, but it is our belief that they are among the most important. To them we add a neutral suggestion: in using either approach don't omit good points suggested by the other. In the Amperian approach, for example, when you get around to quantitative comparisons of ferromagnetic materials, don't hesitate to use as the independent variable in the magnetization curve of a ring specimen the independently controllable number of ampere-turns per meter, alias **H**, even though historically the first reason for its use was a part of the other theory. In either approach don't hesitate to use either a scalar or a vector potential in free space, when you get around to needing it; nor to recognize that the external field of a solenoid can be calculated as if the solenoid was a bar magnet with poles, or *vice versa*. In short, enthusiasm for either system of logic is good; it enlivens the course; but it must be *for*, not against, anything that is true.

Finally, let us repeat: either approach is "right" and the student who goes on in physics must eventually learn both languages. The one thing we can say definitely here, about teaching as distinguished from subject matter, is that we members of the committee have profited greatly by our discussions of these facts and ideas, so we hope to share the profits with other physics teachers.

Radiation Pressure in a Sound Wave

ROBERT T. BEYER
Brown University, Providence, Rhode Island

THE concept of radiation pressure in a sound beam was first presented by Lord Rayleigh[1] nearly fifty years ago. Since that time the concept has frequently been invoked to justify experimental measurements on the absorption of sound in liquids.[2–4]

In a typical experimental set-up,[4] ultrasonic radiation is sent vertically upward in a container of liquid. A flat plate, or detector, is suspended into the liquid from one arm of a microbalance. The plane face of the detector is perpendicular to the direction of propagation of the radiation. When the source is emitting radiation, the apparent weight of the detector is less than it is in the absence of radiation. This loss in weight is attributed to the radiation pressure of the ultrasonic beam, which is, in turn, proportional to the acoustic intensity, so that the change in weight of the detector is also proportional to the intensity. A measurement of the change in weight

[1] Rayleigh, *Phil. Mag.* **3**, 338 (1902); **10**, 364 (1905).
[2] C. Sörensen, *Ann. d. Physik, Leipzig* (5) **27**, 70 (1936).
[3] Claeys, Errera, and Sack, *Trans. Faraday Soc.* **33**, 136 (1937).
[4] E. Hsu, *J. Acous. Soc. Am.* **17**, 127 (1945).

For a long magnet of other shape we may use the previous approximation $\omega/2\pi$ as an estimate of D. For the particular case of dimension ratio 50 the numerical estimate is 2.5×10^{-4}. Since κ_m-1 is of the order of 5×10^{-3} for liquid oxygen, we need not consider cases in which κ_m-1 is greater than 0.01. The smallness of D, D_0, and κ_m-1 then permits us to rewrite Eq. (B–9) for the specific class of cases under consideration as

$$\frac{p_t}{p_0}=\frac{1}{\kappa_m}\left[1+D_0(\kappa_m'-1)-D\left(\frac{\kappa_m'-1}{\kappa_m}\right)\right]$$

$$=\frac{1}{\kappa_m}[1+(D_0-D)(\kappa_m'-1)]. \quad \text{(B–10)}$$

To estimate the probable deviation of D from D_0 let us denote the induced pole-strength due to immersion by p_i and the distance from the center of the magnet to the centroid of p_i by R_i. Then

$$p_tR_t=p_0R_0+p_iR_i. \quad \text{(B–11)}$$

Let us postulate that R_i differs from R_0 by 5 percent, a generous estimate for a magnet of the dimension ratio under consideration. Then R_t/R_0 reduces to $1\pm0.05(1-p_0/p_t)$. The ratio D_0/D is equal to $(R_t/R_0)^2$, or $1\pm0.1(1-p_0/p_t)$. Let us now replace $(1-p_0/p_t)$ by the approximation $1-\kappa_m$, thereby reducing D_0/D to $1\pm0.1(\kappa_m-1)$. The quantity $(D_0-D)(\kappa_m'-1)$ in Eq. (B–10) is now seen to be of the order of magnitude of $10^{-4}(\kappa_m-1)$ and thus of negligible importance. We conclude that for long slender magnets of hard steel the approximation $1/\kappa_m$ for p_t/p_0 is very good.

List of Principal Symbols

F_e, $\mathbf{F}_e$ = net mechanical force of electrostatic origin including contribution from electrostriction.
f_e = direct electrostatic force of one charge on another.
q = electric charge (in formulating Coulomb's law).
q_c = conduction charge.
ρ_c, σ_c = volume and surface densities, respectively, of conduction charge.
ρ_p, σ_p = volume and surface densities, respectively, of polarization charge.
$\rho_t=\rho_c+\rho_p$ = volume density of *net* charge.
V = electric potential.
$\mathbf{P}$ = electric polarization.
κ_e = relative dielectric constant.
ϵ_0 = dimensional constant for D/E in free space.
$\epsilon=\epsilon_0\kappa_e$.
χ_e = electric susceptibility (dimensionless).
I = conduction current.
$\mathbf{J}_c$, $\mathbf{J}_m$ = current densities for conduction currents and uncancelled Amperian currents, respectively.
$\mathbf{F}_m$ = net mechanical force of magnetic origin including contribution from magnetostriction.
p = pole strength (in formulating Coulomb's law for poles).
p_0, p_t = pole strength due to hard polarization only and to total polarization, respectively.
ρ_{m0}, ρ_m = pole density due to hard polarization only and to total polarization, respectively.
$\mathbf{M}_0$, $\mathbf{M}$ = magnetic polarization, hard and total, respectively.
μ_0 = dimensional constant for B/H in free space.
κ_m = relative permeability.
$\mu=\mu_0\kappa_m$.
χ_m = magnetic susceptibility (dimensionless).
χ_m' = differential susceptibility of hard magnet.
$\eta=1/\mu$ = reluctivity.
$\mathbf{n}$ = outward normal to surface.

Errata: The Teaching of Electricity and Magnetism at the College Level. I. Logical Standards and Critical Issues

(Report of the Coulomb's Law Committee of the A.A.P.T.)
[*Am. J. Phys.* 18, 1 (1950)]

Equations in the text of Sec. 2.4 second and third lines after numbered Eqs. (2–28a) and (2–28b) should read "$D=\kappa_e\mathbf{E}$" and "$\mathbf{D}=\epsilon_0\kappa_e\mathbf{E}$" respectively.

Equation (2–38a) should be c curl $\mathbf{B}=4\pi(\mathbf{J}_c+\mathbf{J}_m)$.