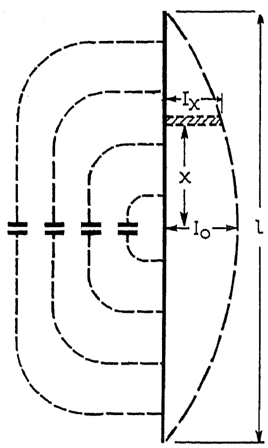
The impedance of the aerial $Z_a = E_o/I_o$. Now by Helmholtz's reciprocal theorem the same e.m.f. E_o induced at x would produce a current I_x at the centre, and therefore an e.m.f. E_o/l induced at x would produce a current I_x/l at the centre. If an equal e.m.f. E_o/l be induced in every centimetre of the length, which would be the case if the received electric field $\mathcal{E}$ in the direction of the aerial had this value, the total current produced at the centre would be the sum of all these l elements of current; in other words, it would be the mean value of I_x taken over the whole length. Hence the received current at the centre is equal to I_o reduced in the same ratio l_{eff}/l, that is it is equal to



$$\frac{E_o}{Z_a}\frac{l_{eff}}{l} = \frac{\mathcal{E} \, . \, l_{eff}}{Z_a}.$$

If, on breaking the receiving aerial at the centre, the p.d. is found to be e_o, then on closing it, Helmholtz's "make and break" theorem says that the current will be e_o/Z_a; hence $e_o = \mathcal{E} \, . \, l_{eff}$. If, instead of merely closing the break in the aerial we insert an impedance Z, the resulting current will be $e_o/(Z + Z_a) = \mathcal{E} l_{eff}/(Z + Z_a)$ where Z_a is the impedance of the aerial measured at the break, i.e. E_o/I_o.

Hence the ratio l_{eff}/l and the impedance of the aerial are the same for transmission and reception, irrespective of the distribution of the aerial characteristics. This proposition forms the basis of the first section of Mr. Burgess' paper, and except that it is concerned with alternating and not direct currents, it is a direct application of the principles propounded by Helmholtz over ninety years ago. Although not so intended, the paper is a worthy tribute to the memory of one of the great pioneers of electrical science. G. W. O. H.

AERIAL CHARACTERISTICS*

Relation Between Transmission and Reception

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SUMMARY.—In Section 1 of the paper it is shown that the impedance of an aerial is the same for reception as for transmission as a consequence of its behaviour as a linear network, which permits the application of the Superposition Principle and Thévenin's Theorem. It is also shown by applying the Reciprocal Theorem, that the effective height and the polar diagram are the same in the two conditions. A generalised definition of effective height is introduced which specifies the radiating or receiving properties of an aerial as a function of the direction of transmission (θ, ϕ) and of the polarisation (α) of the electric vector.

Section 2 comprises a critical discussion of the four methods commonly used in the calculation of aerial impedance, namely

(*a*) the method based directly on the field equations; (*b*) the Poynting vector method; (*c*) the induced e.m.f. method, and (*d*) the transmission line method.

The errors in the papers where differences between the transmitting and receiving impedances have apparently been found are indicated.

In Section 3 the simplifying assumptions usually made in the calculation of aerial impedance are discussed, namely

(*a*) sinusoidal distribution of current; (*b*) zero current at the end of an open aerial; (*c*) concentration of current and charge along the axis, and (*d*) perfect conductivity.

Introduction

It has often been suggested in investigations of aerials that the impedance, effective height and polar diagram may not be the same for reception as for transmission. This present paper is the outcome of an attempt to resolve the problem by reference to the fundamental principles which apply to all linear systems of which an aerial is one.

The first section of the paper is concerned with demonstrating the identity of the impedance and effective height of an aerial for any condition of excitation. An investigation of the arguments

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which led some writers to suggest that the identity may not hold has resulted in a critical examination of the methods of calculation of aerial impedance which is presented in the second section. The third section briefly discusses the simplifying assumptions usually made in these methods of calculations.

