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A CRITICAL STUDY OF THE CHARACTERISTICS OF BROADCAST ANTENNAS AS AFFECTED BY ANTENNA CURRENT DISTRIBUTION*

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Summary—*The action of broadcast antennas with various current distributions is examined in an endeavor to determine the combinations which are most likely to be useful. In each case, the vertical radiation characteristics, the radiation resistance, and the electric field intensity at the surface of the earth one mile from the antenna are determined. The relative merits with respect to fading suppression are also considered.*

Of the several arrangements examined, some have been very disappointing. In no case where the total antenna height is at all reasonable for broadcast use does the field strength at one mile go to exceptionally high values. For heights of the order of a half wavelength, it is hard to find anything better than a straight vertical wire. Some of the arrangements save a small amount in height, but in no other respect do they improve on the straight wire.

The analysis of the antenna with decreased velocity and antennas spread in the horizontal plane shows that both are definitely ruled out of the picture.

The results show that the Franklin antenna would be very useful if it were not for the great heights required.

I. Introduction

In THE past several years, the antenna systems of broadcast stations have received a great deal of attention. A decade ago, the antenna system generally consisted of a T-shaped antenna suspended between two towers. This type of antenna gave no great control over the vertical radiation pattern. In an attempt to gain this control, a single tower was used as the antenna proper. Antennas of this type were extended to great heights compared to previous practice. Recent studies have shown that there is still room for a great deal of improvement in the design of these structures, since the current distributions on these towers generally differ a great deal from the sinusoidal distribution which had been assumed in the theoretical studies which led to the use of these tall structures. Recently, engineers have shown a great deal of interest in the possibilities of so controlling the current distribution on the antenna that desirable characteristics may be obtained with antennas of rather small height. It is the purpose of this investigation to examine the action of antennas with various current distributions, in an endeavor to determine the combinations which are most likely to be useful. In each case, we shall calculate the vertical radiation characteristics, the radiation resistance, and the electric field intensity at the surface of the earth one mile from the antenna when one watt of power is fed into the antenna. In the calculation of the electric intensity, the earth will be considered as a perfect conductor, and the antenna system will be considered to be 100 per cent efficient.

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II. Method of Determining the Antenna Characteristics

Let us suppose that our antenna is a vertical wire above a perfectly conducting earth. The current is distributed as shown in Fig. 1. We are interested in the electric intensity, F_θ, at a remote point, P, a distance,

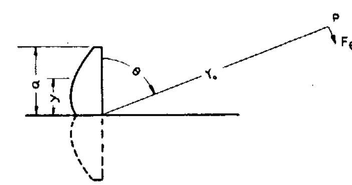

Fig. 1

r_0, from the base of the antenna. The angle between the line drawn from P to the antenna base and the antenna axis is θ. Then we can write

$$F_\theta = +j\frac{60}{r_0} I_0 K f(\theta) \epsilon^{-jkr_0} \tag{1}$$

where I_0 is the current at some reference point along the antenna. This reference point is entirely arbitrary, but is usually chosen either at the base of the antenna or at the point of greatest current. K is called the *form factor* of the antenna and $f(\theta)$ is the *vertical radiation characteristic.* When $f(\theta)$ is plotted on polar coördinate paper, it is sometimes referred to as the *vertical radiation pattern.* The above quantities are defined by the following relations:[1]

$$K = \left[k \sin\theta \int_{y=0}^{y=a} \frac{i_y}{I_0} \cos(ky\cos\theta)dy\right]_{\theta=90°}$$

$$= \left[k \int_{y=0}^{y=a} \frac{i_y}{I_0} dy\right] \tag{2}$$

and,

$$f(\theta) = \left[k \sin\theta \int_{y=0}^{y=a} \frac{i_y}{I_0} \cos(ky\cos\theta)dy\right] \Big/ K \tag{3}$$

where,

λ = wavelength

$k = 2\pi/\lambda$

i_y = current in the antenna at a point y units from the ground.

It should be noted that $f(\theta)$ is unity when θ is 90 degrees. In (1), F_θ is given in volts per centimeter when the current is measured in amperes and r_0 is measured in centimeters. If we replace the constant, 60, by 37.25 and measure r_0 in miles, the field strength is then given in millivolts per meter.

The radiation resistance at the reference point and the field intensity at one mile for a given power input are calculated by methods previously described.[1]

It is difficult to determine the relative merits with respect to fading of a number of antenna arrangements by merely inspecting the vertical radiation characteristics. These merits will be weighed in a slightly different fashion. It is assumed that the Heaviside layer height is 100 kilometers (62.5 miles) and that the layer has an efficiency of reflection of 33.3 per cent. We can then compute the magnitude of the reflected ray at any point on the surface of the earth if we know the vertical radiation characteristic of the antenna. We will designate this intensity as F_2. (Fig. 2.) The value of the direct or ground ray, designated

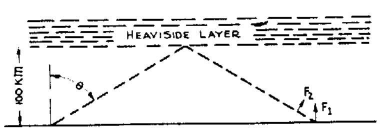

Fig. 2

by F_1, is next computed. We will assume an earth conductivity of 50×10^{-15} electromagnetic units and a wavelength of 300 meters. The ratio F_2/F_1 which is practically zero close to the antenna grows larger as we proceed away from the antenna, until F_1 is negligible compared to F_2. We will define d_1 as the distance along the ground from the antenna to the point where the ratio F_2/F_1 is unity, and call this distance the "100 per cent fading distance." The distance to the point at which the ratio is 0.5 will be designated as $d_{1/2}$, and will be called the "50 per cent fading distance." In most of the cases which we will examine, the values of d_1 and $d_{1/2}$ will be computed. The value of earth conductivity and wavelength used represents average conditions in the broadcast band. While it is admittedly arbitrary to assume a particular Heaviside layer height and reflection coefficient, the procedure is justified in that it does give us a basis for comparing two different antennas.

[1] H. E. Gihring and G. H. Brown, "Tower antennas for broadcast use, Appendix A," Proc. I.R.E., vol. 23, pp. 342–348; April, (1935).

III. Vertical Wire with Sinusoidal Distribution of Current

The characteristics of the vertical wire antenna over a perfect earth have been known for many years. The results will be repeated here since we will use this antenna as a standard of comparison. Let us

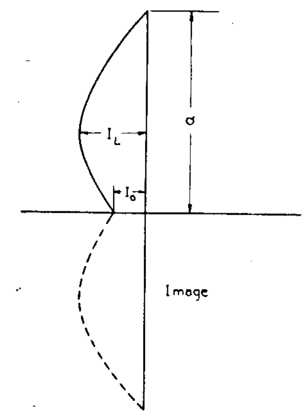

Fig. 3—Vertical wire antenna and image.

suppose that the current is distributed as shown by Fig. 3. Then the current distribution is

$$i_y = \frac{I_0 \cdot \sin(A - ky)}{\sin A} = I_L \sin(A - ky) \tag{4}$$

where I_0 is the current at the base of the antenna, and I_L is the magnitude of the loop current. When a is the height of the antenna and λ is the free space wavelength,

$$A = 2\pi a/\lambda. \tag{4a}$$

For the distribution in question, the vertical radiation characteristic is

$$f(\theta) = \frac{\cos(A\cos\theta) - \cos A}{\sin\theta[1 - \cos(A)]}. \tag{5}$$

This characteristic is plotted in Fig. 4 for a number of values of A. We see that the high angle radiation becomes a minimum when the value of A is about 190 degrees. When the antenna is taller than this, the secondary lobe of radiation becomes prominent. In fact, when the antenna is one wavelength high, there is no radiation along the ground. The form factor, K, referred to the loop of current is

$$K = 1 - \cos(A). \tag{6}$$

The radiation resistance referred to the loop current is

$$R_r(\text{ohms}) = 30\left[-\frac{\cos(2A)}{2}\{C + \log(4A) - Ci(4A)\} + \{1 + \cos(2A)\}\{C + \log(2A) - Ci(2A)\} + \sin(2A)\left\{\frac{Si(4A)}{2} - Si(2A)\right\}\right] \tag{7}$$

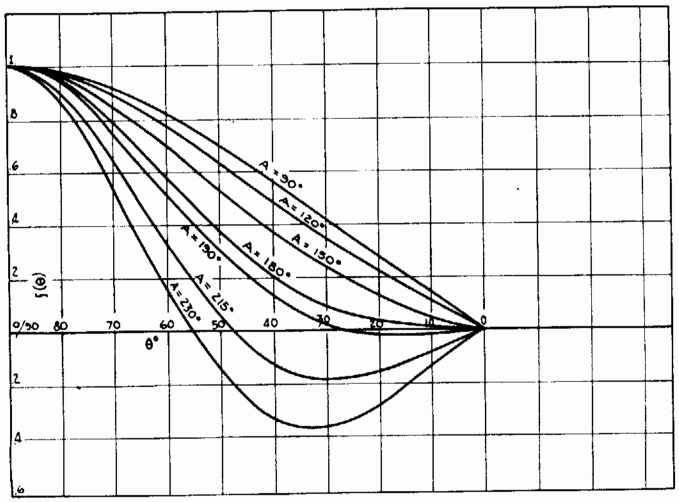

Fig. 4—Vertical radiation characteristics of vertical wire antenna.

where $C = 0.57721$ is Euler's constant, and $Ci(x)$ and $Si(x)$ are respectively the intergral-cosine and the integral-sine as defined on page 19 of the Jahnke-Emde "Funktionentafeln." Fig. 5 shows this radiation resistance plotted as a function of the height, A, in degrees. The field strength at one mile is also shown on this figure. We see that the maximum field strength is obtained when the antenna is 230 degrees tall. However, inspection of Fig. 4 shows that a 230-degree antenna has a rather large high angle lobe of radiation. Examination of the fading distance curve shown on Fig. 5 shows that an antenna whose height is about 190 degrees is the most desirable from the standpoint of fading reduction. For instance, a 190-degree antenna gives a field strength at one mile of 7.8 millivolts per meter for one watt input, while a 230-degree antenna gives 8.7 millivolts per meter, an increase of 11.0 per cent. At the same time, 100 per cent fading occurs at 100 miles if a 190-degree antenna is used, and at 60 miles if a 230-degree antenna is used.

