Electromagnetic energy and power in terms of charges and potentials instead of fields

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Abstract: It has been shown in a previous paper by Konopinski that there are important advantages, particularly to the engineer, in an electromagnetic theory in which the properties, which are customarily associated with the fields, are assigned instead to the source charges, and measured by the potentials. These properties include stored energy and energy flow. The objective of the paper is to reformulate Poynting's theorem in charge-potential terms, and it is shown that the energy flow vector which is obtained simplifies the quantitative description of electromagnetic power transfer. It is closely related to the circuit concepts familiar to all electrical engineers, and, unlike the Poynting vector, is equally useful in communications and in power-frequency applications. It demonstrates the fundamental importance of electrostatic forces in all electromagnetic devices. The paper also shows that practical difficulties in applying field theory, and, in particular, the Poynting vector, have led to the use of hybrids in which two wholly different viewpoints are mixed.

1 Introduction

As magnetism is a consequence of the motion of charge, magnetic energy is equivalent to kinetic energy, and any group of moving charges behaves as if it has inertia. Although this view of magnetism as a kinetic property of the charges themselves, instead of the fields around them, is very well known in general terms, it is usually regarded as an analogy, and somewhat obscured by treating it as another aspect of field theory. But Maxwell [1] demonstrated that it provides an entirely different interpretation of the observable phenomena which is as valid as the explanation in terms of the Faraday concept of magnetic flux, and, as has been shown elsewhere [2], the electrokinetic momentum description has many advantages over field theory for teaching purposes. A similar approach has also been advocated by Konopinski [3] and others [4].

It raises many questions about the role of, and need for, the field concepts, one of which is the flow of energy. The Poynting concept of power flowing through the field, with density $\boldsymbol{E} \times \boldsymbol{H}$, like the electromagnetic momentum vector $\boldsymbol{D} \times \boldsymbol{B}$, appears to be a necessary consequence of electromagnetic propagation, and both have become so familiar a part of electromagnetic theory that they are often regarded as inseparable from it. But they rest on untested assumptions, and are called into question by Maxwell's electrokinetic interpretation of the momentum. As is well known, the curl of the Poynting vector $\boldsymbol{E} \times \boldsymbol{H}$ is arbitrary, so that many different descriptions of energy flow are possible, all equally valid. What is less clearly recognised is that the divergence is arbitrary as well, because it is chosen to match the electric and magnetic field energy densities $\boldsymbol{E} \cdot \boldsymbol{D}/2$ and $\boldsymbol{H} \cdot \boldsymbol{B}/2$, and these are, in turn, unsupported by any experimental evidence. The characteristic of the electrokinetic momentum view is the description of magnetic stored energy as a property of the charges, with density $\boldsymbol{J} \cdot \boldsymbol{A}/2$ instead of $\boldsymbol{H} \cdot \boldsymbol{B}/2$, and the electric stored energy can likewise be removed from the field and attributed to the sources. This obviously affects not only the description of momentum, but also the energy transfer.

The object of the paper is to explore some of the consequences of describing electromagnetic energy flow, like the momentum and stored energy, in terms of the charges and potentials instead of the field vectors. As shown in Reference 2, such a change is of fundamental importance because the Aharonov-Bohm effect [5] provides experimental evidence in favour of the potentials, not the field vectors, as the quantities which are physically significant. But the usual assumption that there is no way of distinguishing experimentally between the different stored energy densities is sufficient for the present purpose, which is limited to examining the relative practical advantages, to engineers, of the different descriptions of power flow in devices which are familiar to them.

Although momentum often receives little attention in electromagnetic texts, because of the way in which field theory obscures its role at low frequency, it is inseparable from the transfer of energy, and one advantage of the charge-potential formulation is that it shows this relationship more clearly [2]. Some of the momentum implications are examined separately* in terms of the application to plane waves, and this also provides an important practical example of the different energy-flow concepts.

2 Charge-potential theory

Electromagnetism rests on the 'electrostatic' interactions between charges due to position, i.e. to their potential energy. This is a system property, usually expressed in the

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form

$$U = q_1\phi_{12} = (q_1\phi_{12} + q_2\phi_{21})/2 \quad (1)$$

for two charges, where ϕ_{12} is the potential at charge 1 due to 2. More generally, it is obtained by summing, or integrating, over all the charges. The energy is customarily assumed to be distributed in the field, with density

$$u'_f = \boldsymbol{E} \cdot \boldsymbol{D}/2 \quad (2)$$

but an equally valid choice of energy density is

$$u'_c = \rho\phi/2 \quad (3a)$$

where ρ is the charge density, either per unit volume, or per unit surface area when applied to conductors. This locates the energy in the regions occupied by the charges and, in this sense, it is stored 'by' the charges, and not in the empty space between them. It represents the pressure energy in a cloud of charge of like sign, analogous to hydraulic pressure in that it is due to the mutual repulsion forces between the constituent charges, and increases as the packing density is increased. It forms the c-equivalent of the Maxwell field stress, an implication which will be examined elsewhere.

Although the two views of energy are very well known, their implications have attracted surprisingly little attention, other than by writers such as O'Rahilly [6]. They provide the basis for two alternative electromagnetic theories, one expressed in terms of the charge densities and potentials, indicated by the subscript c, the other by the field vectors and designated f. They differ fundamentally in viewpoint, even though they may share the same potentials, and one of the incidental objectives of the paper is to attempt to disentangle them. As is well known, the energy is a system property whose distribution cannot be tested by experiment [3, 6–10], so that the statement, in some texts, that the energy of a capacitor is stored in the field and not on the plates is without foundation. If both energy densities are equally valid, then so also are the two theories which follow by adopting either in a self-consistent way.

One obvious advantage of eqn. 3 in place of eqn. 2 is the relative simplicity of the proof. This is because u must be derived from a system, energy U, which is, necessarily, expressed in terms of the charges and potentials, so that the usual proof of eqn. 2 [11] is essentially an exercise in translating from one theory to the other. Replacing eqn. 2 by eqn. 3 makes this unnecessary, and reflects the way in which the charge-potential description is much more closely related to the terminal parameters of circuit theory [2] and to practical measurements.

Likewise the magnetic energy due the current inside a conductor, of density $\boldsymbol{J}$, can be assigned to Maxwell's 'electrical fluid', i.e. to the conduction electrons, by choosing the energy density

$$u''_c = \boldsymbol{J} \cdot \boldsymbol{A}/2 = \rho_- \boldsymbol{u} \cdot \boldsymbol{A}/2 \quad (3b)$$

in place of the field equivalent $\boldsymbol{H} \cdot \boldsymbol{B}/2$. Here, ρ_- denotes the density of electrons in the conduction energy band, $\boldsymbol{u}$ is the mean drift velocity†, and $\boldsymbol{A}$ is the magnetic vector potential of field theory, although the comparison between eqns. $3a$ and $3b$ shows that there is a much more direct relationship between $\boldsymbol{A}$ and ϕ (Section 6). The terms 'kinetic energy' and 'potential energy', or 'pressure energy', are more descriptive and more accurate than 'magnetic energy' and 'electrostatic energy', because

$$u_c = \rho\phi/2 + \boldsymbol{J} \cdot \boldsymbol{A}/2 \quad (4)$$

like

$$u_f = \boldsymbol{E} \cdot \boldsymbol{D}/2 + \boldsymbol{H} \cdot \boldsymbol{B}/2 \quad (5)$$

is assumed to give the system energy, regardless of the motion of the charges.

