COMMENT

INTERACTION OF A MOVING CHARGE AND TOROIDAL COIL

Two recent papers [1, 2] provide a most interesting and stimulating study of a difficult problem, and make a very useful analysis of the details, but I suggest that some of the conclusions are open to question.

The most characteristic feature of the interaction is brought out in [2] by the assumption that the drive current is constant. The coil then exerts no $q\boldsymbol{u} \times \boldsymbol{B}$ force on the charge q, because of the absence of any $\boldsymbol{B}$, although q produces a $\boldsymbol{J} \times \boldsymbol{B}$ force in the toroid in the reference frame R_t, in which the toroid is stationary. But the $\boldsymbol{B}$ field of q disappears in the reference R_q in which q is stationary, so that it cannot produce the $\boldsymbol{J} \times \boldsymbol{B}$ force on which so much of the analysis depends. Energy considerations also suggest no net force on the coil, since the authors confirm the conclusion in their reference 6 [3] that the electrical output is obtained from changes in stored energy, and this allows no force on the moving part in either R_t or R_q. The absence, in R_q, of the $\boldsymbol{A}$ and $\boldsymbol{B}$ vectors also, of course, removes the means by which we account for induction in R_t but, as was pointed out in [3], this can be explained by the magnetic effect of the induced charge. Accounting for $\boldsymbol{J} \times \boldsymbol{B}$ in the same way suggests an interaction between the different parts of the coil, and thus the absence of a net force. It also helps to show the consequences of the neglect of the 'electric' effects described by ϕ. One of the various consequences of the grad ϕ term is that q does not travel at uniform velocity when $q\partial A/\partial t$ and $q\boldsymbol{u} \times \boldsymbol{B}$ are zero, as is stated on p.16 of [2].

The momentum is accounted for by qA and, as remarked in [1], this is equivalent to a change of field momentum, but eqn. 25 gives the mutual term instead of the 'momentum of the field of the charge'. The $\boldsymbol{D} \times \boldsymbol{B}$ equivalent of the mutual qA is given by the product of the $\boldsymbol{D}$ field of q and the $\boldsymbol{B}$ field of the coil, which is confined to the coil interior. None is removed by radiation in the usual way, and we obtain an unusual – indeed unique – picture of a force accounted for by field momentum changes which are confined to the interior of the part on which the force acts. However, the field momentum is given not by the partial, or mutual, components, but by the net $\boldsymbol{B}$, together with a net $\boldsymbol{E}$ which is controlled by the charges induced on the coil, and confirms the importance of ϕ. When a low-resistance coil is supplied at constant current the net $\boldsymbol{E}$ and thus $\boldsymbol{D} \times \boldsymbol{B}$ tend to zero in the interior, and the coil likewise makes no contribution to $\boldsymbol{B}$ in the exterior, suggesting the absence of any net $d\boldsymbol{p}/dt$ force on either part of the system.The distinction between partial and total fields is also relevant to the remarks about the Poynting vector, $\boldsymbol{E} \times \boldsymbol{H}$ in Section 6 of [1], since the net $\boldsymbol{E}$ field is not confined to $\partial A/\partial t$ but includes a large contribution from grad ϕ.

It is helpful to rearrange eqn. 25 of that paper in the form

$$\begin{aligned} d\boldsymbol{p}/dt &= q[(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}] \\ &= q[-\nabla\phi - \partial \boldsymbol{A}/\partial t + \nabla_A(\boldsymbol{u}.\boldsymbol{A}) - (\boldsymbol{u}.\nabla)\boldsymbol{A}] \end{aligned} \quad (1)$$