IV. Vertical Wire with Capacity Hat at the Top

Let us now suppose that the vertical wire has a nonradiating capacity area at the top so that the current distribution is as shown in Fig. 6.

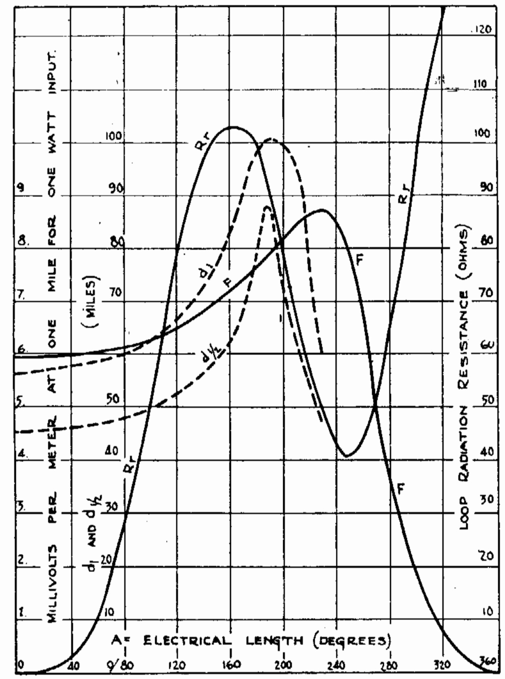

Fig. 5—Characteristics of the simple vertical wire antenna.

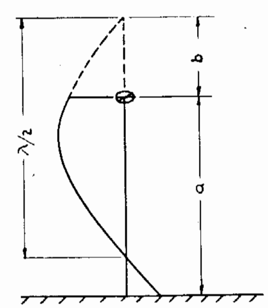

Fig. 6—The vertical wire antenna with a capacity hat at the top.

Then b is the length of the portion of sine wave suppressed by the capacity area. We define the quantities

$$\left.\begin{aligned} B &= 2\pi b/\lambda \text{ radians} = 360b/\lambda \text{ degrees} \\ A &= 2\pi a/\lambda \text{ radians} = 360a/\lambda \text{ degrees} \\ G &= A + B \end{aligned}\right\}. \tag{8}$$

Then $K = \cos(B) - \cos(G)$ (referred to the loop current) (9)

and,

$$f(\theta) = \frac{\cos B \cos(A \cos\theta) - \cos\theta \sin B \sin(A\cos\theta) - \cos G}{\sin\theta[\cos B - \cos G]}. \tag{10}$$

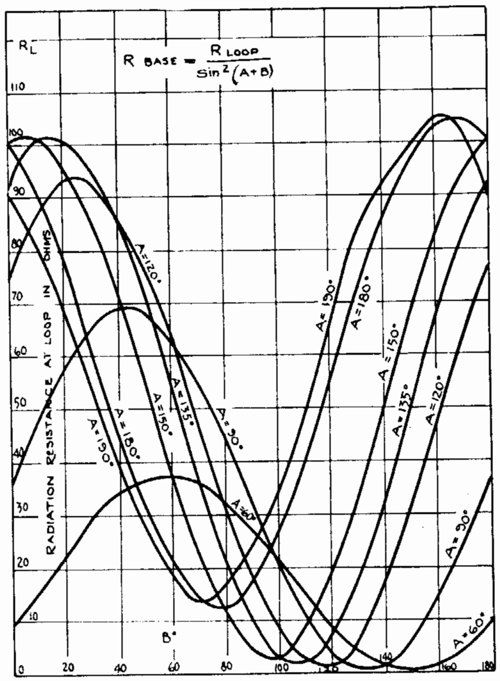

Fig. 7—Loop radiation resistance of the antenna with a hat at the top.

The radiation resistance (referred to the loop current) is

$$R_r(\text{ohms}) = 30\left[\sin^2 B\left\{\frac{\sin(2A)}{2A} - 1\right\}\right.$$
$$-\frac{\cos(2G)}{2}\{C + \log(4A) - Ci(4A)\}$$
$$+\{1 + \cos(2G)\}\{C + \log(2A) - Ci(2A)\} \tag{11}$$
$$\left.+\sin(2G)\left\{\frac{Si(4A)}{2} - Si(2A)\right\}\right].$$

The radiation resistance as a function of B, with A as a parameter, is shown in Fig. 7.

The field strength at one mile is shown by Fig. 8. We see that in all cases where the antenna is less than 200 degrees tall, the field in-

tensity increases with B for a way, and then drops suddenly to zero, rising again almost as abruptly. The zero point occurs when

$$B = 180 \text{ degrees} - A/2. \tag{12}$$

When B is zero, the vertical characteristic is given by the curves of Fig. 4. As B is increased the field pattern changes just as if the height of the antenna were increased. For purposes of illustration, the vertical characteristic of an antenna of height 150 degrees is shown by Fig. 9 for a number of values of B. We see that when B is about 47 degrees,

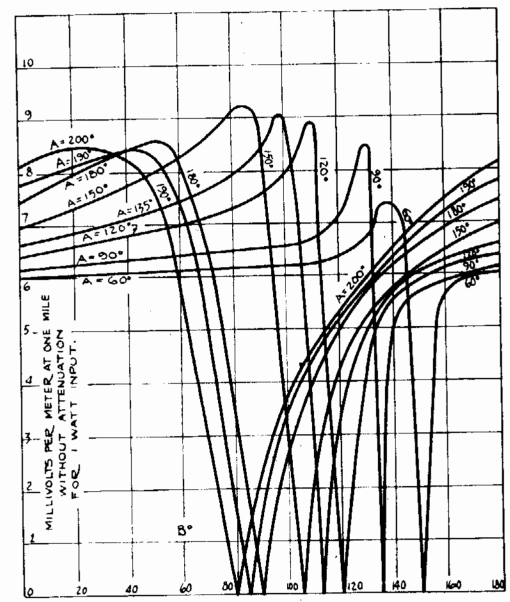

Fig. 8—Field strengths obtainable from the antenna with capacity hat at the top.

the vertical pattern is similar to that obtained with a straight vertical wire 190 degrees tall. For any height, A, we can find the value of B which will give the same vertical pattern as a 190-degree antenna by selecting the value of B which corresponds to a field strength of 7.8 millivolts per meter at one mile on Fig. 8. Since the curve of field intensity peaks, it is important to note that the correct value of B lies on the left side of the peak.

This method seems important, since we are able to simulate a 190-degree antenna with much less height. In many cases, we shall desire a value of B so large that it cannot be obtained by a simple capacity area at the top. The desired result can, however, be obtained by inserting an inductance between the top of the vertical wire and the capacity area. Such a scheme was first disclosed by van der Pol[2] many years ago. To obtain the results predicted by theory, it is necessary that inductance so inserted have a very low resistance, since the current at the coil point may be many times the current at the base. To be specific, let us assume that the coil has one ohm of resistance. Let the capacity area and the inductance of the coil be such that the antenna is adjusted to give a vertical field pattern similar to that of a 190-degree antenna. Let us assume that 1000 watts of power is fed into the an-

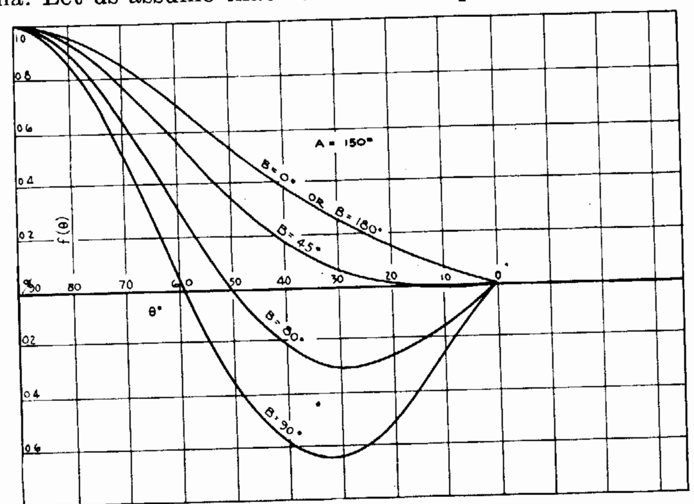

Fig. 9—Variation of the vertical radiation characteristics with top loading.

tenna. Then the conditions for several heights are shown in the following tabulation.