It is this assumption which many authors reject, asserting that field theory is necessary to describe changes in time. But the assertion is not substantiated, and does not appear to have been closely examined. Conventional theory derives from the Lagrangian density [10, 12–14]

$$L_f = (\boldsymbol{E} \cdot \boldsymbol{D} - \boldsymbol{H} \cdot \boldsymbol{B})/2 - \rho(\phi - \boldsymbol{u} \cdot \boldsymbol{A}) \quad (6)$$

in which $\phi - \boldsymbol{u} \cdot \boldsymbol{A}$ is the Schwarzschild 'electrokinetic potential' [15] representing the interaction energy between the fields and the charges, $\rho\phi$ corresponding to the $q_1\phi_{12}$ form of eqn. 1. As the Lagrangian is not limited to static conditions, neither is eqn. 4. Using the alternative form of eqn. 1 is equivalent to

$$L_c = -(\rho\phi - \boldsymbol{J} \cdot \boldsymbol{A})/2 \quad (7)$$

which is given by Schwinger [16]. Charge-potential theory merely substitutes L_c for L_f.

The same point is demonstrated by the momentum. Eqn. $3b$ shows, by inspection, that the kinetic energy requires a momentum density

$$\boldsymbol{g}_c = \rho_- \boldsymbol{A} \quad (8)$$

in place of the field equivalent

$$\boldsymbol{g}_f = \boldsymbol{D} \times \boldsymbol{B} \quad (9)$$

The quantity $\boldsymbol{g}_c$ is familiar to physicists as the electromagnetic part of the canonical momentum, and redefines $\boldsymbol{A}$ as the momentum per unit charge; i.e. as Maxwell's 'electrokinetic momentum' vector. It is less familiar, in this form, to engineers, but can be deduced very simply and directly from the electrostatic interactions between the charges by considering the process by which they are accelerated in a pulse [2]. If the conductor is resistanceless, the conduction electrons are in equilibrium under the influence of **grad** ϕ and momentum forces only, giving

$$\mathbf{grad}\, \phi + \partial \boldsymbol{A}/\partial t = 0 \quad (10)$$

where ϕ and $\boldsymbol{A}$ are observed in the reference frame of the moving charges. It is this equilibrium condition which defines $\boldsymbol{A}$. Relativity theory shows that it describes all the electromagnetic forces which act on the moving charges, at all velocities up to $u = c$, where c is the propagation velocity in empty space. Thus the 'c' description of momentum, like the Lagrangian, applies to radiating as well as nonradiating systems, although it necessarily requires some changes in concepts (Section 12).

3 Energy flow vector

In the c description the moving charges carry the momentum and, thus, also the energy. The Poynting vector

$$\boldsymbol{w}_f = \boldsymbol{E} \times \boldsymbol{H} \quad (11)$$

† The vector symbol $\boldsymbol{u}$ distinguishes velocity from energy u.

can be replaced by‡

$$w_c = J\phi = u\rho_- \phi \quad (12)$$

as this has the required divergence. It describes the transfer of energy, of density $\rho_- \phi$, at the charge velocity u, the negative subscript emphasising the distinction between the moving and the stationary charges when there are two, or more, groups occupying the same space, as in a conductor. The divergence expands to

$$\text{div}\,(J\phi) = (\text{div}\,J)\phi + J \cdot (\mathbf{grad}\,\phi)$$

where

$$\text{div}\,J = -\partial\rho/\partial t = -\partial\rho_-/\partial t$$

Thus

$$\text{div}\,(J\phi) = -\phi(\partial\rho_-/\partial t) + \rho_- u \cdot \mathbf{grad}\,\phi \quad (13)$$

and $J\phi$ accounts for two local actions. The first is the rate of change of potential, or pressure, energy density, expressed in terms of the local changes in ρ, and the second is the work which is required to convey the moving charges through the change in ϕ due to an equivalent stationary charge group (Section 7).

The **grad** ϕ work may be accounted for in many different ways, depending on the various energy sources exerting forces on the charges. The most common example is wire of conductivity σ. Viewed from the stationary crystal-lattice reference frame, the equilibrium condition for the moving electrons becomes

$$\rho_-(\mathbf{grad}\,\phi + \partial A/\partial t - u \times \mathbf{curl}\,A) + J/\sigma = 0 \quad (14a)$$

in place of eqn. 10 or, alternatively,

$$\mathbf{grad}\,\phi + \partial A/\partial t + (\mathbf{curl}\,A) \times u + u/\sigma = 0 \quad (14b)$$

Substituting for **grad** ϕ in eqn. 13 gives

$$\text{div}\,(J\phi) = -\phi(\partial\rho_-/\partial t) - \rho_- u \cdot (\partial A/\partial t) - J \cdot J/\sigma \quad (15)$$

because $u \cdot (u \times \mathbf{curl}\,A)$ is zero (i.e. there is no work done by the magnetic forces). The result shows that, when the balance between the **grad** ϕ and momentum forces on the moving charges is provided by lattice collisions, the energy flow $J\phi$ accounts for the changes in the potential and kinetic energies, plus the rate of energy conversion in collisions. This is the description of the energy exchange process 'seen' from the reference frame in which the wire is stationary, but is easily extended to any sources and sinks of electromagnetic energy. It puts the result in a form which is directly comparable with the Poynting equivalent

$$\text{div}\,(E \times H) = -E \cdot \partial D/\partial t - H \cdot \partial B/\partial t - E \cdot J \quad (16)$$

One important point, which is illustrated by the comparison, is that the vector E is not the same as **grad** ϕ, but represents the difference between the **grad** ϕ and the momentum ($\partial A/\partial t$) forces (Section 7). Another is the more general form of energy density in eqn. 15:

$$U_c = \int \phi\, d\rho + \int J \cdot dA \quad (17)$$

in place of eqn. 4. This, like the Poynting vector equivalent, is valid when the relationships are nonlinear, and shows (Section 11) that the charge-potential description of ferromagnetism is essentially in terms of an electron spin response to A. The first term in eqn. 15 describes directly the forces which have to be exerted on the charges in any volume element in order to remove them, and thus the corresponding energy, from the element. The kinetic energy term takes a different form because the $\partial A/\partial t$ forces are due to changes in velocity of the remote charges, not those causing the local current density J. In general, eqns. 13 and 15 show how the system energy is accounted for by the actual expenditure of work on the charges, whereas, in eqn. 16, the physical significance of the terms is less obvious.

Integrating eqn. 15 over any volume v bounded by a closed surface s gives

$$\oint_s w_c \cdot ds + \int_v \phi \frac{\partial\rho_-}{\partial t}\, dv + \int_v J \cdot \frac{\partial A}{\partial t}\, dv + \int_v J^2/\sigma\, dv = 0 \quad (18a)$$

in place of

$$\oint_s w_f \cdot ds + \int_v E \cdot \frac{\partial D}{\partial t}\, dv + \int_v H \cdot \frac{\partial B}{\partial t}\, dv + \int_v E \cdot J\, dv = 0 \quad (18b)$$

Thus the rate of change of stored energy, plus the energy conversion, within s is obtained by integrating either $E \times H$ or $J\phi$ over s, but the only contribution to w_c is from the intersections of s with the conductors, whereas the Poynting vector $E \times H$ usually has to be evaluated everywhere except at these intersections. A further distinction is that w_f requires a vector cross-product, whereas w_c is in the direction in which the charges are moving, and this mathematical simplicity reflects the conceptual advantages of a description of the energy exchange processes in terms of the charges themselves, instead of the fields around them. The subscript distinguishing the moving (i.e. conduction) electrons ρ_- from the crystal lattice charges in eqn. 18*a* can be omitted, as the only way of producing a change in time is by movement, but its retention helps to emphasise an important point. As the energy transfer and exchange processes depend on the movement of charge, the contribution to eqn. 18*a* from the stationary charges is limited to the potential ϕ (other than the provision of the lattice collision 'obstacles').

The changes in scalar potential are not necessarily the same as the voltage V between any two points, because of what appear as induced electric fields in field theory (i.e. $\partial A/\partial t$ terms). The symbol V was used for potential in Reference 2 because of its familiarity to engineers, and because no distinction was necessary, but the difference between voltage and potential is of fundamental importance when considering power flow.