where the subscript A is used as a reminder that the operation is on $\boldsymbol{A}$, not $\boldsymbol{u}$. The expansion separates out the scalar from the non-scalar components of both $\boldsymbol{E}$ and $\boldsymbol{u} \times \boldsymbol{B}$. The observable force on q, due to its motion in R_t, is given by $q\boldsymbol{u} \times \boldsymbol{B}$ and is zero. $q\nabla_A(\boldsymbol{u}.\boldsymbol{A})$ is calculated in eqn. 27, and in eqn. 15 of [2], and is shown to have the same magnitude as the $\boldsymbol{J} \times \boldsymbol{B}$ force, F, on the coil, so that $q(\boldsymbol{u}.\nabla)\boldsymbol{A}$ is likewise of magnitude F. When $\boldsymbol{u}$ and $\boldsymbol{A}$ are in the same direction, x, the two components are algebraically identical,

$$\nabla_A(\boldsymbol{u}.\boldsymbol{A}) = (\boldsymbol{u}.\nabla)\boldsymbol{A} = u\partial A_x/\partial x \quad (2)$$

The point that both are of the same size as the $\boldsymbol{J} \times \boldsymbol{B}$ force due to q (in R_t) is clearly important, but interpreting them as a charge interaction leads to questions about their physical significance. The sources of both $\nabla_A(\boldsymbol{u}.\boldsymbol{A})$ and $(\boldsymbol{u}.\nabla)\boldsymbol{A}$ are those parts of the coil in which the current is flowing in the axial, or x-direction, whereas the $\boldsymbol{J} \times \boldsymbol{B}$ force acts on the radial wires. Replacing $\boldsymbol{B}$ by $\boldsymbol{A}$ helps to separate out the roles of the different elements of the system, and tends to suggest an equilibrium in the internal forces on the coil rather than a net force, under the unusual conditions of the interaction, in which the $\boldsymbol{A}$ field of q describes the relativistic modification of its ϕ field.

Matters are complicated in [1] by including the $\partial A/\partial t$ term due to changes of current in time, for reasons which are given much emphasis, but are partly called into question, by [2]. Sources with a sufficiently high internal impedance and response time are a laboratory commonplace, and their precise performance hardly matters, since perturbations to the current can be considered as an additional effect. The objection on the grounds that a 'flight-plan' is both necessary and impractical likewise seems difficult to sustain, since the operating cycle can be repeated indefinitely, so that a computer-controlled source could well be 'taught' to respond to any desired accuracy, should we wish to drive the system in this way. More importantly, the analysis in [1] assumes a constant current in calculating the force on q, in eqn. 28, by including only the variation with x when differentiating the right-hand side of eqn. 27 to obtain $q\boldsymbol{u}\, \partial A_x/\partial x$. Two of the components in eqn. 1,

$$q\, d\boldsymbol{A}/dt = q\partial \boldsymbol{A}/\partial t + q(\boldsymbol{u}.\nabla)\boldsymbol{A} \quad (3)$$

are carried to the other side, and ignored by introducing what is called the 'effective force'. The implications are best illustrated by considering the interaction due to the changes of current with time when the charge q is stationary. $q\partial A/\partial t$ is then the only magnetic force on q, and is a very 'real' one, in the sense that it changes the momentum $\boldsymbol{p}$ of q in eqn. 1, as stated in Section 6 of [1]. It is this which we might expect to balance the net force on the coil, but it is excluded by calculating the 'effective force' due to changes of $\boldsymbol{A}$ with x. The result confirms [2] in showing that the components of $q\boldsymbol{u} \times \boldsymbol{B}$ can both be compared with $\boldsymbol{J} \times \boldsymbol{B}$, thus giving an apparent force balance by selecting one. But these components are additional to the $d\boldsymbol{p}/dt$ action of the $q\partial A/\partial t$ force on q due to the current changes.

The practical value and significance of the interaction is clearly much reduced by limiting it, in [1], to the currents induced by spraying electrons at conducting material. However, the energy conversion rate $\boldsymbol{J}.\partial A/\partial t$, in the coil depends directly on the current, and can be increased accordingly by an external current source. As was pointed out in [3], this is why the device has been studied as a possible means of extracting energy (e.g. in Reference 4 of that paper). When operating with a current source, the supply of the output from the stored energy (i.e. from $\boldsymbol{J}.\boldsymbol{A}/2$)

shows that the major problem is one of power factor rather than efficiency. As with a two-winding transformer it is necessary to connect something in series with the secondary to obtain a useful output, and we obtain a rather limited view of the power extraction capability by imposing a short-circuit and studying the loss in winding resistance.