1. Equality of Impedance and of Effective Height for Transmission and Reception

(a) *The equality of the impedance for reception and transmission.*

It is proposed to demonstrate that the fundamental laws which apply to a linear circuit are also applicable to an aerial, and thus that the impedance of a given aerial is unique and independent of its mode of excitation.

The fundamental theorem relating to a linear circuit is the Principle of Superposition which may be stated as follows (*cf.* Ref. 1, p. 50).

" The current produced at any point in a network due to any number and distribution of e.m.fs in the network is the sum of the currents which would be produced if the individual e.m.fs were applied separately."

Thévenin's Theorem follows at once from the Superposition Principle (Ref. 1, p. 55) showing that an active linear two-terminal network is equivalent to an e.m.f. in series with an impedance as regards its external behaviour.

The derivation of Thévenin's Theorem is applicable to an infinite network having distributed constants as well as to one having lumped constants. In fact if the circuit equations are correctly formulated they should be equivalent to the set of electromagnetic field equations which apply to the system considered. This brings out the point that the real basis of circuit theory is the electromagnetic field, and although Carson[2] has stressed this it still remains insufficiently appreciated.

Thus all circuit elements, even of the simplest type (e.g. an inductor) are in reality infinite systems, but it is a matter of practical convenience to designate more or less well defined circuit elements with the names inductor, capacitor, and resistor in order to simplify analysis. Usually these approximations are reasonably accurate, but in some cases they are of limited validity.

The equations of the electromagnetic field apply to all systems of conductors, and the distinction between " lumped " and " distributed " constants just as that between " circuit " and " aerial " is an arbitrary one. In practice the term " aerial " implies that the radiation resistance of the system is not negligible compared with the ohmic resistance, while the term " circuit " implies the converse.

Thus the rigorous theory of circuits rests on the field equations which are linear (in the absence of ferromagnetic media and non-ohmic conductors) for the relations between the currents and charges on the conductors, and the potentials and fields to which they correspond are linear.

It is therefore concluded that an aerial is a linear system to which the Principle of Superposition and thus Thévenin's Theorem applies, and the latter may be expressed as follows for the case of an aerial :—

" If an external impedance Z be connected between any two terminals of an aerial the current i_0 flowing in Z is given by

$$i_0 = \frac{e_0}{Z + Z_0} \qquad \ldots \quad \ldots \quad \ldots \quad (1)$$

where the e.m.f. e_0 is dependent on the mode of excitation but independent of Z and the effective impedance Z_0 of the aerial is independent of both the excitation and Z."

A special mode of excitation is the transmitting condition in which the applied e.m.f. is lumped at the terminals, and it is thus concluded as a corollary that the impedance of an aerial is the same for transmission and reception.

Recently Fränz[3] used similar arguments to demonstrate the uniqueness of the value of the impedance of an aerial. Some writers had already given less general proofs by considering particular models. For example, Colebrook[4] has considered an aerial as a transmission line having uniformly distributed constants with an arbitrary distribution of the exciting field, and found that the system obeyed Thévenin's Theorem. A more general proof was given by Wilmotte[5] who considered the case in which the constants of the aerial are not assumed to be uniformly distributed, with, however, the proviso that " the distribution of the constants of the aerial is independent of the applied e.m.fs." This proviso is unnecessary, since in a linear system the constants must be independent of the excitation, and thus Wilmotte's proof is more general than his wording suggests. Wilmotte considered that as the current distribution and in particular the integral of the current along the aerial was not the same for reception as for transmission, the radiation resistance would not be the same in the two conditions. This argument is incorrect, but unfortunately it has led to the widespread belief that the impedance of an aerial is not the same for reception as for transmission.

Niessen and de Vries[6] in a paper on the impedance of a receiving aerial obtained results which differed from those found by Labus[7] for a transmitting aerial, and concluded that a real difference existed, but this arose from an error in their method of calculation which will be discussed in section 2 (*c*).