4 Hybrid vectors

The $J\phi$ description of energy flow is a return to the ideas which were familiar before Poynting and Heaviside showed that $E \times H$ is a valid equivalent, and conflicts with the usual assumption that all alternatives to $E \times H$ are more complex in form. Slepian [17], for example, has listed nine, and he [18] and others [19] have advocated what we may call the Slepian vector:

$$w_S = J\phi + \dot{D}\phi + H \times \dot{A} \quad (19)$$

‡ Or by $\rho_0 c\phi$, where ρ_0 is the change in charge density in a pulse. This form is closer to $E \times H$. It is examined elsewhere (see the footnote to Section 1).

in which the dots denote differentiation in time. A variant proposed by Macdonald [20] and O'Rahilly [6] has been advocated by Hines [21] and others. Many authors accept $\boldsymbol{E} \times \boldsymbol{H}$ somewhat reluctantly, because of the complexity of such alternatives, which is a direct consequence of their hybrid nature. Whereas $\boldsymbol{E} \times \boldsymbol{H}$ describes energy flow in terms of the field vectors only and $\boldsymbol{J}\phi$ in terms of the charges and potential only, all of the alternatives include mixtures of both.

One reason for this is that $\boldsymbol{w}_S$ attempts to describe power flow in terms of $\boldsymbol{J}\phi$ by utilising the arbitrary nature of the **curl** of $\boldsymbol{w}$, so that energy stored in the field is conveyed by the charges, so far as is possible, the hybrid terms representing the resulting exchange of energy between the two. Slepian also explored the consequences of changing the divergence of $\boldsymbol{w}$, by changing the energy density, but used another form of hybrid (see Appendix 16) which combined the real sources with a surface equivalent, and attributed the energy of one to $\boldsymbol{E} \cdot \boldsymbol{D}/2$ and that of the other to $\rho\phi/2$. Macdonald [20] is one of the few writers to examine the implications of Maxwell's two different theories, and of the two Lagrangian and power densities which they imply. He supported the electrokinetic momentum description, and criticised Maxwell's use of field theory. But he followed Maxwell in including the displacement current, defining the 'total current' as $\boldsymbol{J} + \partial\boldsymbol{D}/\partial t$ in place of $\boldsymbol{J}$, thus attributing some kinetic energy to empty space. He combined the resulting $\boldsymbol{J} \cdot \boldsymbol{A}/2$ description of the kinetic energy with the $\boldsymbol{E} \cdot \boldsymbol{D}/2$ distribution of electric energy, giving yet another hybrid form for $\boldsymbol{w}$. The variety in these results seems to suggest that the ingrained concepts of field theory have tended to obscure alternatives which might otherwise be obvious.

The essence of the problem is shown by the hybrid nature of the usual Lagrangian (eqn. 6). Although the field terms, on their own, provide the whole of the system energy, they are not sufficient, because of the need to predict the observable effects, which are the forces on the charges. Thus the interaction terms, due to ϕ and $\boldsymbol{A}$, are necessary, whereas the field terms are not. The ϕ and $\boldsymbol{A}$ values have to be calculated, of course, although the need to use field concepts is open to question [2]. However, this is of no concern for the present purpose, which is to show that, if we choose the Lagrangian L_c, then $\boldsymbol{J}\phi$ describes the energy flow, and no hybrid terms are necessary.

5 Energy exchange with a capacitor

When viewed from the standpoint of field theory, the weakness of the charge-potential description is its failure to explain how energy is conveyed across empty space from one charge to another; more specifically the transmission of energy between antennas, one of which is radiating. It is this which is central to the objections which many have to a $\boldsymbol{w}$ vector as simple as $\boldsymbol{J}\phi$. However, the mechanisms of the electromagnetic interactions, all of which are remote, are of no interest to the engineer if they are not needed to predict the behaviour of electromagnetic devices, and the general objective of Reference 2 was to show that this is so. As $\boldsymbol{J}\phi$ satisfies the stored energy and force requirements, the point which remains is not whether it is a 'correct' description of energy flow (a question which is clearly meaningless in view of the variety of the alternatives), but whether or not it is sufficient for practical purposes.

Inserting a capacitor into any circuit interrupts $\boldsymbol{J}\phi$, but not the flow of energy from a source in one part of the circuit to a sink in another, so that an example which is commonly used to demonstrate $\boldsymbol{E} \times \boldsymbol{H}$ provides a convenient illustration of the consequences of replacing it by $\boldsymbol{J}\phi$. In general, the plates can be replaced by antennas, because there is no need to restrict the frequency, but this may give a net energy loss (Section 12), and, for the present, it will be assumed that the system, as a whole, does not radiate energy. Its parts, however, necessarily radiate, because changes in any one take time to affect any other, so that the simple process of charging a capacitor (Fig. 1) has wider implications than is sometimes

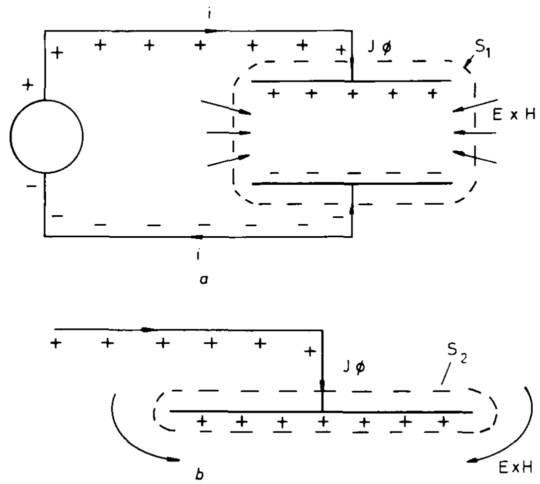

Fig. 1 *Energy flow into a capacitor*
a Integration surface enclosing capacitor
b Surface enclosing one plate

suggested. For the present, it is assumed that there is no dielectric.

Any surface s_1 (Fig. 1*a*), which separates the capacitor from the energy source or sink, provides two lead intersections, through each of which the power flow $\boldsymbol{J}\phi$ gives the rate of change of the energy on the plate to which the lead is connected. The $\boldsymbol{E} \times \boldsymbol{H}$ vector describes the same total energy transfer in terms of a flow around the capacitor edges, where $\boldsymbol{H}$ is due to the current in the plates. If, however, a different surface s_2 is drawn round one plate only, close to the surface (Fig. 1*b*), the two integrals are entirely different. The integral of $\boldsymbol{E} \times \boldsymbol{H}$ gives only the I^2R loss in the plate, because the stored energy flows around the outside of s_2 and is not intercepted by it, whereas the $\boldsymbol{J}\phi$ description is unchanged, and attributes half the total stored energy to each plate, if the ϕ datum is suitably chosen (Section 6). Shrinking s further, so that it encloses only a part of the plate, reduces $\boldsymbol{J}\phi$ to the energy which is expended in transferring charge into that part, and reflects directly the actual forces which have to be exerted on the charges during the transfer process. It gives the energy exchange through s, but (like the $\boldsymbol{E} \times \boldsymbol{H}$ vector) provides no account of the source of energy outside, or of what happens to it inside s. Eqn. 15 is a particular example, illustrating the more general form of energy transfer given by eqn. 13.

In general, the integration of $\boldsymbol{J}\phi$ over surfaces enclosing different parts of the system provides an account of the electromagnetic energy transfer, due to the **grad** ϕ forces, to and from the charges that define the part. The shape and extent of the surface s are irrelevant, in contrast with the integral of $\boldsymbol{E} \times \boldsymbol{H}$ which gives the same information only if s is sufficiently extensive. The capacitor example shows that what is meant by 'sufficiently extensive' is that s has to include the whole of the field which is associated with the part. This definition is not precise, because s necessarily cuts through the leads (i.e. through groups of charges whose individual contributions to the field are difficult to separate), in contrast with $\boldsymbol{J}\phi$ which provides an exact account of the energy transfer process to and from charge groups defined clearly and unambiguously.