One point on which I am in full agreement with Dr. Murgatroyd is in his doubts about using this as an example for students. Nevertheless, I suggest that, whether or not the device has any useful application as a power generator, the underlying issue is of wider interest since the interaction between a moving charge and a current goes to the core of our understanding of electromagnetism. All of what we call 'magnetism', other than the electron-spin contributions, follows by the superposition of the effects of charge motion which are illustrated very directly in this example. The (magnetic) Aharonov–Bohm effect depends on it, although manifest in a different way, and this has generated a great deal of interest, as well as various practical applications, as illustrated by the extent of the literature more than a decade ago [4]. The use of a magnet in place of a coil provides what is, in effect, a constant-current source, and the relevant experimental observations confirm the energy exchange in the absence of any force on q.

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References

1 MURGATROYD, P.: 'Interaction of moving charge and a toroidal coil', *IEE Proc. Sci. Meas. Technol.*, 1999, **146**, (3), pp. 128–134
2 MURGATROYD, P., and SPENCER, A.J.: 'Interaction of moving charge and a toroidal coil supplied by a constant-current source', *IEE Proc. Sci. Meas. Technol.*, 2000, **147**, (1), pp. 15–18
3 CARPENTER, C.J.: 'Electromagnetic energy changes due to charges moving through constant, or zero, magnetic field', *IEE Proc. A*, 1991, **138**, (1), pp. 55–70
4 PESHKIN, M., and TONOMURA, A.: 'The Aharonov-Bohm effect' (Springer-Verlag, 1989)

REPLY

The papers [1, 2] discussed by Dr. Carpenter [3] are about two apparently similar, but in practice different, systems. They were devised by E.G. Cullwick, who, in his analysis of the magnetic forces between moving charges and stationary circuits, saw difficulties which seemed to put the conservation laws for momentum and energy in doubt. The system analysed by Cullwick involves the launching of a charged particle along the major axis of a toroidal coil which in [1] is passive and resistive but in [2] active and supplied by a constant current source (CCS). In both [1, 2] it was shown that classical electromagnetism explains the magnetic interactions fully.

The papers [1, 2] defined the models in some detail, with practical aspects and numbers. Many specific equations were derived for the changing forces, energy flows, dissipation, etc. throughout the interactions and, unlike earlier literature on these problems, the papers also calculated numerical answers. Dr. Carpenter suggests that some of the conclusions of [1, 2] are open to question, but he has not offered alternative equations or different numerical answers.

It was asserted in [1] that the CCS model is impractical. This is not a matter merely of available technology. The analysis in [2], with numbers, shows that the magnetic fields of the moving charge are so small that the needed stability of the CCS is not achievable because it is comparable with single electrons flowing. Dr. Carpenter's suggestion that a permanent magnet might provide the equivalent constant current similarly makes impractical demands on real materials. Any deviation at all from constant current changes the model. A nearly-CCS is not the same problem, and even small current fluctuations will demand solving the resistive [1] problem in some form.

All the literature, so far, has assumed a toroid starting at rest and a charge moving towards it. Dr. Carpenter seeks to widen the discussion by considering systems in which the charge is at rest and the toroid moves. Such systems are impractical, most fundamentally because Earnshaw's theorem precludes stable positioning of the charge before the experiment begins. Even so, considering the moving-CCS case as an imagined and idealised problem only, the equation of motion for q is eqn. 25 of [1]. (This has an error. The term $\nabla\phi$ should be $q\nabla\phi$.) Since q is stationary the last term is zero, and the continued assumption of no charges on the toroid leaves $\nabla\phi$ at zero, so $(\boldsymbol{p} + q\boldsymbol{A})$ stays constant. The stationary q has no magnetic field, so there is no magnetic force on the toroid. As the toroid closes with the charge, $\boldsymbol{A}$ increases at q, so $\boldsymbol{p}$ changes in the opposite direction and q also begins to move.