A Degrees	B Degrees	Radiation Res. (at loop)	Radiation Res. (at coil)	Total Res. (at coil)	Watts Radiated	Watts Lost in Coil	F
150	45	63.5	127.0	128.0	992.0	8.0	246.0
135	77	26.2	27.3	28.3	965.0	35.0	243.0
120	97	11.0	11.2	12.2	918.0	82.0	236.0
90	125	1.5	2.23	3.23	690.0	310.0	205.0

In the above, we have assumed a constant coil resistance. Actually, this resistance will probably be much greater than one ohm for the lower heights. Practically, it is seen that it might be possible to use this scheme for values of A as low as 135 degrees, although a height of 150 degrees would be much more desirable. When the antenna is this tall, it is not necessary to insert the coil at the top. An interesting practical example of an antenna of this type is that at Breslau, Germany. Breslau is a sixty-kilowatt station. The antenna is a straight vertical wire supported by a wooden tower. The suppression of current at the top is attained by a ring, ten meters in diameter placed at the top of the tower. This ring suppresses about forty meters or one-eighth of a wavelength.[3] The antenna height is 455 feet or 138.5 meters, while the wavelength is 325 meters. The current node is nineteen meters from the ground. The maximum current comes at 100 meters from the ground. No energy is radiated at sixty to sixty-five degrees to the horizon. The horizontal radiation is increased about twenty-five per cent while the radius free from fading is increased about forty per cent. We see that this antenna is 150 degrees tall with forty-five degrees of loading at the top. The intensity along the horizon in the ideal case is twenty-seven per cent greater than that due to a short antenna. This agrees well with the above observed value of twenty-five per cent.

[2] Balth. van der Pol, *Jahr. der draht. Telegraphie und Telephonie*, Band 13, Heft 3, pp. 217–238, (1918).

When a steel tower is used for the antenna, it will be necessary to use a larger outrigger or larger coil to attain the same amount of suppression of current at the top of the antenna. This will be especially true if the tower is of nearly uniform cross section from bottom to top. If this last-named condition is not fulfilled, the current distribution will no longer be sinusoidal, and it would then become a rather formidable task to adjust the coil to the correct value to simulate a 190-degree antenna. A method has been devised which will enable one to determine the correct coil setting from measurements made on the ground close to the antenna.

V. Sectionalized Antenna

As stated in the previous section, it may become difficult to place a big enough outrigger on the tower to attain the effect wanted and still have the tower nearly uniform in cross section. One way out of this situation is the so-called "sectionalized antenna." The coil is placed some distance from the top of the antenna, and no outrigger is used. Thus the section above the coil radiates. The current distribution is as shown in Fig. 10. The total antenna height is d, while a is the distance from the earth to the coil. The length of sine wave which would be above the coil point if the coil and top section of antenna were replaced by a straight wire is b. Then we define

$$\begin{aligned} D &= 2\pi d/\lambda \text{ radians} = 360d/\lambda \text{ degrees} \\ A &= 2\pi a/\lambda \text{ radians} = 360a/\lambda \text{ degrees} \\ B &= 2\pi b/\lambda \text{ radians} = 360b/\lambda \text{ degrees.} \end{aligned} \tag{13}$$

[3] *Telefunken Zeit.*, August, (1933).

The vertical radiation characteristic, referred to the loop current, is

$$f(\theta) = \Big[\cos B \cos (A \cos \theta) - \cos \theta \sin B \sin (A \cos \theta) - \cos (A + B) + \frac{\sin B}{\sin (D - A)}\{\cos (D \cos \theta) - \cos (D - A) \cos (A \cos \theta) + \cos \theta \sin (D - A) \sin (A \cos \theta)\}\Big]/K \sin \theta. \quad (14)$$

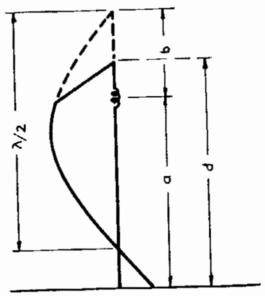

Fig. 10—The sectionalized antenna.

The form factor referred to the same point is

$$K = \Big[\cos B - \cos (A + B) + \frac{\sin B}{\sin (D - A)}\{1 - \cos (D - A)\}\Big]. \quad (15)$$

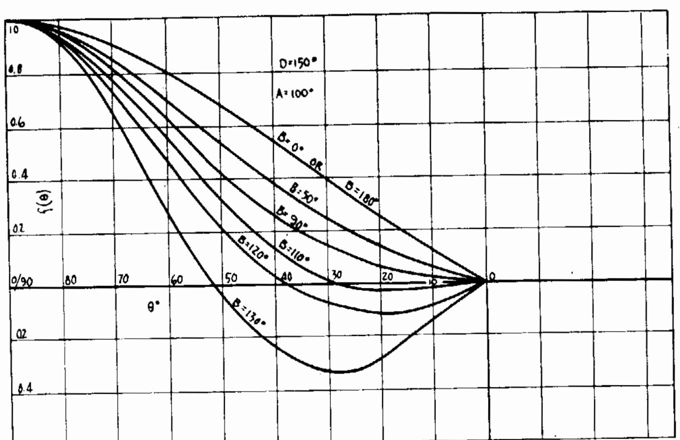

Fig. 11.—Vertical radiation characteristics of a sectionalized antenna.

Let us consider a particular case. We shall choose a total height of $D = 150$ degrees, with the coil placed at $A = 100$ degrees. Then Fig. 11 shows the vertical radiation characteristics for a number of values of B. This antenna thus has the same characteristic variation as the antenna with the loaded top. The field intensity at one mile and the loop radiation resistance is given in Fig. 12. The resistance referred to the coil point is found from this resistance curve by

$$R_{cp} = R_{loop}/\sin^2 B. \quad (16)$$

Likewise,

$$R_{base} = R_{loop}/\sin^2 (A + B). \quad (17)$$

Thus, in the above case, when B is 110 degrees (for best fading suppression) the resistance at the coil point is 23.6 ohms and the base resistance is 84.0 ohms.

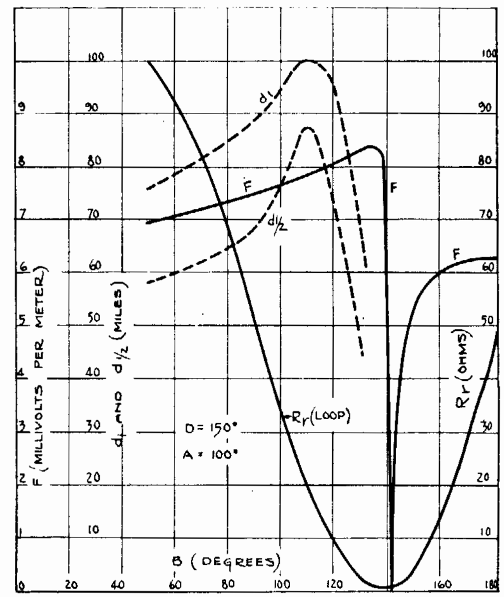

Fig. 12—The dependence of the characteristics of a sectionalized antenna upon the amount of sectionalizing coil.

It is of interest to examine the constants for other coil positions. Let us suppose that for each coil position the coil is readjusted so that the antenna simulates an ordinary 190-degree antenna. The variation of the quantities in question when D is 150 degrees is shown by Fig. 13. Fig. 14 shows a similar set of quantities when D is 120 degrees. From considerations of this sort, we find that the 150-degree height is quite good, while it is very unlikely that we could obtain good efficiencies with a height of 120 degrees.

VI. Antenna with Constant Current

The next case to come to our attention is the antenna with constant current. By this, we mean a vertical antenna so loaded that the current at any point along the antenna has the same magnitude and phase position as the current at any other point. Then if the total height of the antenna is a, we can define

$$A = 2\pi a/\lambda \text{ radians} = 360a/\lambda \text{ degrees}. \tag{18}$$

The form factor is

$$K = A = 2\pi a/\lambda \tag{19}$$

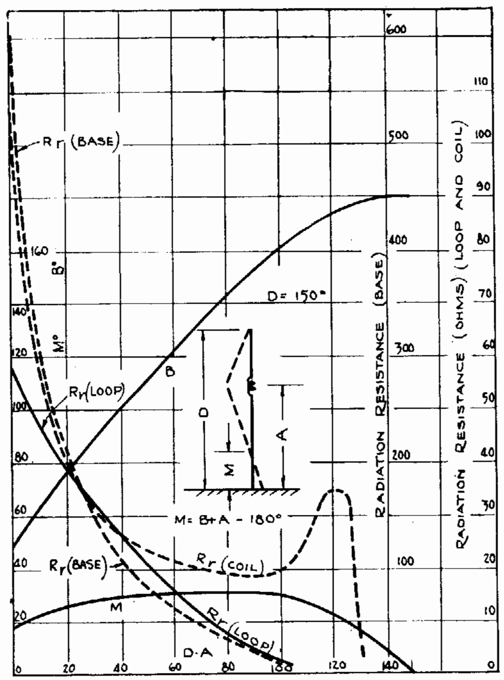

Fig. 13—The dependence of the characteristics of a sectionalized antenna upon sectionalizing coil position. (In each position, the coil is adjusted so that the vertical radiation characteristic is the same as that of a simple 190-degree antenna.)

and the vertical characteristic is

$$f(\theta) = \frac{\tan\theta \sin(A\cos\theta)}{A}. \tag{20}$$

The family of vertical radiation characteristics is shown in Fig. 15. The field strength at one mile and the radiation resistance is shown by Fig. 16. It is seen that this antenna must be practically 190 degrees tall to simulate a 190-degree antenna with sinusoidal distribution. Thus this scheme does not appear to be very promising, for it does not seem reasonable to go to all the trouble involved in getting a constant current along the antenna when the same vertical pattern can be obtained by merely using a straight vertical wire antenna of the same height.