When the potentials of both plates are raised, by adding an energy sink in series, power which enters one reappears at the other, in accordance with what is observed. Introducing any form of $\boldsymbol{E} \times \boldsymbol{H}$ probe changes the system, by adding additional charges, and the energy flow measured is the new $\boldsymbol{J}\phi$. The charge-potential prediction of the actual events accords with the field prediction, regardless of the rapidity, or frequency, of the process, because both give the same forces on the charges. The potential ϕ and momentum vector $\boldsymbol{A}$ quantify an action-at-a-distance theory which accepts that charges on one plate cause forces on the other, and the disappearance or reappearance of energy is a direct consequence of the spatial (and temporal) separation of those forces.

6 Lorentz gauge

ϕ and $\boldsymbol{A}$ are usually defined in terms of the fields, and rejected as quantities of any physical significance because of the arbitrary choice of datum and gauge, or div $\boldsymbol{A}$. The charge-potential approach suggests powerful reasons for choosing the definitions

$$\phi = \int \frac{\rho}{4\pi\varepsilon_0 r} dv \tag{20}$$

and

$$\boldsymbol{A} = \phi \boldsymbol{u}/c^2 \tag{21}$$

where the charges of density ρ are moving at velocity $\boldsymbol{u}$, both retarded values, evaluated at time $t - r/c$. These relationships, which are valid for all velocities ($u < c$) and accelerations, define both $\phi = 0$ and $\boldsymbol{A} = 0$, and also the gauge

$$\text{div } \boldsymbol{A} = -\frac{1}{c^2}\frac{\partial \phi}{\partial t} \tag{22}$$

When a group of charges of like sign is assembled, the resulting pressure energy density $\rho\phi/2$ is a consequence of the assembly forces, causing, for example, tensile stresses in the crystal lattice of a charged conductor. Transferring a charge Q from one conductor to another leaves both in an equal state of strain, if both have the same size and shape and the geometry is symmetrical. Because the strain is observable, the energy definition of ϕ requires a ϕ datum in accordance with eqn. 20. The resulting pressure energy $\rho_- \phi/2$ in Maxwell's 'electrical fluid' determines the rate of flow of energy, in eqn. 12, and shows that the distribution of power between the conductors of an n-wire system is not arbitrary, even though it may be convenient to take one wire as the datum when measuring total power.

The relationship between energy and momentum provides another illustration. The motion of any group of charge, of density ρ_1, at velocity $\boldsymbol{u}_1$, will cause a mutual momentum of density

$$\boldsymbol{g}_{21} = \rho_2 \boldsymbol{A}_{21}$$

at a second group, of density ρ_2, where

$$\boldsymbol{A}_{21} = \boldsymbol{u}_1 \phi_{21}/c^2$$

relates the $\boldsymbol{A}$ and ϕ values at group 2 due to group 1. The momentum reflects the forces on ρ_2 due to changes in $\boldsymbol{u}_1$. From eqn. 20 (with due regard to simultaneity considerations)

$$\rho_2 \phi_{21} = \rho_1 \phi_{12}$$

so that $\boldsymbol{g}_{21}$ represents a mass of density

$$m_{12} = \rho_1 \phi_{12}/c^2$$

moving at velocity $\boldsymbol{u}_1$, where $\rho_1\phi_{12}$ includes equal potential-energy and kinetic-energy contributions. The same energy density defines the mutual energy flow rate

$$\boldsymbol{u}_1 \rho_1 \phi_{12} = \boldsymbol{J}_1 \phi_{12}$$

in eqn. 12 (or, alternatively, an energy and charge transfer at velocity c, if the charges in a conductor are replaced by the change in charge density which is caused by a pulse [2]). By separating, or uncoupling, the potential and kinetic energies, the Lorentz gauge provides a simple and direct link between energy, mass, momentum and energy flow, with the proviso that the motion of one group defines $\boldsymbol{J}\phi$ in it, but assigns the resulting mutual momentum to the other.

It is helpful to compare the retarded ϕ and $\boldsymbol{A}$ values with those obtained from the Coulomb gauge, defined by

$$\text{div } \boldsymbol{A}' = 0$$

which is the most common alternative. As shown in Reference 2, all changes, regardless of supply frequency, can be described in terms of superposed pulses or surges (Fig. 2*a*). We take long parallel wires, so as to limit reflections, and assume, for the present, that the spaces around the wires are empty, giving the corresponding propagation velocity c. Nothing happens at any point P until the disturbance arrives, and there are, likewise, no changes in ϕ or $\boldsymbol{A}$, as defined by the Lorentz gauge. The Coulomb gauge, on the other hand, gives a potential ϕ' in accordance with eqn. 20, but with ρ interpreted as the instantaneous, not retarded, value, thus changing both the energy density and the energy flow. The corresponding Coulomb field is electrostatic, travelling ahead of its sources, so that the magnitude of ϕ' at P rises progressively to half its final value at the instant at which the disturbance arrives (assuming this to be steep-fronted). The changes in **grad** ϕ' are matched by corresponding changes in $\boldsymbol{A}'$, which is defined by

$$\nabla^2 \boldsymbol{A}' - (1/c^2)\, \partial^2 \boldsymbol{A}'/\partial t^2 = -\mu_0 \boldsymbol{J} + (1/c^2)\nabla(\partial\phi'/\partial t) \tag{23}$$

(Reference 12, p. 221). This shows how $\partial\phi'/\partial t$ acts as a source of $\boldsymbol{A}'$, so that $\boldsymbol{A}'$ also 'travels ahead' of the surge, despite the retardation term on the left-hand side. The additional sources generate components of $\boldsymbol{A}'$ at right angles to $\boldsymbol{J}$, as necessary to make the divergence zero, in contrast with the retarded $\boldsymbol{A}$, which is confined to the direction of current flow and is discontinuous in the plane of the wavefront.

In the region behind the wavefront, the radial $\boldsymbol{E}$-vector (Fig. 2*b*) is $-$**grad** ϕ in the Lorentz gauge (Fig. 2*c*§), whereas it contains contributions from both ϕ' and $\boldsymbol{A}'$. The $\boldsymbol{H}$, or $\boldsymbol{B}$, vector is zero before the wave arrives, but $\boldsymbol{A}'$ is not, providing another example of the difference

§ The retarded ϕ values are needed only on the wires, but are shown for comparison with the field description. The magnitude of $\boldsymbol{A}$ varies in much the same way (eqn. 21).

between 'kinetic' and 'magnetic' which results from the change of gauge. From the charge-potential viewpoint, the 'Coulomb', 'transverse' or 'radiation' gauge (because

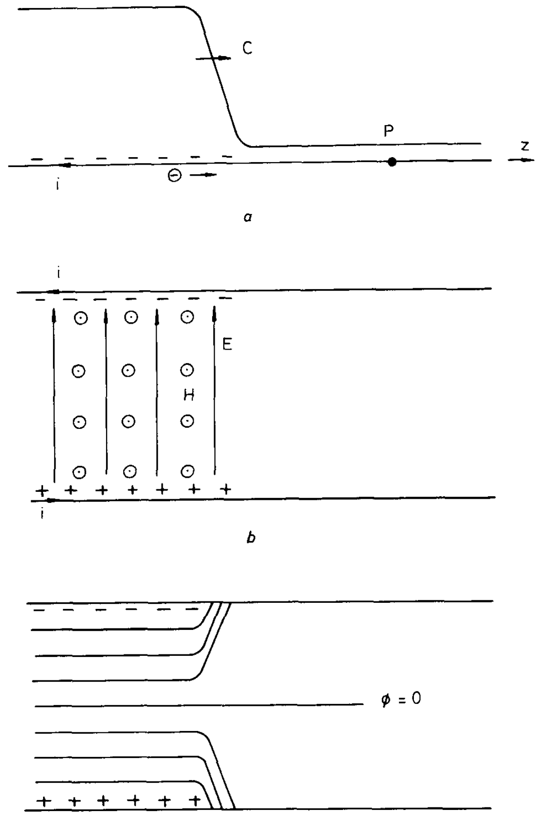

Fig. 2 *Surge travelling along parallel wires*

a Surge waveform
b **E** and **H** (or **B**) vectors
c Retarded ϕ values in space between wires
ϕ varies linearly for all z

it describes radiation in terms of A only) is essentially artificial in that the potential ϕ' travels at infinite velocity, and has to be compensated by opposing terms in A', because the condition div $A = 0$ is entirely arbitrary, and conflicts with the requirements imposed by retardation. This is of paramount importance because it is directly observable. The simple relationships between g and w are lost because they depend on eqn. 21. It is considerations such as these, following from relativistic properties of the wavefront (compared with those of ϕ' and A') (Reference 14, p. 178), which make the Lorentz gauge the obvious choice in the charge-potential formulation. It separates the potential energy from the kinetic energy in a physically significant way (Section 7), and the acceleration term emerges as a direct consequence of the retardation.