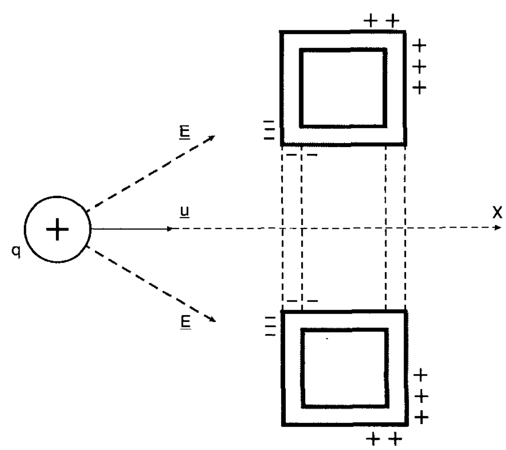

Fig. 1 *Image effects in resistive toroid*

Dr. Carpenter remarks on the neglect of $\nabla\phi$ terms in the equation of motion of q. This neglect was clearly stated in [1] after eqn. 25, and two reasons were given for it. Firstly, the foregoing literature was mostly about magnetic interactions and [1] sought to balance up the momentum and energy effects of these interactions only. Secondly, looking at the toroid in a practical way as made of continuous copper as described in [1], it became clear that there was another mechanism involved, as will now be outlined. Fig. 1 depicts the charge q approaching the stationary resistive toroid as in [1]. Regarded as a quasi-static system, rings of surface charge are induced all around the toroid, such as to cancel exactly the $\boldsymbol{E}$ field of q inside, while the toroid as a whole remains charge-neutral. These surface charges have an $\boldsymbol{E}$ field (the $\nabla\phi$ terms) affecting the motion of q, which will be attracted. There is an active literature reviewed in [4] on what might be termed 'image' systems of moving charges. When q moves, the induced charges have to change, both in magnitude and distribution. This requires and dissipates energy, and it has been shown in other geometries that this energy is taken from the moving charge by a friction-like force opposing the motion. The underlying physics of these interactions includes the conductivity of the metal (which does not feature in the CCS model), and the calculations are difficult, even in simpler geometries than the toroid. However, some long-range ($x >> b$) scaling properties can be predicted. The 'image' forces attracting q scale as x^{-5} compared with $\boldsymbol{u}^3x^{-8}$ for the magnetic repulsion in [1]. The friction-like dissipation scales as $\boldsymbol{u}^2$ in the 'image' movements, whereas the magnetically induced current losses scale as $\boldsymbol{u}^4$ in [1]. So the $\nabla\phi$ terms arising from 'image' effects have to balance energy and momentum separately from the 'magnetic' effects. While there seems no reason to suppose they will not balance in the toroid problem, the likely effort required is daunting.

Finally, Dr. Carpenter mentions the Aharanov–Bohm Effect. Interestingly, the literature cited in [4] indicates that the 'image' studies were partly motivated by a search for a classical model to explain that quantum effect, but the search seems not to have succeeded. Cullwick's original problem was with classical electromagnetism, and there still seems no reason to go beyond classical theory to explain it.

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References

1 MURGATROYD, P.: 'Interaction of a moving charge and a toroidal coil', *IEE Proc., Sci. Meas. Technol.*, 1999, **146**, (3), pp. 128–134
2 MURGATROYD, P., and SPENCER, A.J.: 'Interaction of moving charge and a toroidal coil supplied by a constant-current source', *IEE Proc., Sci. Meas. Technol.*, 2000, **147**, (1), pp. 15–18
3 CARPENTER, C.J.: 'Comment: Interaction of a moving charge and toroidal coil', *IEE Proc., Sci. Meas. Technol.*, 2000, **147**, (5), pp. 266–267
4 BOYER, T.H.: 'Understanding the penetration of electromagnetic velocity fields into conductors', *Am. J. Phys.*, 1999, **67**, pp. 954–958