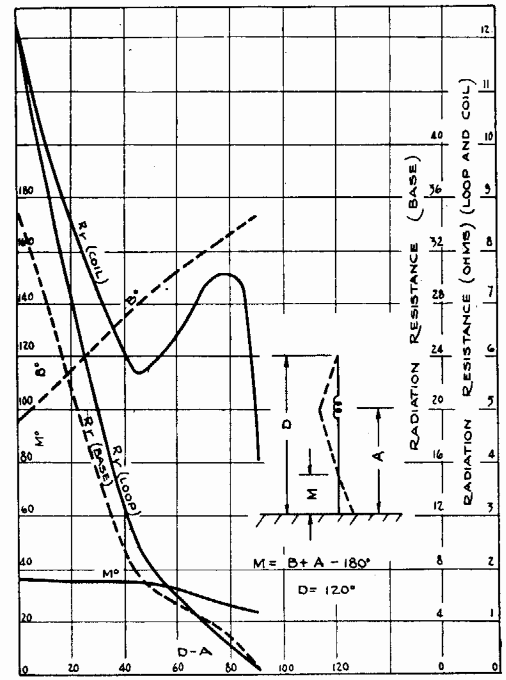

Fig. 14—Similar to Fig. 13, but for a different total height.

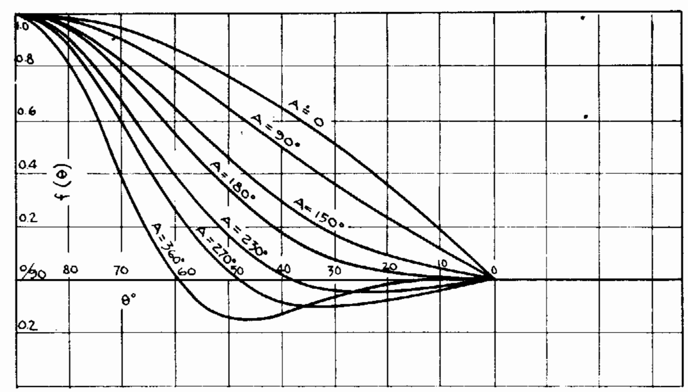

Fig. 15—Vertical radiation characteristics of the antenna with constant current.

VII. Antenna Element Elevated from the Earth

Another possibility of antenna design which suggests itself is to raise an antenna element above the surface of the earth. For convenience, let us assume that the element in question is a half-wave antenna, as shown in Fig. 17. Let h be the distance from the ground to

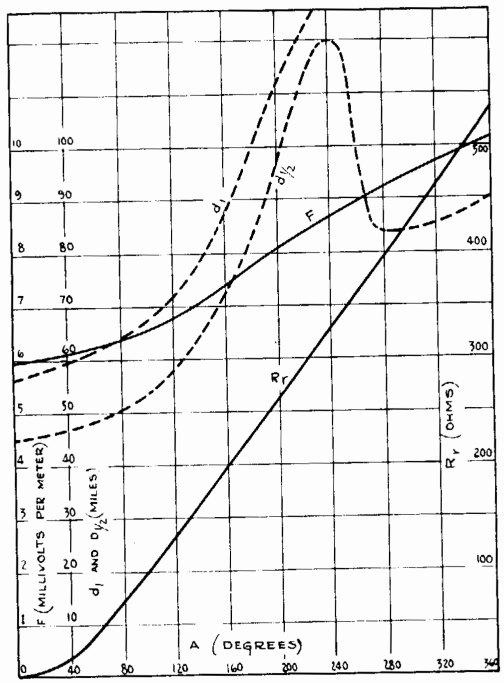

Fig. 16—The characteristics of the antenna with constant current.

the mid-point of the antenna. Then the distance from the ground to the lower end of the antenna is $d = h - \lambda/4$. The form factor of the

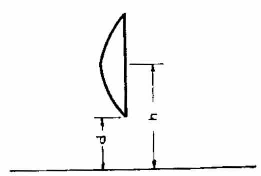

Fig. 17

system is $K = 2$, and the vertical radiation characteristic is

$$f(\theta) = \frac{\cos\left(\dfrac{\pi}{2}\cos\theta\right)\cos(H\cos\theta)}{\sin\theta} \tag{21}$$

where $H = 2\pi h/\lambda$ radians $= 360h/\lambda$ degrees. Since we are considering a half-wave element, the lower end is at the earth's surface when H is

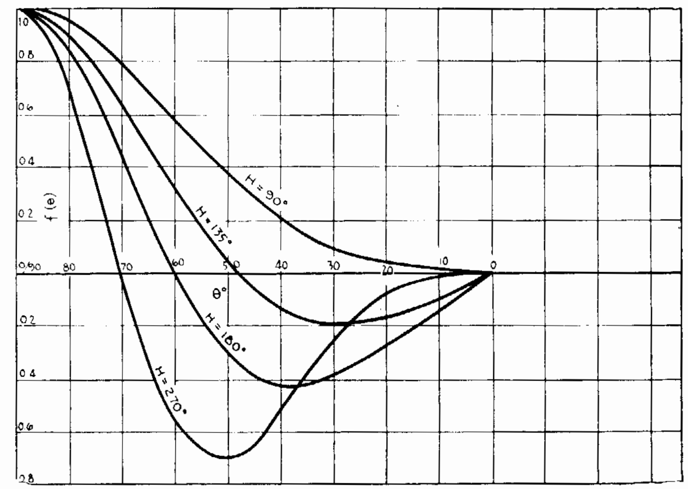

Fig. 18—Vertical radiation characteristics of a half-wave element elevated above the earth.

90 degrees. Fig. 18 shows the vertical radiation characteristic for a number of values of H. The field intensity at one mile and the radiation resistance is shown by Fig. 19. From the fading distance curve on this

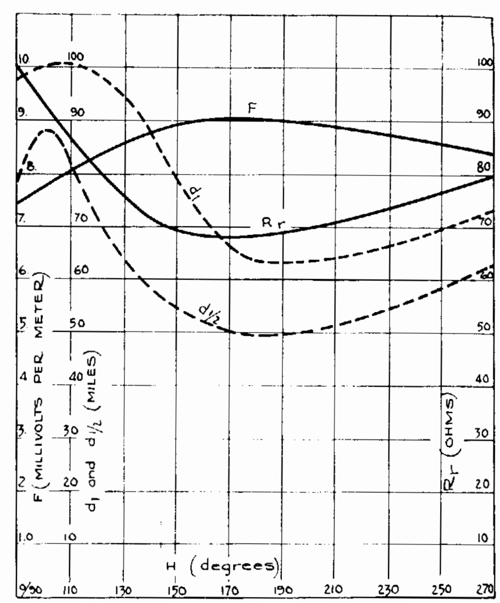

Fig. 19—Characteristics of an antenna element elevated above the earth.

figure, we see that this antenna simulates a 190-degree antenna when H is 102 degrees. This means that the top of the antenna is 192 degrees from the ground.

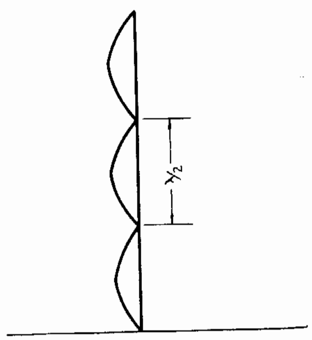

Fig. 20—The Franklin antenna.

Since the vertical characteristic of the half-wave element in space, $\cos\left[(\pi/2)\cos\theta\right]/\sin\theta$, is essentially equivalent to $\sin\theta$, we can take the above vertical characteristics and the field intensity at one mile as indicative of that obtained if the element were shorter than one-half wavelength, where H is still the distance to the center of the element from the surface of the earth.

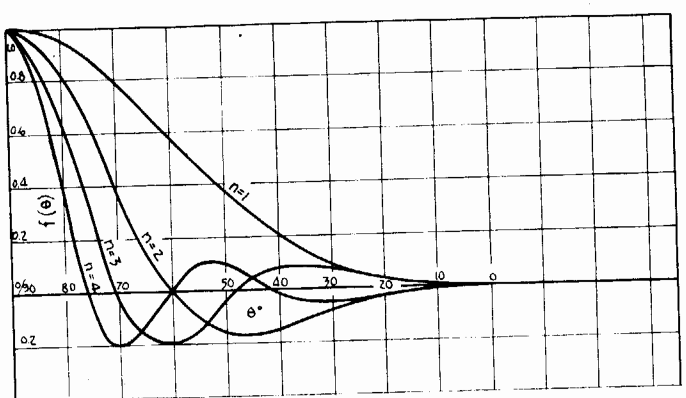

Fig. 21—Vertical radiation characteristics of the Franklin antenna.

VIII. The Franklin Type Antenna

The Franklin antenna, which has been rather useful in short-wave work, consists of a number of half-wave antennas placed end to end on a vertical line, and so fed that the currents in each element are equal and in phase. (See Fig. 20.) Let n be the number of half-wave elements

above the earth. Then the form factor is

$$K = 2n. \tag{22}$$

The vertical radiation characteristics for several values of n are shown in Fig. 21.