7 Separation principle

If we take two charge groups, 1 and 2, of equal but opposite densities,

$$\rho_1 = -\rho_2$$

travelling at equal but opposite velocities, and superpose them, the two current densities are equal:

$$J_1 = J_2$$

giving an A-vector, but no ϕ because there is no net charge. Thus, any electromagnetic system, defined as a set of charges whose positions, velocities and accelerations are all specified, may have superposed on it two additional sets, 1 and 2, chosen so that

$$J + J_1 + J_2 = 0$$

at all points, where J is the original current density. Removing the currents removes the A-vector, leaving the net charge density at every point, and thus the potential ϕ (eqn. 20) is unchanged. We may, likewise, redefine the charge groups 1 and 2, so that their two current densities are everywhere equal but opposite, and

$$\rho + \rho_1 + \rho_2 = 0$$

at every point, where ρ is the original charge density. Superposing this removes all the sources of ϕ in eqn. 20, leaving

$$A = \int \frac{J}{4\pi\varepsilon_0 c^2}\, dv \tag{24}$$

unchanged, where J is the retarded value, as in eqns. 20 and 21.

Any system may be separated in this way into two independent components. One, defined by $A = 0$, carries all of the potential energy $\rho\phi/2$ and the other, defined by $\phi = 0$, all the kinetic energy $J \cdot A/2$. This is merely the well known 'decoupling' property of the Lorentz gauge, but expressing it in physical terms helps to illustrate its significance. $J\phi$ is zero in each of the components, showing that changes in stored energy with time are entirely accounted for by the work done by the restraints which have to be imposed on the additional charge groups, two of which are needed for each of the separated ϕ and A sources. Thus, the $J\phi$-vector in the original system describes the work done on the 'moving' charges by the 'stationary' charges, and these terms can be simply, but precisely, defined (so that the symbol w is not inappropriate, although it does not indicate the work per unit volume, which is div w). ρA likewise represents a momentum transfer between the two system components, if ρ is interpreted as the net charge density. As ρ and J (like ϕ and A) are changed by a Lorentz transformation, the components also change and the reference frame must be specified, but the principle remains valid, regardless of the (inertial) reference which is chosen.

Hammond [22] has stated the same principle, although, in a rather different way, and shown how it is used implicitly in practical applications of $E \times H$. As E is zero at the surface of an ideal conductor, the power in the transformer, for example, flows around the ends of, and along the gap between, the windings, so that the energy transfer from the primary to the secondary cannot be found by integrating $E \times H$ over any surface s drawn between them. The copper acts as an energy waveguide, which causes practical problems in measuring $E \times H$ in electrical machines (Section 8) and means that the power radiated by an antenna cannot be found by integrating $E \times H$ over its surface. Resolving E into its components:

$$E = -\mathbf{grad}\ \phi - \partial A/\partial t$$

is equivalent to separating out the applied voltage from the back EMF, in the transformer or other device. If we remove the charges which cause ϕ, the charges which are left will not be in equilibrium and will require some other forces equivalent to the missing electric field. This is what Hammond calls the 'partial field', and is equal to

$-\mathbf{grad}\ \phi$. The Poynting vector, due to the part of the system which is left, becomes

$$\int \boldsymbol{E} \times \boldsymbol{H} \cdot ds = -\int \boldsymbol{H} \cdot \dot{\boldsymbol{B}}\, dv - \int \boldsymbol{E} \cdot \dot{\boldsymbol{D}}\, dv - \int (\boldsymbol{J} \cdot \boldsymbol{J}/\sigma + \boldsymbol{J} \cdot \mathbf{grad}\ \phi)\, dv$$

and gives a power flow into the transformer winding which accounts for the resistance loss, together with the energy conversion due to the partial field. Choosing s so as to cut through the supply leads, the last term integrates through the volume of the winding to give $\boldsymbol{J}\phi$ at the intersection with s. The partial-field $\boldsymbol{E} \times \boldsymbol{H}$ vector, which is defined in this way, is another form of hybrid, because it assigns the energy conversion to the charges, but the stored energy to the field. It predicts the same energy conversion in the winding as does $\boldsymbol{J}\phi$, but differs from it in explaining how the energy gets there.

One example of the separation approach is the customary description of energy flow into a capacitor (Fig. 1). The field is usually assumed to be $-\mathbf{grad}\ \phi$, and the $\partial A/\partial t$ term is ignored, an assumption which is sometimes stated explicitly and justified by arguing that the induced fields are negligible if the changes are sufficiently slow. This selects a partial field, but is valid only if resistance effects take the place of the missing term; either one can be ignored, but not both. The resulting discrepancy is the well known energy anomaly which appears when one capacitor is charged from another, so that the total charge Q is shared, giving a final $Q\phi/2$ energy which is half the initial value when the capacitances are equal. Eqn. 10 shows that the underlying charging process necessarily involves surges [2], and the final condition is an oscillation, not a static charge distribution, if there is no loss.

8 Measurement of energy transfer

Although a study of conductors in relative motion goes beyond the scope of this paper, the problem of measuring energy flow across the airgap in an electrical machine [23] provides a convenient illustration of the application of $\boldsymbol{J}\phi$. Consider, for simplicity, a linear version of the Faraday disc generator, consisting of a conducting sheet moving past an iron-cored DC coil (Fig. 3a), and supplying a load via brushes in contact with the opposite sides of the sheet. The $\partial A/\partial t$ (i.e. mutual momentum) forces on the conduction electrons in the sheet (Reference 2, Section 10) produce the EMF which, in field theory, is given by $\boldsymbol{u} \times \boldsymbol{B}$, and is opposed by the $\mathbf{grad}\ \phi$ forces due to the surface charges. The latter determine the potential difference between the brushes, so that integrating $\boldsymbol{J}\phi$ over any surface which intersects the connecting leads will give the power transfer to or from the machine. Extending the surface to infinity gives the same result from the integral of $\boldsymbol{E} \times \boldsymbol{H}$, provided that no other conductors are intersected. This view of the machine from a surface external to it illustrates the point that it is the $\mathbf{grad}\ \phi$ forces which determine the power flow into the machine (when motoring), regardless of the nature of the forces which oppose them.