(*b*) *Equality of effective height for reception and transmission.*

The radiating properties of an aerial are only completely known when the polarisation and field intensity produced at all points on a surface surrounding the aerial are specified. For simple aerials the properties can be specified in terms of an "effective height." Thus if a linear aerial carries a current distribution $i_t(z)$ for a terminal current i_0 the effective height for transmission normal to the aerial is given by

$$h_t = \frac{1}{i_0}\int i_t(z)\,.\,dz \qquad \ldots \qquad \ldots \qquad \ldots \qquad (2)$$

The effective height of a simple aerial for reception is the ratio of the induced e.m.f. e_0 appearing at its terminals to the uniform inducing field E parallel to the aerial:—

$$h_r = \frac{e_0}{E} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (3)$$

Since any aerial behaves as a linear circuit, the Reciprocal Theorem (Ref. 1, p. 52) can be applied to it. This theorem states that if any e.m.f. at one point in a circuit produces a certain current at any other point in the circuit, the same e.m.f. acting at the second point would produce the same current at the first point. Now an e.m.f. $i_0 Z_0$ applied to the terminals of an aerial produces the transmitting current distribution $i_t(z)$ having value i_0 at the terminals, and thus in reception of a wave of field distribution $E(z)$ parallel to the conductor, the e.m.f. $E(z)\,dz$ induced in the element dz at z produces a current

$$di = E(z)dz\frac{i_t(z)}{i_0 Z_0}$$

at the terminals which is equivalent to an e.m.f. $Z_0\,.\,di$ at these terminals. Thus the total induced e.m.f. appearing at the terminals is given by

$$e_0 = \frac{1}{i_0}\int E(z)\; i_t(z)\,.\,dz \qquad \ldots \qquad \ldots \qquad (4)$$

where, it must be remembered i_t is the current distribution in the *transmitting* condition for a terminal current i_0.

Equation (4) is a useful form of the Reciprocal Theorem which is of particular value calculating the e.m.f. e_0 induced by any arbitrary distribution of exciting field E. When E is uniform we find

$$\frac{e_0}{E} = \frac{1}{i_0}\int i_t(z)\,.\,dz$$

and thus $h_r = h_t$ $\qquad \ldots \qquad \ldots \qquad \ldots \qquad (5)$

Hence the Reciprocal Theorem leads to a general proof of the equality of effective height for reception and transmission.

A specialised proof of this equality was given by Wilmotte[5] who considered the aerial as a transmission line with arbitrarily varying parameters along its length, but he again introduced the unnecessary proviso that the constants should be independent of the excitation.

For complex aerial systems the simple conception of effective height is no longer applicable, but it may be extended in the following manner:— Let P be a point in the aerial system and let a sphere of radius r large compared with the wavelength and the dimensions of the system be drawn about it, and let Q be any point on the sphere having polar co-ordinates (r, θ, ϕ). If a current i_0 of angular frequency ω is flowing at the terminals due to a transmitter connected to them, the electric field E at Q will in general be elliptically polarised with the plane of ellipse in the spherical surface. Let the electric field component of polarisation α be $E(\alpha)$ at Q. Then the effective height h of the aerial system is given by

$$\frac{\omega h i_0}{c^2 r} = E(\alpha) \qquad \ldots \qquad \ldots \qquad \ldots \qquad (6)$$

This definition of effective height h like $E(\alpha)$ is a function of the direction of transmission (θ, ϕ) and of polarisation (α) and may be termed the "generalised effective height"; for simple aerials it coincides with the usual significance of the term given by equation (2). For example the effective height of a short dipole of length $2l$ is equal to l in the usual notation, while the generalised effective height has the form

$$h(\theta, \phi, \alpha) = l \cos\alpha \sin\theta$$

which for maximum transmission or reception ($\alpha = 0$ and $\theta = 90°$) agrees with the usual form.

The "generalised effective height" for reception is defined as the ratio of the e.m.f. e_0 appearing at the terminals of the aerial system to the incident plane polarised field intensity E producing it, where E is due to a distant source having co-ordinates (θ, ϕ) and polarisation (α). It is seen that the generalised effective height is also the same for reception and transmission as a consequence of the Reciprocal Theorem by considering transmission between the aerial system and a short dipole at (θ, ϕ) on the sphere with inclination (α).

