Electromagnetic Induction

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THE TWO most widely used laws of electromagnetic induction are the flux-linking law, as given in rationalized meter-kilogram-second units

$$\mathcal{E} = -\frac{d\phi}{dt} \tag{1}$$

which states that the electromotive force $\mathcal{E}$ induced in a circuit is equal to the negative time derivative of the magnetic flux ϕ linking the circuit, and the flux-cutting law

$$d\mathcal{E} = v \times B \cdot dl \tag{2}$$

which states that the electromotive force across an element dl moving with a velocity v through a magnetic field B is equal to their triple scalar product. Upon integrating over the circuit, this gives the induced electromotive force

$$\mathcal{E} = \int v \times B \cdot dl \tag{3}$$

For uniform and mutually perpendicular B, v, and l the flux-cutting law is simply

$$\mathcal{E} = Bvl \tag{4}$$

In many simple cases both induction laws lead to the same value of induced electromotive force. This has been the source of much confusion, since it then appears that the two induction laws are merely alternate expressions for the same thing, or that one of them is completely general and the other merely a special case. The most outstanding by-products of this misunderstanding are the innumerable d-c generators without moving contacts which have been invented (and do not work). It is a simple matter to construct examples in which the correct value of induced electromotive force is given by: (a) equation 1 and not by equation 3, (b) equation 3 and not by equation 1, (c) the sum of equations 1 and 3, neither equation 1 nor equation 3 nor the sum of the two.

There are actually two kinds of electromagnetic induction: motional and transformer induction. The electromotive force generated by motion through a constant magnetic field is a pure case of motional induction. The electromotive force generated in a stationary circuit by the variation of the magnetic field with time is a pure case of transformer induction. In general, both types of induction are involved in a given case. One source of confusion is that there is *not* a one to one correspondence between the two types of induction and the two common induction laws.

The flux linking law, $\mathcal{E} = -d\phi/dt$, and the flux cutting law, $\mathcal{E} = Bvl$, often erroneously are considered as merely different ways of expressing the same phenomena. This article attempts to dispel the confusion surrounding the subject of electromagnetic induction.

The flux-cutting law gives exactly the electromotive force caused by motional induction. The flux-linking law sometimes includes both types of induction and sometimes does not. Hence, if the flux-cutting law is used alone, the transformer induction will be lacking. If the flux-linking law is used alone, part or all of the motional induction will be lacking. If both laws are used together, part or all of the motional induction may be counted twice, and sometimes the flux-linking law calls for an electromotive force when there is none at all. The rule for foolproof application of the flux-linking law, which does not seem to have been enunciated previously, will be given later. The case in which the flux-linking law calls for an induced electromotive force which does not exist have been classified by one means or another as a substitution of circuit. It has been stated *ad hoc*, as a fundamental principle, that "no voltage can be induced by a substitution of circuit."[1] Actually, this will appear as an obvious deduction from the fundamentals discussed in this article.

Another source of confusion, rather less well recognized, can be traced back to the choice of "field theory" underlying the analysis of the problem. When the source of the magnetic field is moving, the question arises as to what the motion of the magnetic field itself is. Does the magnetic field move as if rigidly attached to its source, does it remain stationary while the source moves through its own field, does it behave with a motion in between these two, or is there some entirely different alternative? Each different answer is the basis of a different field theory. The two most used theories for the solution of ordinary engineering problems are the moving field theory, which assumes that the field moves as if rigidly attached to its source, and the stationary field theory, which assumes that the field remains fixed while the source moves through its own field. In addition, there is, of course, relativity theory in which the field is not an entity which can be characterized by the property of motion. The purpose of this article is to clarify the phenomena of electromagnetic induction and not to discuss the scope and limitations of the various field theories; all the examples treated here fall well within the scope of the moving and stationary field theories.

The total induced electromotive force is a measurable quantity and, as such, any correct theory must lead to the same value. The total electromotive force is the sum of electromotive forces caused by both motional and transformer induction. The defining concepts and formulas for transformer and motional induction are the same for both

types of field theories. However, calculated by different field theories, in general, they will have different values. (An analogy about distance between two fixed points in two different co-ordinate systems may be helpful. While the individual co-ordinate differences are different in the two co-ordinate systems, the distance itself is variant.) Hence, for a clear understanding of motional and transformer induction, the underlying difference between field theories must be understood. For successful solution of a problem, it is essential not to change field theories used in the middle of the problem. In some circles it is believed that presence of transformer or motional induction depends merely on the choice of reference axis.[1] That this is erroneous will become obvious from the examples treated. The change, if any, is actually caused by a change of theory and not a change of axis.