An important characteristic of the charge-potential description is that ϕ, and hence $\boldsymbol{J}\phi$, is unchanged when moving from the stationary system to the reference frame of the sheet, moving at velocity $\boldsymbol{u}$ (provided that u/c is small), in direct contrast to the $\boldsymbol{E}$-vector in $\boldsymbol{E} \times \boldsymbol{H}$, which is $-\mathbf{grad}\ \phi$ in a stationary reference frame, but falls to zero in the moving sheet if this has no resistance. These rapid variations in the Poynting vector have led to

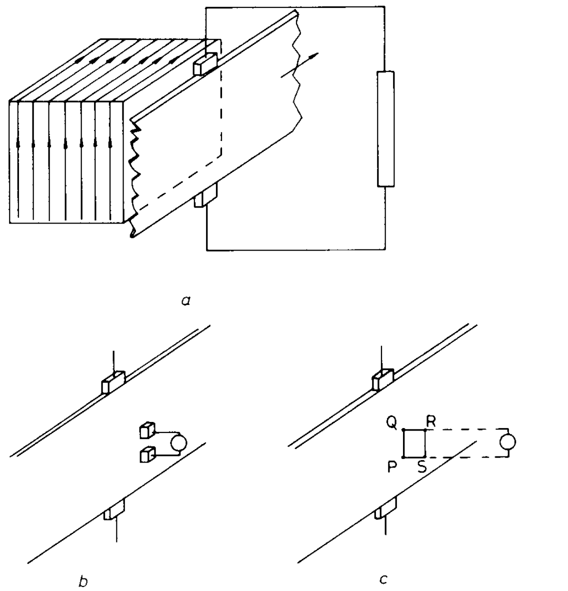

Fig. 3 *Linear version of Faraday disc generator*
a Generator arrangement
b **grad** ϕ probe
c Probe used to measure $\boldsymbol{A}$ and $\boldsymbol{J}$

another hybrid (including field and potential terms) devised specifically to solve the measurement problem [23]. The $\mathbf{grad}\ \phi$ forces can be observed with a voltmeter connected between two brushes mounted close together at the point of interest (Fig. 3b), and translated into $\boldsymbol{J}\phi$ by measuring the momentum vector $\boldsymbol{A}$, due to $\boldsymbol{J}$, with an integrating voltmeter connected to a short wire PQ mounted close to the plate surface (Fig. 3c). In practice, the brushes might be replaced by a capacitive pick up, and the A-probe by a multi-turn search coil PQRS giving the difference in A between PQ and RS, but these changes do not affect the measurement principle, which can be used to obtain a detailed picture of the $\boldsymbol{J}\phi$ power exchange with the sheet.

The output of the search coil used to measure $\boldsymbol{A}$ can equally be interpreted in terms of the component of $\boldsymbol{H}$ tangential to the sheet. Thus the probe can also be viewed as a means of measuring the normal component of $\boldsymbol{E}_p \times \boldsymbol{H}$ (where $\boldsymbol{E}_p = -\mathbf{grad}\ \phi$ is the partial field), providing another example of the way in which this gives the same description as $\boldsymbol{J}\phi$ of the power conversion in the sheet, although not the means by which it is conveyed there. The energy conversion process is an action which tends to conserve the mutual momentum $\rho\boldsymbol{A}$, between the conduction electrons in the sheet and the field currents, so that the signs of both the induced EMF and the drag force follow by inspection (in marked contrast to the usual cross-product, or 'right-hand' rules), and the result integrates directly to the mutual inductance expressions of modern machine theory.

The interaction between the conduction electrons in the two circuits is bilateral in ϕ, so that setting the sheet in motion causes an immediate reaction on the electrons in the field coil, because of the $\mathbf{grad}\ \phi$ forces due to the charge displacement in the sheet. This causes a rapid transient, leaving a surface charge on the field coil which modifies $\boldsymbol{J}\phi$, and hence the transfer of power into the field circuit. It can be argued that such a description of the machine behaviour provides a clearer view of the energy transfer mechanism than the customary description in terms of magnetic flux, and this description is better able to cope with unusual operating conditions such as fast-rise pulses.

9 The endless dance of energy

A point at which many texts seriously question the assumptions of field theory is in describing the effect on the Poynting vector of placing a charge on a magnet, or current-carrying coil. The radial $\boldsymbol{E}$- and axial $\boldsymbol{H}$-fields produce an $\boldsymbol{E} \times \boldsymbol{H}$-vector pointing in the circumferential direction, and thus a continuous flow of energy around the source, extending out to infinity. In general, the flow of mutual energy around closed paths is typical of any static combination of a magnet and a charged body. Although this is an immediate and necessary consequence of the assumption that the magnetic and electric energies are stored in the field, it is one which causes much unease, and is often held to have no physical significance.

The momentum density $\boldsymbol{D} \times \boldsymbol{B}$ may produce a force, or torque, when it changes, and it is, perhaps, surprising that this receives less attention than $\boldsymbol{E} \times \boldsymbol{H}$. Its implications will be discussed elsewhere. Replacing $\boldsymbol{E} \times \boldsymbol{H}$ suggests that there is nothing remarkable about energy flow in closed paths, as $\boldsymbol{J}\phi$ depends on the motion of charge and gives a picture of coils and magnets which is not static. The pressure in the 'electrical fluid' in a current-carrying coil necessarily controls the local energy transfer rate and, as in hydraulics, it is irrelevant where the 'pipe' comes from, or goes to, or whether or not any energy is being extracted. The point is, of course, of fundamental importance to the concept of energy transfer. Substituting a magnet for the coil replaces the current by electron spins, but these are equivalent to a surface current, so that only the details of the $\boldsymbol{J}\phi$ description are changed, not its essential features.

10 Dielectrics

Adding a dielectric (e.g. in Fig. 1) adds the polarisation vector $\boldsymbol{P}$, due to the bound charges, and hence an additional current of density

$$\boldsymbol{J}_d = \partial \boldsymbol{P}/\partial t = \dot{\boldsymbol{P}}$$

giving a corresponding flow of energy through the dielectric at the rate of $\boldsymbol{J}_d \phi$. This provides the change in the potential energy stored in the dielectric, of density

$$u_c' = (\boldsymbol{P} \cdot \mathbf{grad}\ \phi)/2 \tag{25}$$

which is comparable to the field energy distribution, as the field description acquires a direct physical significance inside polarised materials. But the direction of energy flow is to and from the plates into the material, and is at right angles to the $\boldsymbol{E} \times \boldsymbol{H}$ vector, because it is the charge displacement which both stores and conveys the energy. $\boldsymbol{P}$ measures the amount of the displacement, and $\mathbf{grad}\ \phi$ the electrical force which is necessary to produce it, requiring a restraint acting in opposition and a corresponding energy store. The description in terms of charges, as in conductors, provides a more direct picture of what is happening than that in terms of fields, and it avoids the ambiguities which are inherent in the conflict between the choice of the flux vector $\boldsymbol{D}$, or the vector $\boldsymbol{P}$, as the more fundamental concept: i.e. between the alternative treatments of dielectrics as 'conductors of flux' and as assemblies of charges. These can be a source of much confusion, because the equivalent surface charge, which plays a key role in the latter, is ignored in the definition of $\boldsymbol{D}$.

Taking the divergence,

$$\begin{aligned} \operatorname{div} (\dot{\boldsymbol{P}}\phi) &= \phi \operatorname{div} \dot{\boldsymbol{P}} + \dot{\boldsymbol{P}} \cdot \mathbf{grad}\ \phi \\ &= -\phi\dot{\rho} + \dot{\boldsymbol{P}} \cdot \mathbf{grad}\ \phi \end{aligned} \tag{26}$$

as in eqn. 13, but the charges are now subject to different constraints. Expressed in field terms, these are described by

$$\boldsymbol{P} = (\varepsilon_r - 1)\varepsilon_0 \boldsymbol{E} = \kappa\varepsilon_0 \boldsymbol{E} \tag{27}$$

where κ, rather than relative permittivity ε_r, measures the fundamental dielectric property. Translated into charge-potential terms, the equilibrium condition in a loss-free dielectric is

$$\mathbf{grad}\ \phi + \partial \boldsymbol{A}/\partial t - \boldsymbol{u} \times \mathbf{curl}\ \boldsymbol{A} + \boldsymbol{P}/\kappa\varepsilon_0 = 0 \tag{28}$$

compared with eqn. 14 in a conductor. The losses add a further component which can be represented in various ways, including an equivalent conductivity. In the absence of losses, eqn. 18*a* is replaced by

$$\oint_s (\dot{\boldsymbol{P}}\phi) \cdot d\boldsymbol{s} + \int_v \phi\dot{\rho}\ dv + \int_v \dot{\boldsymbol{P}} \cdot \dot{\boldsymbol{A}}\ dv + \int_v \frac{1}{\kappa\varepsilon_0} \boldsymbol{P} \cdot \dot{\boldsymbol{P}}\ dv = 0 \tag{29}$$

in which the last two terms represent changes in the kinetic energy

$$u_c'' = \dot{\boldsymbol{P}} \cdot \boldsymbol{A}/2 \tag{30}$$

of the displaced charges, and in the displacement energy u_c'.