The field intensity at one mile and the total antenna resistance is given by Fig. 22. We see that very high field strengths are obtained when many sections are used, provided that the losses in the phasing

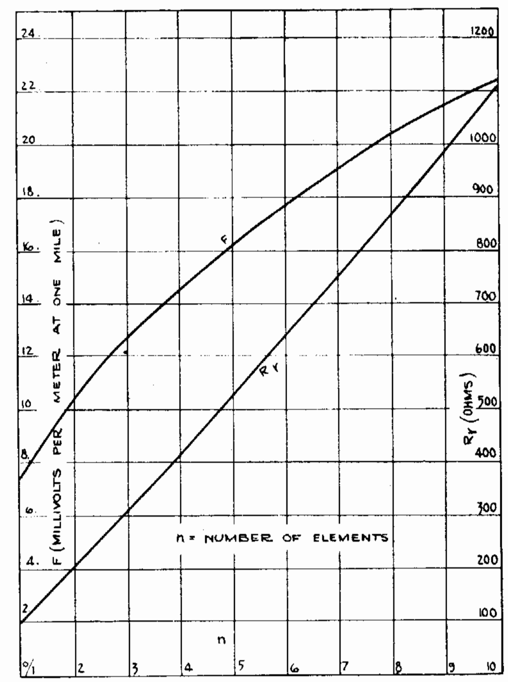

Fig. 22—Field intensity and radiation resistance of the Franklin antenna.

equipment do not become large. It is also extremely unlikely that antennas whose height is more than one wavelength could be built economically.

IX. Antenna with Decreased Velocity

Another arrangement which has been proposed by various engineers from time to time is the antenna with decreased velocity. The idea is to decrease the velocity of propagation of the current waves on the antenna wires so that current nodes are separated by less than one half of the free space wavelength. For instance, if the wire were so loaded that the velocity were but one half the velocity of light in free space, we would have a half sine wave distribution occurring in one fourth of a free space wavelength. Several methods of loading might be utilized. One arrangement might be to arrange the antenna wire in a spiral. The velocity of propagation might also be decreased by surrounding a vertical wire with a mass of dielectric. We shall not interest ourselves in the methods of accomplishing this condition, but shall note the consequences of such an arrangement.

Let us now examine Fig. 23. This shows an antenna of height a loaded at the top with a capacity area. The distance, b, is the actual

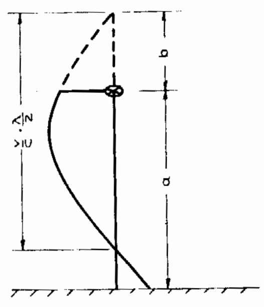

Fig. 23

length of sine wave suppressed by the loading at the top. We shall now define the following quantities:

a = actual height of antenna
b = actual length of sine wave suppressed by loading at the top
c = velocity of propagation of light (or radio waves) in free space ($=3\times10^{10}$ centimeters)
f = frequency of the transmitter
$\lambda = c/f$ = free-space wavelength
v = velocity of current wave on the wire
$A_0 = 2\pi a/\lambda$ (radians) $= 360a/\lambda$ (degrees)
$B_0 = 2\pi b/\lambda$ (radians) $= 360b/\lambda$ (degrees)
$A = A_0/(v/c)$
$B = B_0/(v/c)$
$G = A + B$

Then the form factor (referred to the loop current) is

$$K = \frac{v}{c}[\cos B - \cos G] \tag{23}$$

and the vertical radiation characteristic

$$f(\theta) = \frac{v}{c}\sin\theta\left[\cos(B)\cos(A_0\cos\theta) - \frac{v}{c}\cos\theta\sin B\sin(A_0\cos\theta) - \cos G\right]\Big/K\left[1 - \left(\frac{v}{c}\right)^2\cos^2\theta\right]. \tag{24}$$

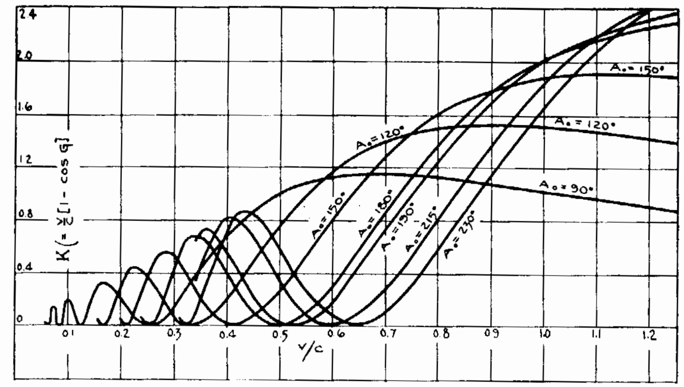

Fig. 24—The form factor of the antenna with decreased velocity.

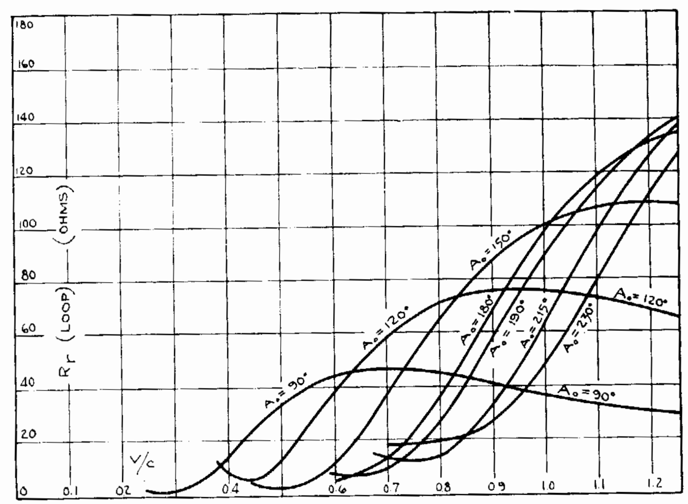

Fig. 25—Loop radiation resistance of the antenna with decreased velocity.

Let us first examine the case when no loading is placed at the top. Then $B = B_0$ = zero degrees. The form factor for this case is shown as a function of v/c for a number of values of A_0 by Fig. 24. Fig. 25 shows the radiation resistance for the same cases. This radiation resistance is referred to the loop current. To find the radiation resistance at the base, we would use the following relation:

$$R_{\text{base}} = R_{\text{loop}}/\sin^2 (G).$$

The radiation resistance at the base for a number of cases is shown by Fig. 26.

The field strength at one mile for one watt input is given by Fig. 27. We that see for a fixed antenna height, the field strength at one mile increases as the velocity is decreased until the field strength reaches about 8.5 millivolts per meter. The field strength then drops rapidly to zero

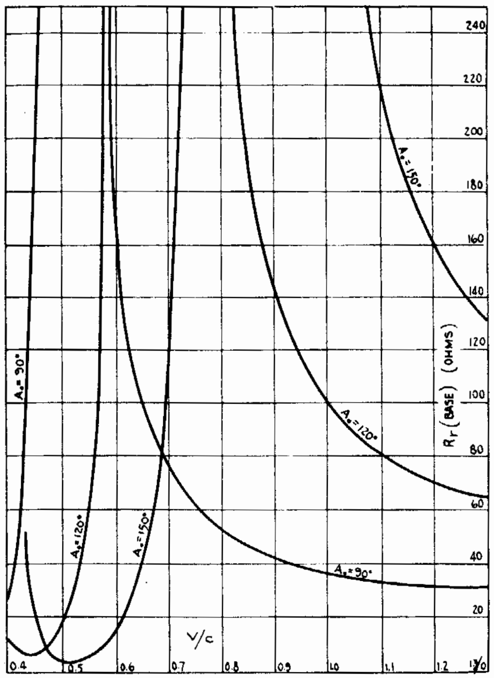

Fig. 26—Base radiation resistance of the decreased velocity antenna.

with further decreases in velocity. It is important to note that the maximum values of field strength occur at the point where the base radiation resistance is extremely small.