It is concluded that the polar diagram of an aerial system and the various measures of it (directivity, gain) are identical for reception and transmission. This remark applies even in the presence of the earth, for this does not affect the linearity of an aerial system.

2. Summary of the Methods of Calculation of Aerial Impedance

The four main methods of calculating aerial impedance are now briefly discussed. For a useful survey of the theory of antenna impedance the reader is referred to a recent paper by Schelkunoff[8].

(a) Method based directly on the field equations.

This is the classical method which was used by Abraham[9] and recently by Page and Adams[10] to consider the electrical oscillations of a perfectly conducting prolate spheroid. The technique is that of expressing Maxwell's equations in terms of the curvilinear co-ordinates appropriate to the shape of conductor, and on satisfying the boundary conditions at the conductor, solving the resulting differential equation. The natural modes of oscillation are thus determined, and from the damping of these oscillations the radiation resistance can be found. The main difficulty of the method is to express the equations for the oscillations in terms of known functions or of rapidly converging series.

The chief limitation of the method is that of shape, since the spheroidal conductor is the only one amenable to exact analysis, as other shapes involve a discontinuity at the ends which make the analysis either approximate or intractable. The shape which is of the greatest practical interest is the cylinder, but an exact analysis of this shape would appear to be impossible, although a number of investigators have endeavoured to develop a reasonably accurate theory.

For example, Hallen[11] has calculated the impedance of an imperfectly conducting dipole, with the assumptions that the current vanishes at the end, and that the ratio of length to radius is large although the logarithm of the ratio is regarded as finite. The integral equation for the current is solved by taking the method of successive approximation two stages and the convergence and hence accuracy of this method is better the thinner the aerial.

The difficulty in applying the results for the spheroidal aerial to the cylinder is to know how to choose the equivalent dimensions for it seems that any *a priori* choice is somewhat arbitrary, since exact equivalence cannot hold.

The method based on the field equations is capable of greater accuracy than other methods since it usually involves fewer simplifying assumptions. For example the current distribution is not chosen beforehand, but is determined by the boundary condition of zero tangential electric force at the surface of a perfect conductor, and thus certain inconsistencies of other methods are avoided.

(b) The Poynting vector method

This method consists of integrating the Poynting vector $\frac{c}{4\pi}(E \times H)$ over a closed surface (usually spherical) described about the aerial, at a large distance from it. Thus if i_0 is the terminal current the power radiated is given by

$$P = R_0 i_0^2 = \frac{c}{4\pi}\int (E \times H)dS \quad \ldots \quad (7)$$

where R_0 is the radiation resistance at the terminals. It is usually assumed in calculating E and H that the current distribution is sinusoidal.

The disadvantage of this method is that it only gives the resistive component of the terminal impedance, since it is concerned with the distant field. The degree of accuracy with which it gives the radiation resistance depends upon the closeness of the postulated current distribution to the actual distribution, and the thinner the aerial the better this approximation is. The error is most obvious for a dipole which is a multiple of a wavelength, for the predicted radiation resistance is then infinite since radiation occurs although the terminal current is zero; in actual fact the radiation resistance reaches finite maxima in these regions.

(c) The induced e.m.f. method

This method, as outlined by Pistolkors[12] consists of evaluating the integral of the product of the current $i(z)$ and the electric force $E'(z)$ at the surface of the aerial which is set up by the postulated current distribution. The current $i(z)$ in the element dz at z does work against the induced e.m.f. $E'(z) \cdot dz$ and the complex Poynting vector integrated over the aerial gives the power which is supplied to it. In the case of a perfectly conducting aerial with zero external impedance between its terminals this (complex) power has an active part which corresponds to the radiation resistance R_0 of the aerial and a reactive part corresponding to the reactance X_0: considering the case where i is in phase with i_0 along the aerial, we have

$$(R_0 + jX_0)i_0^2 = Z_0 i_0^2 = -\int E'(z)i(z)dz \quad (8)$$

This method gives the same value of radiation resistance as the Poynting vector method for the same postulated current distribution, and has the advantage of giving the reactance as well.