PARADOXES

Unipolar Generator. The Faraday disk is one of the oldest and best known of the experiments which sharply differentiate between the flux-cutting and flux-linking laws. This form of unipolar generator consists of a circular conducting disk rotating in a constant uniform magnetic field perpendicular to it, as shown in Figure 1. A voltmeter and leads complete the circuit through brush A on the conducting axle and brush B on the rim. Calculating the induced electromotive force by means of the flux-cutting law, equation 3:

$$\mathcal{E} = \omega B \int_0^R r dr \tag{5}$$

$$\mathcal{E} = (1/2)\omega B R^2 \tag{6}$$

Using the flux-linking law, equation 1, however, leads to

$$\mathcal{E} = 0 \tag{7}$$

because there is no flux-linking the circuit. Even if the generator were tilted so that the flux linked the circuit, the time derivative is zero and hence there is still no induced
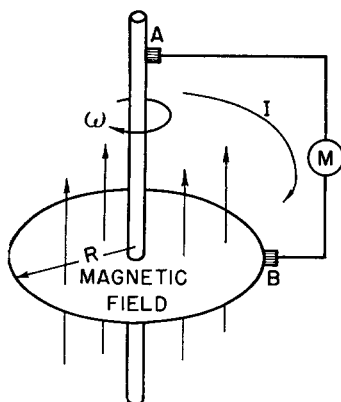

Figure 1. Faraday's disk

electromotive force. Which is correct? Experiment verifies equation 6, hence this example is a pure case of motional induction. It is ironical that the induction law attributed to Faraday does not hold for his own device.

D-C Generator.[2] Consider the scheme shown in Figure 2. The conductor in the magnet air gap swings back and forth like a trapeze bar about the horizontal leads which are connected to the meter. Its velocity is given by $v = v_m$

cos ωt. The amplitude of swing is taken as small, so that the error made by considering the velocity to be horizontal is negligible. An alternating current is supplied to the exciting coil so that the magnetic field in the air gap is $B = B_m \cos \omega t$ and is uniform over the path of swing. The field in the gap is positive when it is directed upward. The velocity of the bar is positive when swinging from left to right. When the bar is swinging to the right, the magnetic field is in the upward direction. When the bar reaches the right extremity of its swing, the magnetic field has decreased to zero. The direction of the field reverses at the same time as the velocity of the bar. Therefore, on the back-swing the bar travels through a magnetic field directed downward; as a result, the electromotive force induced in the wire is alway in the same direction, thereby giving a pulsating direct voltage.

The motionally induced electromotive force, from the flux-cutting law, equation 4, is

$$\mathcal{E} = B_m v_m l \cos^2 \omega t \tag{8}$$

or

$$\mathcal{E} = 1/2 B_m v_m l + 1/2 B_m v_m l \cos 2 \omega t \tag{9}$$

Hence this d-c generator has a constant voltage component of $1/2 B_m v_m l$ and a double-frequency a-c component of $1/2 B_m v_m l \cos 2 \omega t$. It is interesting to notice that there is an alternating electromotive force over and above the constant electromotive force generated by motional induction.
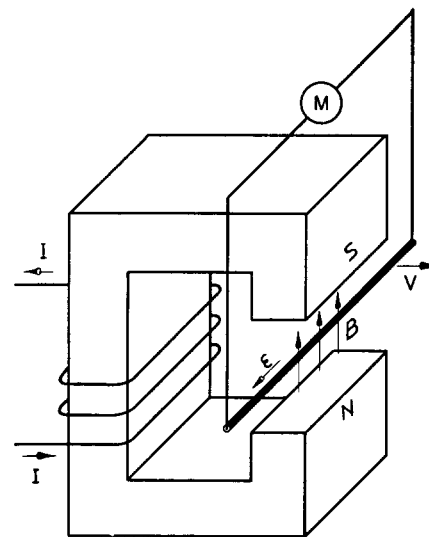

Figure 2. D-c generator

Let us calculate the induced electromotive force by the flux-linking law

$$\phi = B \cdot A \tag{10}$$

$$\phi = (B_m \cos \omega t)(lr)\cos \theta \tag{11}$$

$$\phi = B_m lr \cos \omega t \sin \omega t \tag{12}$$

$$\mathcal{E} = B_m lr\omega (\cos^2 \omega t - \sin^2 \omega t) \tag{13}$$

$$\mathcal{E} = B_m v_m l \cos 2 \omega t \tag{14}$$

This is not in agreement with results of the flux-cutting law, equation 9. Experiment verifies that the flux-linking law gives the correct result.