The vertical radiation characteristics will now be examined. Let $A_0 = 150$ degrees. Fig. 28 shows the vertical radiation characteristic for a number of values of the ratio, v/c. It is seen that a decrease in velocity for a fixed height changes the vertical characteristics in the same manner as an increase in height of a straight vertical wire. This is true for any value of A_0 which we choose. This fact can be made use of to show the characteristics of the decreased velocity antenna. We shall define a quantity, A_s, in the following fashion: Suppose we have under con-

sideration a specific antenna of height A_0 degrees and a velocity ratio, v/c. Then A_s is the height in degrees of a straight vertical wire antenna

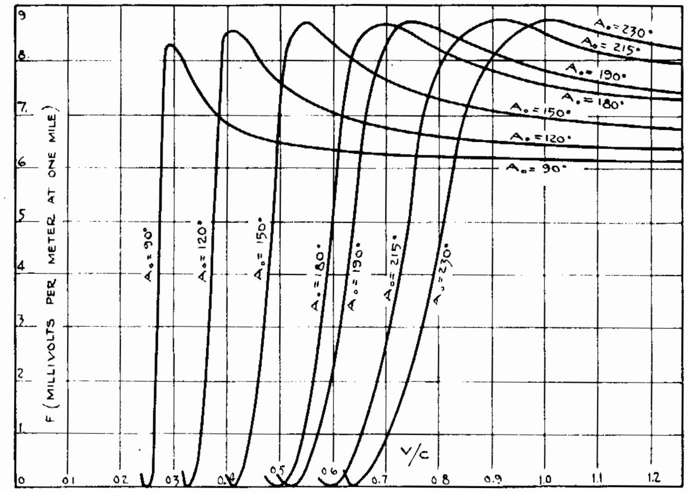

Fig. 27—Field strengths obtainable from the decreased velocity antenna.

which has the same vertical characteristic and field strength at one mile as the decreased velocity antenna under consideration. This quan-

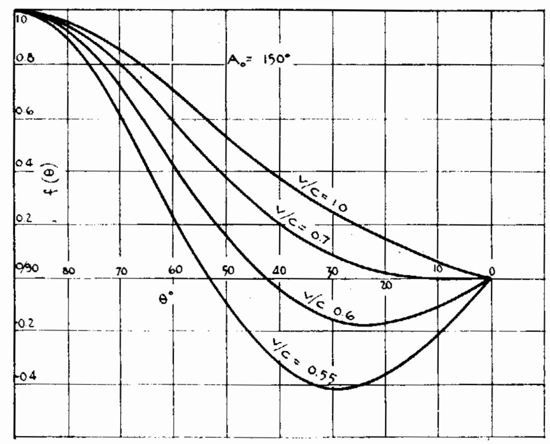

Fig. 28—Vertical radiation characteristics of a decreased velocity antenna.

tity, A_s, is shown as a function of v/c for a number of values of A_0 in Fig. 29. The broken line on this diagram indicates the values of v/c where the current node occurs at the base of the antenna.

The above relations can be used to examine another interesting case. It has been shown previously that it was desirable to build steel tower antennas so that the cross section is substantially constant over the entire length rather than to have the tower much smaller at

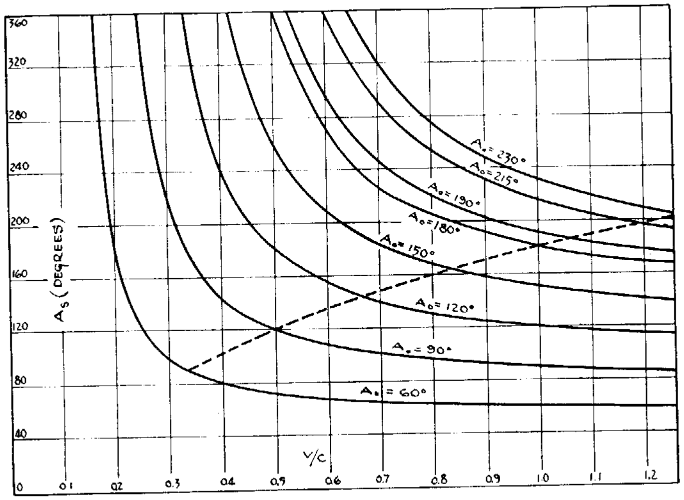

Fig. 29—Curves of similitude of the decreased velocity antenna. A_0 is the actual height of the decreased velocity antenna. A_s is the height of a vertical wire antenna which has the same vertical radiation characteristic and yields the same field intensity at one mile.

the top than at the base. It has been suggested that it is desirable to go still further by making the tower flare out at the top, thus moving the current maximum up the tower. We shall examine an extreme case for purposes of illustration.

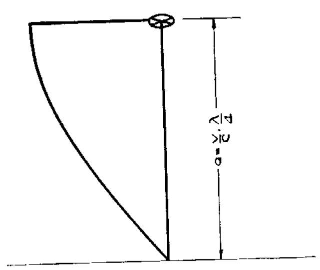

Fig. 30

Suppose that for any particular height antenna, the velocity is so altered and the loading at the top so altered that the current loop occurs at the top and the current node at the bottom of the antenna. This condition implies a velocity greater than the velocity of light when the antenna is greater than one quarter of a free-space wavelength in height. This current distribution is shown by Fig. 30. Fig. 31 shows the

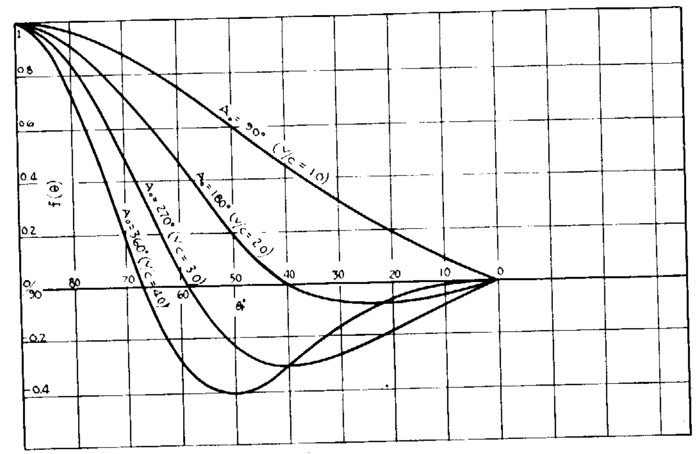

Fig. 31—Vertical radiation characteristics of the current distribution shown in Fig. 30.

vertical radiation characteristic for a number of antenna heights. The corresponding field intensity at one mile and the radiation resistance referred to the current at the top of the antenna is given in Fig. 32.

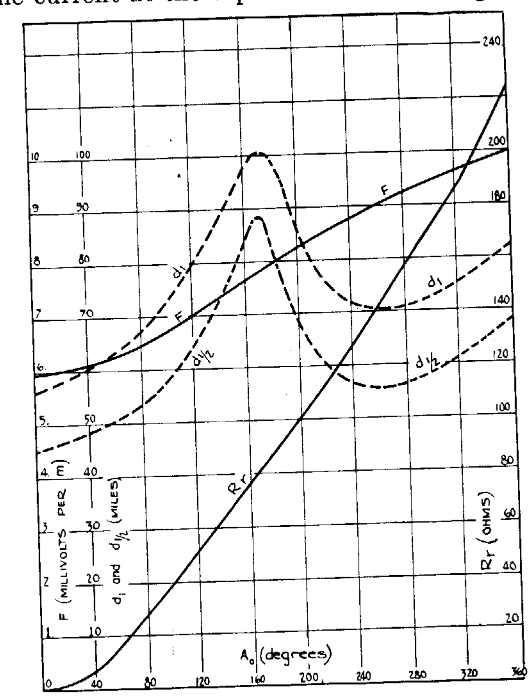

Fig. 32—Field intensity, radiation resistance, and distance to the fading zone of the current distribution shown in Fig. 30.

The distance to the fading zone is also shown. It is seen that if we wish to simulate a vertical wire 190-degree antenna, this type of antenna must be 170 degrees high. This is not a very great saving in height when it is considered that it would be both expensive and difficult to attain the current distribution in question.

It should be further pointed out that all radiation resistances were computed with reference to the vertical component of antenna current. When the velocity of propagation is decreased by spiraling the antenna wire, the radiation resistance referred to the actual current in the wire is the above calculated values multiplied by $(v/c)^2$. This substantially reduces the resistance in all cases where v/c is less than unity.

X. Franklin Type Antennas with Decreased Velocity

We have previously shown that the Franklin antenna composed of half-wave sections gave substantial increases in field strength, but that at the same time the heights became excessive. It has been suggested that the velocity on each element be reduced to one half the velocity

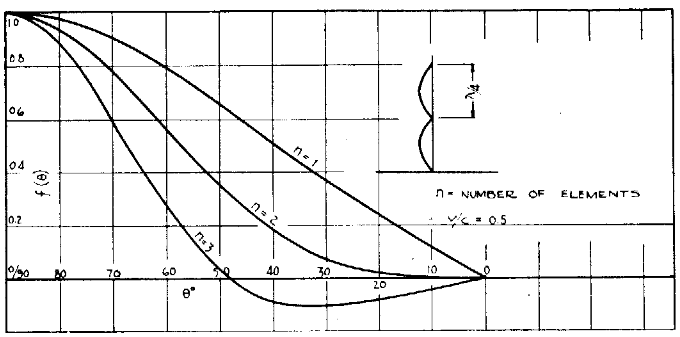

Fig. 33—Vertical radiation characteristics of the Franklin antenna with decreased velocity.

of light so that one half a sine wave of current will occur in a quarter wave of space. This arrangement is supposed to yield large gains in field strength. A few computations have been made concerning this case. Each element is 90 space degrees in length. The ratio, v/c, is 0.5 while n is the number of elements. The vertical radiation characteristics are shown for one, two, and three elements by Fig. 33. We see that the single element gives a vertical characteristic similar to that obtained from a vertical wire, 120 degrees tall. The two-element arrangement yields a vertical characteristic similar to that obtained from an ordinary half-wave antenna, while the three-element antenna is similar to a conventional 215-degree antenna. The following table shows the actual results:

n	mv/m at 1 Mile	R_1 (ohms)	R_2 (ohms)	Total Height (measured in free-space wavelengths)	Height of Equivalent Vertical Wire (measured in free-space wavelengths)
1	6.30	35.0	8.75	0.25	0.333
2	7.67	94.0	23.5	0.5	0.514
3	8.58	169.8	42.45	0.75	0.61

In the above table, R_1 is the total radiation resistance of the system referred to the vertical component of current. If the decreased velocity is obtained by spiraling the conductor, $R_2 = R_1/4$ is the resistance with reference to the actual current in the wire.