The method is frequently misunderstood and incorrectly formulated, and inconsistencies are found in many papers on the subject. The important point is that at the surface of a perfectly conducting aerial the resultant longitudinal electric force must vanish. (The gradient of the scalar potential along an aerial must not be confused with this electric force, since the former represents only the contribution of the charges to the field). Now a sinusoidal distribution of current would give rise to a non-zero value of electric field at the surface of the conductor, and thus this distribution which is usually assumed to hold in a transmitting aerial, violates the boundary conditions. This point is discussed in Section 3(a) in greater detail.

Hence in a transmitting aerial, the current distribution must in fact depart from the sinusoidal to just the extent that makes the field vanish everywhere along the aerial. In a receiving aerial the distribution of current must be such as to produce at every point on the aerial a field exactly equal and opposite to the longitudinal component of the incident field.

The incorrect formulation of the induced e.m.f. method leads to the "Radiation Paradox" as Schelkunoff[8] has termed it. For if a sinusoidal distribution of current be assumed to exist on the transmitting aerial, the resultant field is not zero, implying the existence of a distributed impedance along the aerial which is known to lead approximately to the impedance Z_0 appearing at the terminals from comparison of experiment with the values deduced by this method. And yet if i is given the correct distribution for transmission the field must vanish, leading as it should to zero distributed impedance. Schelkunoff then asks why the incorrect sinusoidal distribution should lead so nearly to the right value when such a fundamental discrepancy is present, but he offers no answer. The following considerations may serve to explain the apparent paradox.

Let a generator of e.m.f. e be connected to the aerial terminals where the current produced is i_0 and the distribution along the aerial is $i_t(z)$ appropriate to the transmitting condition. The complex Poynting vector integrated over the whole system gives the active and reactive parts of the power supplied by the generator. Now the longitudinal field and hence the Poynting vector is zero along the aerial itself, and thus the power *appears* to emerge entirely at the source, for there we have

$$Z_0 i_0^2 = P = \int E i_0 \,.\, dz = e i_0$$

This only tells us that $Z_0 i_0 = e$ which is perfectly correct but of no assistance in calculating the impedance Z_0.

Now consider the case of reception in which the incident field has the distribution $E(z)$ and the terminals are short-circuited, the current at this point being i_0. Then if $i_t(z)$ is the current distribution in the transmitting condition corresponding to the terminal current i_0, we have from equation (4) that the induced e.m.f. is

$$e_0 = \frac{1}{i_0} \int E(z) i_t(z) \,.\, dz$$

and thus $i_0 = \dfrac{e_0}{Z_0} = \dfrac{1}{i_0 Z_0} \displaystyle\int E(z) i_t(z)\, dz$

$$\text{giving } i_0^2 Z_0 = \int E(z) i_t(z) \,.\, dz \qquad \ldots \qquad (9)$$

This equation gives the relation between any distribution $E(z)$ of incident field and the current i_0, it produces at the aerial terminals. Now consider the field $E'_s(z)$ which would be produced by the sinusoidal current distribution i_s having the value i_0 at the terminals ($x = 0$). This field must be equal and opposite to the incident field $E(z)$ required to support i_s on the aerial in the receiving condition and thus we can write (10) as

$$Z_0 = -\frac{1}{i_0^2} \int E'_s(z) i_t(z) \,.\, dz \qquad \ldots \qquad (10)$$

It should be noted that $i_t(z)$ is the true transmitting current distribution even though E'_s is derived from a sinusoidal distribution of current.

Now since $i_t(z)$ is approximately sinusoidal we see that the impedance is given *approximately* by

$$Z_0 = -\frac{1}{i_0^2} \int E'_s(z) i_s(z) \,.\, dz \qquad \ldots \qquad (11)$$

where $i_s(z)$ is the sinusoidal distribution corresponding to i_0 at the terminals. Equation (11) is the form which is usually used for the calculation of impedance by the induced e.m.f. method and the error it gives is seen to depend on the departure of $i_s(z)$ from $i_t(z)$. Because the true distribution does not usually depart considerably from the sinusoidal, the usual form (11) of the induced e.m.f. method gives a value of impedance which may not be too seriously in error, at least for the resistance.