It is interesting to note that Maxwell's equation for transformer induction

$$\mathcal{E} = -\int\int \frac{\partial \boldsymbol{B}}{\partial t} \cdot d\boldsymbol{S} \tag{15}$$

gives

$$\mathcal{E} = -1/2 B_m v_m l + 1/2 B_m v_m l \cos 2 \omega t \tag{16}$$

as the electromotive force generated by transformer induction. There is a constant component of induced electromotive force caused by transformer action. If this electromotive force is omitted, this device appears to be a d-c generator without moving contacts. The same, of course, would be true if only the transformer induction were considered. If the electromotive force caused by both types of induction are added together, however, the d-c components cancel and the a-c components add to give the result obtained from the flux-linking law.

Commutating Magnet. Consider a magnet with a loop around one leg, as shown in Figure 3. In the loop is a voltmeter *M*. The upper end of the loop is completed by means of a pair of spring clips which are capable of acting as a pair of sliding contacts. The magnet is moved upward, separating the spring contacts and completing the circuit through the magnet. The magnet continues to move upward until the magnet and its flux are separated from the loop.

Before calculating the induced electromotive force, if any, we must decide whether to use the moving field theory or the stationary field theory, since the source of the field, the magnet, is in motion. For the moment let us avoid this decision by considering the equivalent problems of holding the magnet stationary and moving the meter, leads, and spring clip instead. The flux linking the circuit is

$$\phi = Blx \tag{17}$$

The flux-linking law, equation 1, gives

$$\mathcal{E} = -Bl \frac{dx}{dt} \tag{18}$$

or

$$\mathcal{E} = Blv \tag{19}$$

The flux-cutting law, equation 4, gives

$$\mathcal{E} = 0 \tag{20}$$

because the portion of the circuit in the magnetic field is not moving and the moving portion is not in the magnetic field. Since the magnetic field does not vary with time, there is no transformer induction; this is pure motional induction, which, in this case, is zero. Although the flux-linking law states that there is an induced electromotive force, experiment verifies that there is none.[3] The breakdown of the flux-linking law in this case has been excused and dismissed by the statement "no voltage can be induced by a substitution of circuit."[1] The exact cause of this failure will be explained later.

Let us now return to the original problem with the magnet moving as shown in Figure 3. First, we will analyze the problems using the moving field theory and, second, with the

stationary field theory. The moving field theory calls for the field to move as if rigidly attached to the magnet. Therefore, the portion of the circuit in the magnetic field, the magnet leg itself, has no velocity through the magnetic field and, hence, there is no motional induction, equation 20. The flux-linking law again gives the erroneous result of equation 19. Analyzing the problems with the stationary field theory is more complex. Here the field is assumed to be stationary, so that the magnet leg actually moves
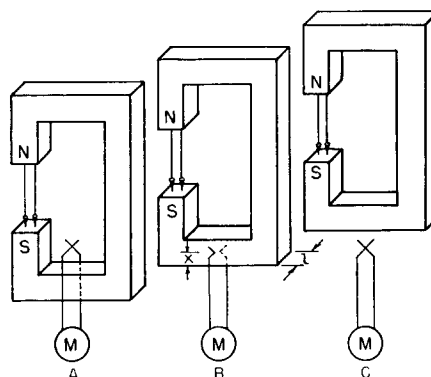

Figure 3. Commutating magnet

through its own magnetic field. A motionally induced electromotive force, given by equation 4, is produced and has the value

$$\mathcal{E} = Bvl \tag{21}$$

The magnetic field is building up from zero to full strength in the leading edge of the magnet leg and collapsing from full strength to zero in the trailing edge of the leg. Since the trailing edge is inside the circuit, the time variation of magnetic field at that location causes a transformer induced electromotive force in the circuit and as given by equation 15 is

$$\mathcal{E} = -Bvl \tag{22}$$

The total induced electromotive force is zero since the transformer component exactly cancels the motional component. Here again the flux-linking law, which is insensitive to choice of theory, gives the erroneous result given by equation 19.