XI. Antennas Extended in the Horizontal Plane

We have seen that the process of modifying the current distribution along a vertical axis has not shown any current distribution to be outstanding, when the antenna height is less than a wavelength. The next obvious procedure is to extend the antennas over the horizontal plane. Since we are interested in broadcast antennas which radiate uni-

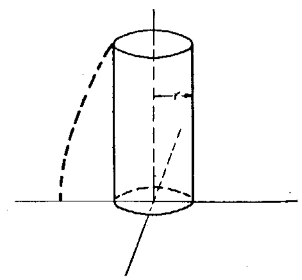

Fig. 34

formly in all directions along the horizontal plane, the antenna elements must be circularly symmetrical. In other words, each element of the antenna must itself be a solid ring of antennas. Of course, this ring can be simulated to a certain extent by placing several vertical antennas on the circumference of a circle. The ideal case, however, is that in which each element is a cylinder. We shall examine this case in detail, since the results of this case will be applicable to the case of several antennas on a circumference. This type of antenna was first discussed by Bohm.[4]

Let us suppose our antenna to be a cylinder, of radius, r. The cylinder is one-quarter wavelength tall and is placed over a perfect earth. (Fig. 34.) It is assumed that the current on this cylinder is sinusoidally distributed. The total current at the base of this cylinder is I_1. Then the form factor is

$$K = J_0(2\pi r/\lambda) \qquad (25)$$

[4] O. Bohm, *Zeit. für Hochfrequenz. u. Elect.*, p. 137, October, (1933).

where J_0 is Bessel's function of the first kind and zero order. The vertical radiation characteristic is

$$f(\theta) = \frac{\cos\left(\dfrac{\pi}{2} \cos \theta\right)}{\sin \theta} J_0(2\pi r \sin \theta/\lambda)/K. \tag{26}$$

Fig. 35 shows a plot of $J_0(2\pi r/\lambda)$ as a function of r/λ.

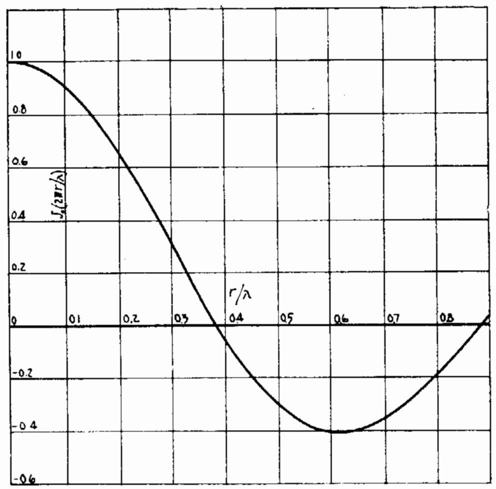

Fig. 35—Form factor of the cylindrical current sheet of Fig. 34.

Fig. 36 shows the vertical radiation characteristics for a number of cylinder radii.

The radiation resistance and the field intensity at one mile for one watt input is shown on Fig. 37. We see that the field strength at the horizon goes to zero whenever $2\pi r/\lambda$ is a root of the Bessel function. The first root occurs when r equals 0.382 wavelength.

While the above results show a rather interesting characteristic, it is evident that a ring of antennas used alone is of little use. We shall next examine the case where another quarter-wave antenna is placed at the center of the ring. The current at the base of this antenna is I_0. The currents in the inner antenna and the outer ring may be related to each other in any phase or ratio. Suppose that this relation is

$$I_1 = I_0(a + jb). \tag{27}$$

Then the form factor (referred to the center antenna) is

$$K = \sqrt{\left[1 + aJ_0\left(\frac{2\pi r}{\lambda}\right)\right]^2 + \left[bJ_0\left(\frac{2\pi r}{\lambda}\right)\right]^2} \tag{28}$$

where r is the radius of the ring of antennas.

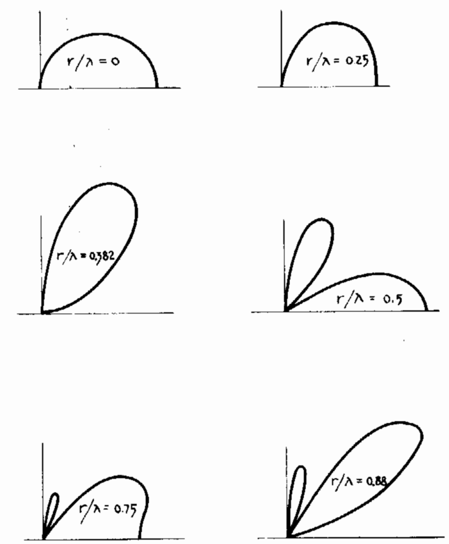

Fig. 36—Vertical radiation patterns of cylindrical current sheets.

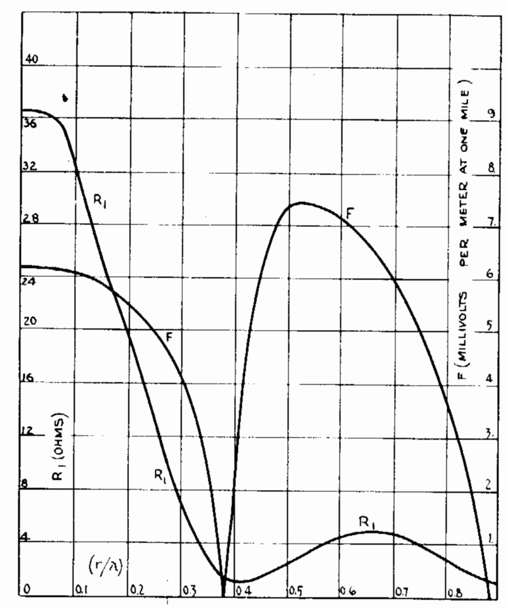

Fig. 37—Field strength and radiation resistance of cylindrical current sheet.

The vertical radiation characteristic is

$$f(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{K\sin\theta}\sqrt{\left[1 + aJ_0\left(\frac{2\pi r}{\lambda}\sin\theta\right)\right]^2 + \left[bJ_0\left(\frac{2\pi r}{\lambda}\sin\theta\right)\right]^2}. \quad (29)$$

It is obvious from the above expressions that the quantity b must be

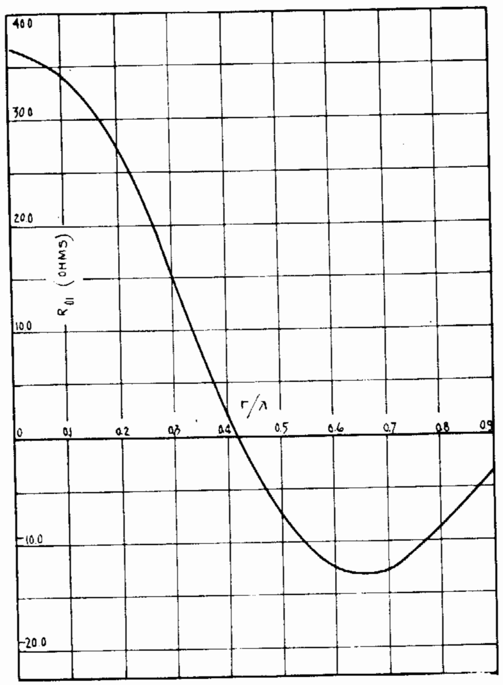

Fig. 38—Mutual resistance between cylinder and concentric wire.

zero to secure sky-wave suppression. It also seems evident that this condition must also be true to secure the maximum field strength along the horizon. That this is true will be shown by the following analysis.

The resistance of the ring of antennas has already been shown. This resistance will be called R_1. The resistance of the center antenna is $R_0=36.6$ ohms. The mutual resistance between the center antenna and the ring of antennas, R_{01}, is given as a function of r/λ by Fig. 38. The power input into the antenna is then

$$P = [a^2R_1 + 2aR_{01} + b^2R_1 + R_0]I_0^2 \text{ (watts)}. \quad (30)$$

Then the field strength at one mile for one watt input is

$$F = \frac{37.25\sqrt{\left[1 + aJ_0\left(\frac{2\pi r}{\lambda}\right)\right]^2 + \left[bJ_0\left(\frac{2\pi r}{\lambda}\right)\right]^2}}{\sqrt{[a^2R_1 + 2aR_{01} + b^2R_1 + R_0]}} \text{ (mv/m)} \quad (31)$$

or,

$$F^2 = \frac{37.25^2\left\{\left[1 + aJ_0\left(\frac{2\pi r}{\lambda}\right)\right]^2 + \left[bJ_0\left(\frac{2\pi r}{\lambda}\right)\right]^2\right\}}{a^2R_1 + 2aR_{01} + b^2R_1 + R_0}. \quad (32)$$

To find the value of b which gives the maximum field strength along the ground, we differentiate the last expression with respect to b and set equal to zero. This yields

$$\frac{dF^2}{db} = \frac{37.25^2\left\{2J_0{}^2\left(\frac{2\pi r}{\lambda}\right)[a^2R_1 + 2aR_{01} + R_0] - 2R_1\left[1 + aJ_0\left(\frac{2\pi r}{\lambda}\right)\right]^2\right\}b}{[a^2R_1 + 2aR_{01} + b^2R_1 + R_0]^2} \quad (33)$$

$$= 0$$

or,

$$b = 0. \quad (34)$$

Thus we see that the current in the inner conductor must be either in phase or in phase opposition with the current in the outer ring.