The thin dipole has been analysed using the induced e.m.f. method by Labus[7]. Comparison with the more accurate theory of Hallen shows agreement except in the region where the aerial is a multiple of a wavelength long where Labus predicts an infinite resistance while both the resistance and reactance found by Hallen remain finite, as in fact they should.

Niessen and de Vries[6] used the induced e.m.f. method to calculate the impedance of a receiving aerial, but applied the method incorrectly and on obtaining values which differed from those of Labus concluded that these differences really existed between the impedances for reception and transmission. They took as the current distribution of a uniform field parallel to the aerial

$$i_r = i_0 \frac{\cos kz - \cos kl}{1 - \cos kl}$$

and deduced the corresponding longitudinal field which would be set up by this current. But instead of using the corresponding sinusoidal *transmitting* current i_s in the integral (equation 11) they used the receiving current i_r and so obtained incorrect values for the impedance, which agree with those of Labus only when i_r and i_s are equal, that is when the total length of the dipole is an odd number of half-wavelengths.

(d) The transmission line method

In this method the aerial is represented by a transmission line with uniformly distributed constants and has the advantage of simplicity and of involving conceptions which are already familiar to radio engineers.

Colebrook[4,13] considered the qualitative behaviour of the impedance of an aerial with uniform distribution of constants.

Siegel and Labus[14] placed the method on a quantitative basis by using the results found by Labus[7] for a transmitting aerial carrying a sinusoidal distribution of current and making the assumptions that :

(i) the characteristic impedance is the mean of the value deduced from the scalar potential along the aerial, and

(ii) the attenuation constant has the value which makes the input resistance of the equivalent line equal to the radiation resistance ; this assumption implies that the effect of radiation can be included in the transmission line equations in the same way as ohmic resistance.

Both these assumptions are of doubtful validity, and the complete analysis is elaborate, for it involves a preliminary calculation of the potentials along the aerial and of the radiation resistance by the induced e.m.f. method.

Schelkunoff[8] has improved on the transmission line method by regarding an aerial as a line with parameters varying slowly along its length. Using Carson's solution of the differential equation of such a system he has been able to express the aerial impedance explicitly in terms of tabulated functions, but the usual assumption of zero current at the end of the aerial is made and thus the results are only applicable to thin aerials.

3. Simplifying Assumptions used in the above Methods

This section of the paper comprises a discussion of the assumptions which are customary in some or all of the four methods of calculating aerial impedance.

(a) Sinusoidal distribution of current

This assumption is encountered in the last three methods but is absent from the first by its very nature for the sinusoidal distribution leads to a non-zero component of electric field parallel to the surface of the conductor. It is seen from equation (10) that in the induced e.m.f. method a knowledge of the true current distribution is required for an accurate evaluation of the impedance. However, except when the aerial is near a condition of high resonant impedance ($2l$ = odd number of $\lambda/2$ for a dipole) the value of resistance given on the assumption of a sinusoidal distribution is fairly accurate, but this is not necessarily true of the reactance which is always more sensitive to departures from the truth.

It is known[8,11,15], that in a linear uniform perfectly conducting aerial the scalar and vector potentials are propagated sinusoidally, i.e. there is no damping of the potential waves due to radiation. Thus either of these potentials can be written in the form $(A_1 \cos kz + A_2 \sin kz)$ and an integral equation for the current distribution is obtained. One method of solving this equation explicitly is by successive approximations, such as Hallen has carried out, but the complexity of the analysis prevents its being taken more than a few stages. If the exact expressions for the current distribution or a close approach to it is used in the induced e.m.f. method the latter becomes formally identical with the field theory method.

(b) Zero current at the end of an open aerial

It is usually assumed that in cylindrical aerials the current vanishes at the flat end, but owing to the accumulation of charge which must occur there this represents an approximation which grows worse as the ratio of radius to length increases. This limitation has been recognised by most writers, and the results obtained using this assumption are usually claimed to be applicable only to thin aerials.

The remedy for thick aerials would apparently be to regard the flat end as equivalent to a capacitance loading at the end proportional to the radius.