The vector $\dot{\boldsymbol{P}}\phi$ in the dielectric, like that in the conductor, represents the interchange of energy between the 'stationary' charges and the local moving charges. It is of less practical interest than is $\boldsymbol{J}\phi$ in the conductors, because the relevant energy sources are not as accessible. There is no $\boldsymbol{w}$-vector in the airgaps between the different materials, because the charge-potential description of the energy transfer across them is in terms of remote actions.

11 Magnetic materials and transformers

Viewed in charge-potential terms, a transformer consists essentially of two long conductors (Fig. 4*a*), wound in a spiral to maximise their mutual momentum, usually with the assistance of an iron core (Fig. 4*b*). An example of design calculations carried out in these terms, without reference to flux, has been given in Reference 2. The $\rho\phi/2$ pressure originating in the supply transports charges along the primary winding, carrying with them a $\boldsymbol{J}\phi$ energy from each terminal. This energy is dissipated partly in lattice collisions, and partly by the $\partial \boldsymbol{A}/\partial t$ forces, where $\boldsymbol{A}$ represents the coupling with the core and the secondary. The core, represented largely by an equivalent surface current $\boldsymbol{J}_s$, has the effect of a flywheel, into which the primary injects and recovers momentum by the remote action effect of $\boldsymbol{A}$. The electron stream in the secondary conveys energy into the load, which generates a $\rho\phi/2$ pressure in the winding, and hence controls the rate at which energy is extracted from the 'fluid'. The resulting

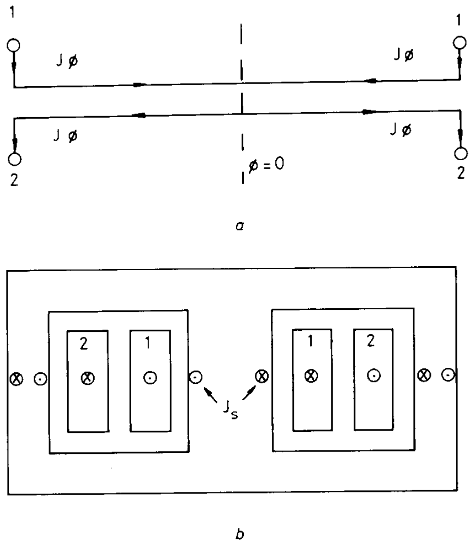

Fig. 4 *Energy flow in a transformer*
a Primary and secondary conductors
b Winding and core arrangement
Equivalent current $\boldsymbol{J}_s$ distributed over the core surface

drag tends to reduce $\partial A/\partial t$ in the secondary, and hence in the primary, thus increasing the energy conversion rate and the $\boldsymbol{J}\phi$ energy supplied.

This summary illustrates the essential simplicity of the different viewpoint, and also the way in which the energy is conveyed across the gap by the kinetic coupling due to the movement of the electrons, described quantitatively by the vector $\boldsymbol{A}$. The transformer would still work if ϕ were removed by the separation principle. This is in contrast to the energy transfer between the plates in Fig. 1, due to the 'static' forces and quantified by ϕ. In the transformer, as in the capacitor, the assumption that the relevant energy is stored in the copper makes any enquiry into the 'mechanism' of the transfer essentially meaningless. Everything which is observable, however the observation is made, is accounted for by $\boldsymbol{J}\phi$, which is sufficient to provide a complete and detailed account of the energy transfer in the device, regardless of the supply frequency.

Although the purpose of the transformer is to transfer power, few texts give any explanation of how this occurs, and the Poynting vector does not transfer it between windings. Most engineers would probably regard the description of the output energy as flowing around the end-windings as entirely academic. Yet it is commonly asserted that the energy is stored in the field, and some form of energy flow giving very high-power densities in empty space (in a large transformer) necessarily follows. The student is surprised to learn that, although the core is the seat of much of the action, it accounts for practically none of the energy (or energy flow), a point which is explained more simply and directly in momentum terms, as is the relationship between the stored energy and the equivalent circuit model. It is doubtful whether many students gain any real understanding of the role of the 'magnetic' energy, although the transformer works by transforming potential energy into kinetic energy, and the $\boldsymbol{J}\phi$ vector explains exactly how this is done.

The energy flow in the core, like that in a dielectric, is of limited practical interest (forces can be calculated more conveniently in terms of the charge-potential equivalent of the Maxwell stress, as will be shown elsewhere). The customary dipole model of magnetic materials can be translated into charge-potential terms [2], but is essentially foreign to it because the dipole, as an element, is based on the concept of flux. A self-consistent description requires the introduction of an electron spin vector as the source, which is loosely equivalent to a reversal of the roles of the $\boldsymbol{B}$- and $\boldsymbol{H}$-vectors in field theory. However, it presents the magnetisation curve in an unfamiliar way, and requires a more extended treatment than is possible here.

12 Radiation

Although the $\boldsymbol{J}\phi$ description is well recognised at low frequencies, it is customarily treated as an approximation which becomes invalid as the frequency is raised. The failure to question this assumption shows how deeply rooted is the field theory philosophy, as the surge (Fig. 2) provides an obvious example of the equivalence of the field and charge-potential descriptions of energy, at all frequencies.

The radiation of energy from an antenna is commonly taken as direct evidence of $\boldsymbol{E} \times \boldsymbol{H}$, but the power is usually measured by connecting some form of wattmeter into the leads through which the antenna is supplied with energy, so that the measurement provides equal evidence for $\boldsymbol{J}\phi$. It is, of course, the radiation lobes which matter in antenna design, and these may be described by the $\boldsymbol{E} \times \boldsymbol{H}$ vector, but they are calculated by using the retarded ϕ and $\boldsymbol{A}$, and measured with a second receiving antenna; i.e. by observing not the field vectors, nor $\boldsymbol{E} \times \boldsymbol{H}$, but by moving the receiving antenna around and measuring either $\boldsymbol{J}\phi$ or ϕ with a voltmeter. The different descriptions of energy flow form part of two theories which differ in philosophy, but share the same ϕ and $\boldsymbol{A}$, and predict the same forces on the charges:

$$\boldsymbol{f} = \rho(-\mathbf{grad}\ \phi - \partial \boldsymbol{A}/\partial t + \boldsymbol{u} \times \mathbf{curl}\ \boldsymbol{A}) \qquad (31)$$

and thus the same observable effects in antennas and in waveguides, as in all other electromagnetic devices.

Once $\boldsymbol{f}$ is known everywhere, then the energy transferred by the moving charges is given by $\boldsymbol{u}\rho_{-}\phi$, in accordance with eqn. 13, and the antennas are no different in this respect from the plates in Fig. 1, or from the wires in Fig. 4*a*. Both examples illustrate the way in which removing the energy from empty space also removes the energy flow vector which is needed to account for it, and makes the question of the 'mechanism' of energy transfer wholly irrelevant. The mutual energy is divided between the charges, so that they necessarily become the vehicle by which the energy is transferred. It is, perhaps, remarkable that, although Maxwell's attempts to use field theory to develop a mechanistic aether model have been universally abandoned, and with them the possibility of any useful meaning to the concept of the field as providing a force transfer mechanism, the properties of mass, energy and momentum in empty space are still interpreted literally, and go largely unchallenged. The resulting point of view is essentially ambivalent, as we cannot have the field energy, or momentum, without the Maxwell stress, a point which is discussed in more detail elsewhere. Most engineers have no difficulty in accepting the idea of electrostatic forces between capacitor plates, or magnetic forces between coils, as remote actions, and the concept of remote energy transfer is a direct and necessary consequence.