To find the best value of a, we again differentiate, this time with respect to a, and set equal to zero. This yields the value of a, which will give the greatest field strength along the ground as

$$a_m = \frac{R_{01} - R_0J_0\left(\frac{2\pi r}{\lambda}\right)}{R_{01}J_0\left(\frac{2\pi r}{\lambda}\right) - R_1} \quad (35)$$

and the value of a, which gives zero field strength along the ground, is

$$a_0 = -\frac{1}{J_0\left(\frac{2\pi r}{\lambda}\right)}. \quad (36)$$

The values of a_m were substituted back into (31) and the values of maximum field strengths computed. It was found that the largest field strength obtainable for ring diameters less than two wavelengths was about 9.0 millivolts per meter for one watt input into the system, or a gain over a single quarter-wave antenna of forty-six per cent.

Fig. 39 shows the field strength at one mile for one watt input as a function of a, for a number of values of r/λ. Fig. 40 shows the varia-

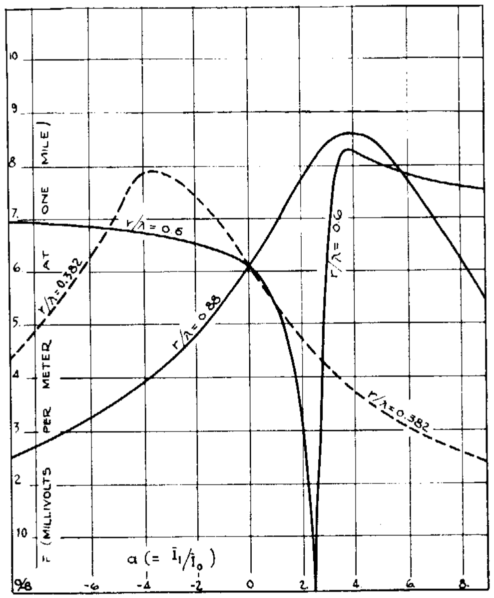

Fig. 39—Field strengths obtained from a cylinder and a concentric wire.

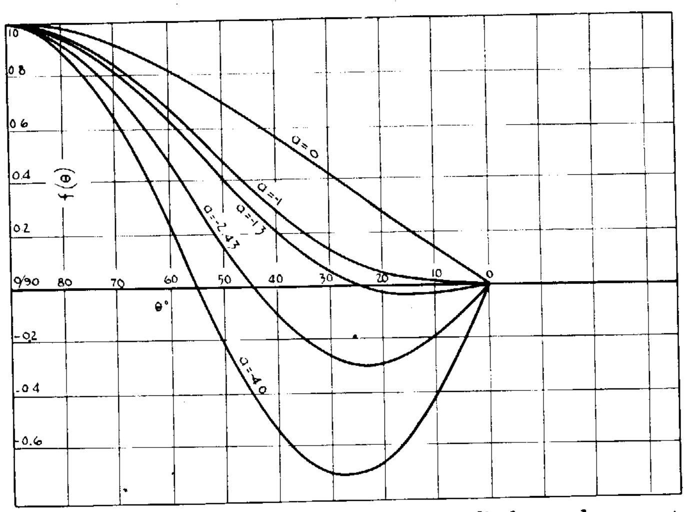

Fig. 40—Vertical radiation characteristics of a cylinder and concentric wire when the cylinder has a radius of 0.382 wavelength.

tion of the vertical characteristic with a when the radius of the outside ring is $r=0.382$ wavelength. We see that in the neighborhood of

$a=-4.0$, the secondary sky lobe is excessive. This is the point at which the field strength along the ground is greatest. It is invariably true for this type of antenna that the greatest ground signal strength is accompanied by a large secondary lobe.

XII. Conclusion

We have now examined a number of antenna arrangements and current distributions. Some of these arrangements have been very disappointing. In no case where the total antenna height is at all reasonable for broadcast use does the field strength at one mile go to exceptionally high values. For heights of the order of a half wavelength, it is hard to find anything better than a straight vertical wire. Some of the arrangements save a small amount in height, but in no other respect do they improve on the straight wire.

In the case of a straight vertical wire, Fig. 5 shows that an antenna length of 190 degrees (0.528 wavelength) is the most desirable when the power of the station is sufficient so that the night service radius is limited by fading rather than by ground signal deficiency. While it is seen from Fig. 5 that a 230-degree antenna gives ten per cent more signal at one mile, the daytime range is only increased about five per cent. It is apparent that the fading reduction accomplished by the 190-degree antenna far offsets the slight decrease in daytime service range suffered by not extending the antenna height to 230 degrees.

The antenna with the capacity hat at the top is an arrangement which enables a saving in height of antenna. By varying the loading at the top of the antenna, the vertical characteristic of a straight wire antenna of greater physical height can be obtained. This arrangement can be used on antennas of height 135 degrees or better to simulate the pattern of a 190-degree antenna. For antenna heights lower than 135 degrees, the antenna radiation resistance goes to rather small values as the loading at the top is made sufficient to suppress the sky wave. When the antenna itself is a steel tower, it will probably be difficult to keep the tower of nearly constant cross section and still build a large enough capacity hat to be effective with a reasonably small coil.

The sectionalized tower was developed to overcome the difficulties encountered by the tuned hat. Sectionalizing down the tower enables one to build the tower of uniform cross section, so that the results can be predicted in advance. The outstanding advantage of this arrangement is evident when the power is high. In general, the voltage occurring across the sectionalizing insulators is considerably less than occurs across the insulators of the tuned hat on a tower of the same height.

Examination of the curves pertaining to the antenna with constant current shows that it requires a total height of 190 degrees to give results equivalent to those obtained from a 190-degree vertical wire antenna. A further increase in height moves the fading wall still further. Since the distance to the fading wall is already 100 miles, this further increase would be important only in very special cases.

The gain in signal strength obtained by raising a half-wave element from the earth is very slight. It is seen that the best suppression of fading occurs when the top of the half-wave element is 190 degrees off the ground.

The Franklin antenna seems the most promising, in spite of the extreme heights required. It is not unreasonable to speculate on the possibilities of this antenna as a result of improved tower design. Such an arrangement would probably require a wooden tower. A three-section Franklin antenna would be 985 feet high if the operating frequency were 1500 kilocycles. Such an arrangement would give a field intensity at one mile twice as large as that obtained from a simple quarter-wave antenna. This would be the equivalent of quadrupling the power into the lower antenna.

It is of interest to note that the two-section Franklin antenna of total height one wavelength has the same vertical characteristic and field intensity at one mile as obtained from the antenna with constant current of the same height.

The results indicate that the antenna with decreased velocity has no advantage over simpler types. This antenna in general requires a wooden supporting tower. When a wooden tower is used, a straight vertical wire with a capacity hat would be the best arrangement. It has been shown that the Franklin type antenna needs no decreased velocity. In fact, for any given antenna height, it seems that equivalent results can be obtained with a straight vertical wire, either loaded at the top with a capacity hat or sectionalized some distance from the top.

The ring of antennas, while an interesting subject of analysis, is obviously too elaborate an arrangement for the small advantages it offers. In any event, this ring and central antenna can be replaced by a single vertical wire, 150 degrees tall, with the proper amount of loading at the top.

In the above discussion, it was found that most arrangements gave field strengths at one mile of less than 9.0 millivolts per meter. This was due to the fact that the vertical characteristic was not concentrated sufficiently. A simple calculation shows the necessity of concentrating the radiation close to the horizon. Let us suppose that the vertical radiation characteristic is constant within the angle ϕ measured from the earth's surface and is zero at all higher angles, as shown by

the sketch in Fig. 41. Then Fig. 41 shows the field strength at one mile for one watt of power into the system as a function of the angle ϕ. It is seen that the effective concentration must be within seventeen degrees to obtain 9.0 millivolts per meter at one mile. To obtain twice

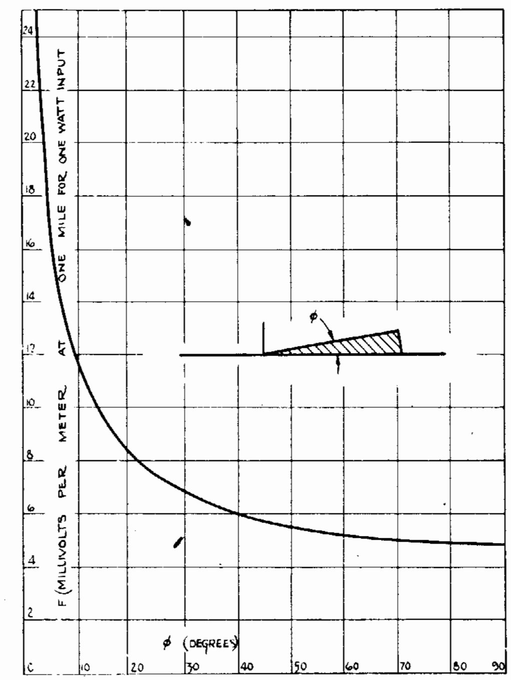

Fig. 41

the signal obtained from a simple quarter-wave antenna, the concentration must be within nine degrees of the earth.

No attention has been given to the relative merits of the various arrangements in reducing ground losses, since these merits will only be present when the ground system is very small. When adequate ground systems are provided, the earth losses can always be reduced to negligible quantities.