(c) Concentration of current and charge along the axis

The scalar and vector potentials corresponding to a given current distribution $i(z)$ are frequently calculated on the (implicit) assumption that the current and charge are concentrated along the axis whereas in fact the distribution is practically superficial at radio-frequencies. For example the longitudinal component of the vector potential at the surface of the aerial is usually written

$$A(z) = \frac{1}{c}\int \frac{i(z_0)e^{-jkr}}{r}\,dz_0$$

where $r = \sqrt{(z - z_0)^2 + r_0^2}$ in which r_0 is the radius of the aerial. This leads to an error which is most pronounced near the end of the aerial and for thick aerials. By considering the actual distribution of charge and current taking into account the skin effect, Zinke[15] has shown that the distance r should be written

$$r = \sqrt{(z - z_0)^2 + r'^2_0}$$

where the "effective radius" r'_0 lies between about $0.4\, r_0$ for $z - z_0 = 0$ and $1.4\, r_0$ for $|z - z_0| > 6r_0$.

(d) Perfect conductivity

The assumption of perfect conductivity is usually made for simplicity and it leads to a very small error in the impedance of resonant aerials which are those most used in practice. When the conductivity is not perfect the electric force parallel to the conductor does not vanish but is of such a value to balance the internal field $(R' + j\omega L')i(z)$ at every point along the aerial, R' being the ohmic resistance and L' the internal inductance, per unit length. In the analysis of Hallen[11] this relation is taken into account before solving the integral equation for the current and thus his results include the effect of ohmic resistance in a general and fairly exact form.

To a first approximation, however, the impedance for perfect conductivity can be modified by including an additional term calculated on the assumption that the presence of ohmic resistance causes a negligible change in the current distribution. When the conductivity is imperfect the Poynting vector is no longer zero along the aerial but has the value $(R' + j\omega L')i_t^2(z)$ directed inwards, and thus to a first approximation the terminal impedance has an additional term:

$$\Delta Z_0 = +\frac{1}{i_0^2}\int (R' + j\omega L')i_t^2(z)\,.\,dz \quad (12)$$

An equivalent derivation of this expression from circuit theory is obtained by applying the Compensation Theorem (Ref. 1, p. 56), considering the presence of ohmic resistance and internal inductance as equivalent to the introduction of generators at points all along the aerial.

To show the small effect of imperfect conductivity, a numerical example is given. A thin half-wave dipole has an impedance of $(73 + j\,42.5)$ ohm according to the approximate form of the induced e.m.f. method; if such a dipole is made of copper and has a length of 1 m. (λ 2m or 150 Mc/s). and radius of 0.5 cm. then R' and $\omega L'$ both have the value of 0.1 ohm/m.

For a half-wave dipole of length $2l$

$$\frac{1}{i_0^2}\int_{-l}^{+l} (R' + j\omega L')i_t^2\,.\,dz = (R' + j\omega L')l$$

very approximately, and thus in the example considered the additional impedance is $(0.05 + j0.05)$ ohm, which is clearly insignificant.

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Institution of Electrical Engineers

Wireless Section.—Dr. G. L. Sutherland will open a discussion on "Metals and their Finishes in Radio Construction" at the meeting at 5.30 on April 18th. The Silver Jubilee Commemoration Meeting of the Section has been arranged for May 3rd. The meeting, which will be preceded by a reception and tea, begins at 5.30 and will include a series of six short addresses by past chairmen giving a review of wireless progress. The speakers will be Col. Sir A. Stanley Angwin, Dr. W. H. Eccles, Prof. G. W. O. Howe, Admiral Sir Charles E. Kennedy-Purvis, H. Bishop and Dr. R. L. Smith-Rose.

Cambridge and District Wireless Group.—B. J. Edwards will give a "Survey of the Problems of Post-war Television" at a meeting to be held at 5.30 on April 17th at the Cambridgeshire Technical School, Collier Road, Cambridge. A discussion on "Training for the Radio Industry" will be opened by C. R. Stoner, and R. W. Wilson at a meeting at 5.30 on May 1st, at the Cambridgeshire Technical School.