Linear Motion Inductor.[4] A cylindrical metal tube moving axially along a straight conductor, carrying a steady current, is shown in Figure 4. On the inner and outer surfaces are two fixed brushes making contact with the cylinder in order to complete the voltmeter circuit through the cylinder. Will the induced electromotive force depend on the permeability of the cylinder?

First calculate the induced electromotive force with the flux-linking law. Let $\boldsymbol{B}_o$ be the magnetic field in free space, $\boldsymbol{B}$ the field in the cylinder, and $\boldsymbol{B}_i$ the component of the field contributed by the atoms composing the cylinder. Then $\boldsymbol{B} = \boldsymbol{B}_o + \boldsymbol{B}_i = \mu \boldsymbol{B}_o$. The flux linking the circuit is

$$\phi = \int\int \boldsymbol{B} \cdot d\boldsymbol{S} \tag{23}$$

$$\phi = x \int_{r_1}^{r_2} B \, dr + (l-x) \int_{r_1}^{r_2} B_o \, dr + C \tag{24}$$

$$\phi = x \int_{r_1}^{r_2} \frac{\mu I}{2\pi r} \, dr + (l-x) \int_{r_1}^{r_2} \frac{I}{2\pi r} \, dr + C \tag{25}$$

$$\phi = \frac{I}{2\pi} \ln \frac{r_2}{r_1} (\mu x - x + l) + C \qquad (26)$$

The flux-linking law then gives

$$\varepsilon = -\frac{I}{2\pi} (\mu - 1) v \ln \frac{r_2}{r_1} \qquad (27)$$

Next calculate the induced electromotive force by the flux-cutting law, equation 3. According to the moving field theory, the field $\boldsymbol{B}_i$ moves with the cylinder and the field $\boldsymbol{B} - \boldsymbol{B}_i = \boldsymbol{B}_o$ does not. Hence the moving elements of
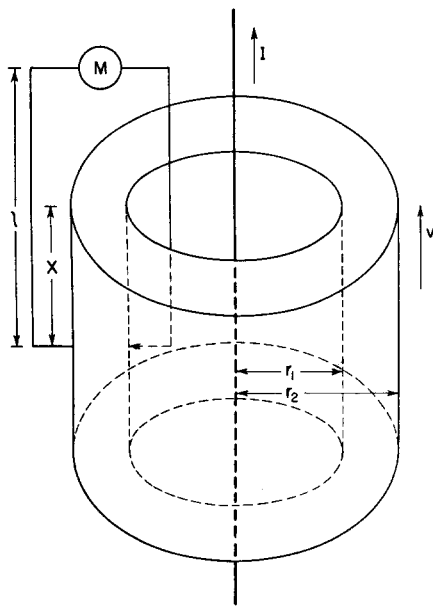

Figure 4. Linear motion inductor

the circuit only move through the field $\boldsymbol{B}_o$ and the motionally induced voltage is, by equation 3

$$\varepsilon = v \int_{r_1}^{r_2} B_o dr \qquad (28)$$

$$\varepsilon = \frac{I}{2\pi} v \ln \frac{r_2}{r_1} \qquad (29)$$

Since there is no time variation of magnetic field there is no transformer induction by equation 15. If the stationary field theory is used to analyze this example, the transformer induction due to the building up of the field from $\boldsymbol{B}_o$ to $\boldsymbol{B}$ in the leading edge of the cylinder is given by equation 15 as

$$\varepsilon = \frac{I}{2\pi} (1 - \mu) v \ln \frac{r_2}{r_1} \qquad (30)$$

The motional induction is given by equation 3 as

$$\varepsilon = \frac{\mu I}{2\pi} v \ln \frac{r_2}{r_1} \qquad (31)$$

Addition of equation 30 and 31 gives the total electromotive force based on the stationary field calculations and is exactly the same as equation 29, the total electromotive force as calculated by the moving field theory. Experiment verifies equation 29, and reveals another failure of the flux-linking law.

ELECTROMAGNETIC INDUCTION THEORY

Forces. To calculate the behavior of an electrodynamic system, the forces exerted on charges must be known.

Charges experience certain forces which are related to their magnitudes

together with their

(*a*). Positions.
(*b*). Velocities.
(*c*). Accelerations.