Hammond [22] has given examples comparing the usual $\boldsymbol{E} \times \boldsymbol{H}$ integral with $\boldsymbol{J}\phi$ (expressed in terms of the 'back EMF') to obtain the energy radiation from simple dipoles, showing that the $\boldsymbol{J}\phi$ calculation can be simpler. He points out that the usual integral is in terms of a partial field, because the actual $\boldsymbol{E}$-field is complex, and $\boldsymbol{E} \times \boldsymbol{H}$ describes the energy as coming not from the dipole itself, but from the stream flowing in around the supply leads. In consequence, the standard textbook treatment substitutes a hypothetical dipole consisting of isolated charges; i.e. uses the separation principle. This has the desired effect of making the dipole appear to radiate, but its use is potentially confusing to students and increases the mystery which tends to surround $\boldsymbol{E} \times \boldsymbol{H}$. The need for such hybrids illustrates the practical difficulties faced by communication as well as power-frequency engineers in applying 'pure' field theory.

If we remove the leads which supply the dipole it is possible to draw a closed surface s around it, and to include the energy source (i.e. the hypothetical restraints) inside s. $\boldsymbol{J}\phi$ allows the same surface, with the leads in place. It describes energy as coming into the dipole through s and disappearing, whereas $\boldsymbol{E}_p \times \boldsymbol{H}$ describes it as appearing inside s and flowing out. The two energies are the same because they are different descriptions of the same forces. If we make the connection leads sufficiently long to separate the dipole from the generator, the local conditions around it become irrelevant, and the dipole appears as any other electrical load. Its radiation resistance can be calculated from the local ϕ- and $\boldsymbol{A}$-values (in the Lorentz gauge), showing that the concept of mutual energy (Fig. 1) is now limited to the different parts of the dipole, and leading to an examination of the nature of the charge conditions causing the radiation which is at least as instructive as 'explaining' it in terms of fields. The retardation shows that the energy loss depends on acceleration and makes radiation resistance a simple extension of the concept of electron stream momentum.

13 Conclusions

It has been shown that conventional electromagnetism has failed to distinguish sufficiently clearly between two entirely different viewpoints, one of which attributes energy, energy flow and momentum to the fields around the charges, whereas the other attributes the same properties to the charges themselves. Both were recognised by Maxwell, who developed the idea of the 'electrical fluid' momentum in some detail and contrasted the two approaches, but the clarity of Maxwell's treatment has been lost in modern accounts. Some of the consequences have been examined.

'Pure' field theory treats conductors as no more than passive waveguides, so that energy does not flow from the primary to the secondary of a transformer, and is not radiated from the surface of an antenna. Although such a theory is simple and self-consistent, it is of limited practical value, as is shown by the very selective way in which it is used. Engineers need to predict the behaviour of the field sources (i.e. the charges) rather than what happens in the empty spaces between them, and this has led to the use of various forms of hybrid in which the field and charge-potential descriptions are mixed. The hybrids include the energy flow vectors suggested by Slepian and others, and the partial-field approach which is commonly used to predict dipole radiation. It has been shown that a theory which is both simple and self-contained is obtained by abandoning the field description altogether, and attributing all electromagnetic properties to the charges. The energies densities are $\rho\phi/2$ and $\boldsymbol{J} \cdot \boldsymbol{A}/2$, in place of $\boldsymbol{E} \cdot \boldsymbol{D}/2$ and $\boldsymbol{H} \cdot \boldsymbol{B}/2$, and the energy flow and momentum vectors are $\boldsymbol{J}\phi$ and $\rho\boldsymbol{A}$ in place of $\boldsymbol{E} \times \boldsymbol{H}$ and $\boldsymbol{D} \times \boldsymbol{B}$. The retarded potential ϕ and momentum vector $\boldsymbol{A}$ take the place of the field vectors (or, more specifically, the fluxes) as the quantities of fundamental physical significance.

Reformulating Poynting's theorem in charge-potential terms shows that replacing $\boldsymbol{E} \times \boldsymbol{H}$ by $\boldsymbol{J}\phi$ requires a change in divergence as well as curl. It confines the energy flow in an antenna to the interior of the wires, and predicts the radiation from the input instead of the output power, so that it is $\boldsymbol{J}\phi$, not $\boldsymbol{E} \times \boldsymbol{H}$, which describes the antenna wire as radiating. The $\boldsymbol{J}\phi$-vector formalises a view of power flow which is used intuitively by probably the great majority of engineers and is, in essence, a return to ideas which were generally accepted before Poynting and Heaviside showed the validity of the field alternative.

It is, perhaps, remarkable that so simple and general an alternative to the Poynting vector has escaped attention in modern work, but this may reflect an artificial difficulty, created by the ingrained concepts of field theory. The disappearance of energy into one winding of a transformer, and its reappearance in the other, is a direct consequence of the interaction forces between the various charges and currents, and it is no more necessary to provide an 'explanation' for the 'mechanism' of the transfer of energy than it is to provide one for the transfer of force. The $\boldsymbol{J}\phi$ description is more in accord with practical measurement than is $\boldsymbol{E} \times \boldsymbol{H}$, and simplifies the calculations, in both power-frequency and communication applications, by avoiding the complexity of the $\boldsymbol{E} \times \boldsymbol{H}$ energy-flow pattern around end-windings and antennas. It also avoids the extremely high-power densities, in the empty spaces of large machines, which are generally regarded as conceptually obnoxious, and it clarifies other conceptual difficulties, such as the 'endless dance of energy' around a charged magnet.

The $\boldsymbol{J}\phi$-vector illustrates the fundamental importance of the electrostatic forces in all electromagnetic devices, including those labelled magnetic. The flow of energy is intimately associated with the momentum vectors $\rho\boldsymbol{A}$ and $\boldsymbol{D} \times \boldsymbol{B}$, which are compared elsewhere. As Maxwell pointed out [1], the concept of electrokinetic momentum follows naturally from Lagrangian dynamics, which are of increasing importance in modern physics [24], showing the possibilities offered by the charge-potential approach as a means of integrating the treatment of mechanical and electromagnetic systems.

14 Acknowledgments

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16 Appendix: Slepian energy-flow vectors

In Reference 17, Slepian defined nine different energy-flow vectors $\boldsymbol{P}$ at any surface s, by considering the sources which would be needed to terminate the field on s and calculating the power which these sources would require. He showed that various forms of source were possible, defining $\boldsymbol{P}$ vectors with different curls, and that other variants depended on the choice of $\rho\phi/2$ in place of $\boldsymbol{E} \cdot \boldsymbol{D}/2$ and $\boldsymbol{J} \cdot \boldsymbol{A}/2$ in place of $\boldsymbol{H} \cdot \boldsymbol{B}/2$. But he limited these changes in energy density to the equivalent sources on s, and ignored the actual field sources in the region enclosed. The resulting $\boldsymbol{P}$ vectors gave energy densities such as

$$T_6 = T_1 - \boldsymbol{H} \cdot \boldsymbol{B} + \boldsymbol{J} \cdot \boldsymbol{A}$$

in his notation, where

$$T_1 = \boldsymbol{E} \cdot \boldsymbol{D}/2 + \boldsymbol{H} \cdot \boldsymbol{B}/2$$

is the usual field energy density associated with the Poynting vector $\boldsymbol{E} \times \boldsymbol{H}$. Slepian pointed to the negative field energy which results, but did not appear to realise that this indicates the need to reallocate the energy of one of the components of the hybrid. The consistent choice of a single energy density, instead of a hybrid, gives the results obtained in Section 3. It is remarkable that, having come so close to his objective of a $\boldsymbol{J}\phi$ vector [18], Slepian failed to achieve it.