The differential formulas for these forces are, respectively,

$$d^2\boldsymbol{F}_1 = \frac{dq_1 \, dq_2 \, \boldsymbol{r}}{4\pi\epsilon r^3} \qquad (32a)$$

$$d^2\boldsymbol{F}_1 = \frac{\mu dq_1 dq_2 \boldsymbol{v}_1 \times (\boldsymbol{v}_2 \times \boldsymbol{r})}{4\pi r^3} \qquad (32b)$$

$$d^2\boldsymbol{F}_1 = -\frac{\mu dq_1 dq_2 \boldsymbol{a}_2}{4\pi r} \qquad (32c)$$

where $d^2\boldsymbol{F}_1$ is the differential force exerted on charge dq_1 by charge dq_2, $\boldsymbol{v}_1$ is the velocity of dq_1, $\boldsymbol{v}_2$ is the velocity of dq_2, $\boldsymbol{a}_2$ is the acceleration of dq_2, and μ is the permeability. When the discrete charges are sufficiently concentrated, the relation

$$I d\boldsymbol{l} = \boldsymbol{v} dq \qquad (33)$$

allows equations 32b and 32c to be expressed in the normal form involving current elements

$$d^2\boldsymbol{F}_1 = \frac{\mu I_1 I_2 d\boldsymbol{l}_1 \times (d\boldsymbol{l}_2 \times \cdot)}{4\pi r^3} \qquad (34b)$$

$$d^2\boldsymbol{F}_1 = -\frac{\mu dq_1 d\boldsymbol{l}_2}{4\pi r} \frac{\partial I_2}{\partial t} \qquad (34c)$$

Charges also experience forces due to radiation, thermal, chemical, and contact phenomena. As these are outside the scope of the subject of this article they will not be discussed here.

Fields. Fields may be considered as mathematical constructs. Their purpose is to simplify the determination of the forces experienced by charged particles. For each type of force there is a type of field suitable for its specification. These are, respectively,

(*a*). Electrostatic field.
(*b*). Magnetic field.
(*c*). Vector-potential field.

The differential formulas for these fields are, respectively

$$d\boldsymbol{E} = \frac{dq_2 \boldsymbol{r}}{4\pi\epsilon r^3} \qquad (35a)$$

$$d\boldsymbol{B} = \frac{\mu dq_2 \boldsymbol{v}_2 \times \boldsymbol{r}}{4\pi r^3} \qquad (35b)$$

$$d\boldsymbol{A} = \frac{\mu dq_2 \boldsymbol{v}_2}{4\pi r} \qquad (35c)$$

or in terms of current elements

$$d\boldsymbol{B} = \frac{\mu I_2 d\boldsymbol{l}_2 \times \boldsymbol{r}}{4\pi r^3} \qquad (36b)$$

$$d\boldsymbol{A} = \frac{\mu I_2 d\boldsymbol{l}_2}{4\pi r} \qquad (36c)$$

By expressing the force formulas of the preceding section in terms of the foregoing fields (and integrating once), consider-

able simplification of formulas is obtained. The force formulas are then, respectively

$$dF_1 = dq_1 E \tag{37a}$$

$$dF_1 = dq_1 v_1 \times B \tag{37b}$$

$$dF_1 = -dq_1 \frac{\partial A}{\partial t} \tag{37c}$$

The "field concept" allows the "divide and conquer" technique to be applied to the problem of finding the forces.

Electromagnetic Induction. When the forces which are exerted on charged particles by magnetic and vector-potential fields are explained in terms of an (hypothetical) electric field, this electric field is said to be generated by electromagnetic induction. In other words, if a charged particle experiences a force, we may consider that this force is caused by an electric field. If the force is actually caused by a magnetic or vector-potential field, we may consider this equivalent electric field as being generated by electromagnetic induction. This definition reduces all the force formulas to the form

$$dF = dqE \tag{38}$$

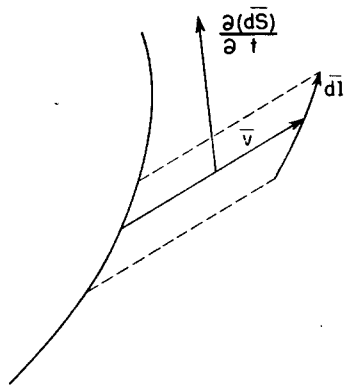where the magnetic field is replaced by an equivalent electric field given by

$$E_M = v \times B \tag{39}$$

and the vector-potential field is replaced by an equivalent electric field given by

$$E_T = -\frac{\partial A}{\partial t} \tag{40}$$

The reduction of all fields to equivalent electric fields

Figure 5. Differential area swept out by motion of circuit element

enables the force to be calculated from one resultant field. The total electric field is then given by

$$E = E_s + E_M + E_T \tag{41}$$

Substituting equations 39 and 40 into equation 41 gives

$$E = E_s + v \times B - \frac{\partial A}{\partial t} \tag{42}$$

(This formula is actually the Lorentz force formula for a unit charge, which physicists usually write as $F = E + v \times B$. The E of Lorentz's formula is the $E_s - \frac{\partial A}{\partial t}$ of the equation and is written out for the purpose of emphasizing the conceptual basis.)

E_s is the electric field given by

$$E_s = -\nabla \psi \tag{43}$$

Where ψ is the scalar potential, E_M is the electric field generated by motional induction, and E_T is the electric field generated by transformer induction.

Transformer Induction. In a case of pure transformer induction there is no motion of material bodies. The formulation of this type of induction is given by

$$E_T = -\frac{\partial A}{\partial t} \tag{40}$$

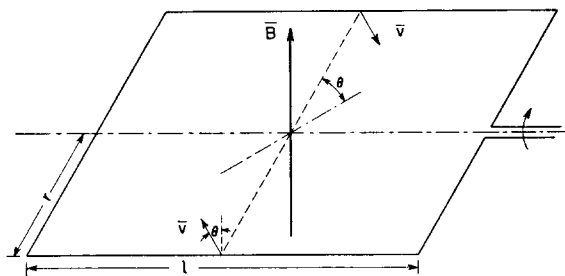Transformer induction may be formulated in terms of the

Figure 6. Rotating loop in a constant uniform magnetic field

magnetic field since the vector-potential and magnetic fields are intimately related as follows

$$B = \nabla \times A \tag{44}$$

Taking the curl of both sides of equation 40, permuting the ∇x with $\frac{\partial}{\partial t}$, and substituting equation 44 gives

$$\nabla \times E_T = -\frac{\partial B}{\partial t} \tag{45}$$

which is simply Maxwell's differential formulation of Faraday's induction law. Integrating both sides of equation 45 over the surface bounded by a circuit and transforming the left hand side by Stoke's theorem gives

$$\oint E_T \cdot dl = -\int \int \frac{\partial B}{\partial t} \cdot dS, \tag{46}$$

which is Maxwell's integral formulation of Faraday's induction law. The electromotive force generated by transformer induction is the line integral of E_T

$$\varepsilon_T = \int E_T \cdot dl \tag{47}$$

In terms of the vector potential it is

$$\varepsilon_T = -\int \frac{\partial A}{\partial t} \cdot dI \tag{48}$$

In terms of the magnetic field it is

$$\varepsilon_T = -\int \int \frac{\partial B}{\partial t} \cdot dS \tag{15}$$

Note that the formulation in terms of the magnetic field can give only the total induced electromotive force in circuits which are essentially closed loops. The vector-poten-

tial formulation is capable of giving the voltage induced in any individual element of a circuit.

Motional Induction. In a case of pure motional induction there is no variation of magnetic field with time. The formulation of this type of induction is given by

$$E_M = v \times B \tag{39}$$

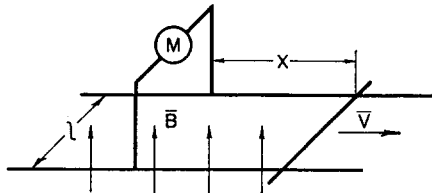This law was derived by Lorentz from the force formula of

Figure 7. Example of motional induction covered by the combined formula

Biot and Savart.

The electromotive force generated by motional induction is the line integral of E_M

$$\mathcal{E}_M = \int E_M \cdot dl \tag{49}$$

$$\mathcal{E}_M = \int v \times B \cdot dl \tag{50}$$

A Combined Induction Law. In the general problem, both types of induction, transformer and motional, occur. It is therefore convenient to have both types combined in one formula.

The total electric field intensity due to both types of induction is the sum of equations 39 and 40

$$E = v \times B - \frac{\partial A}{\partial t} \tag{51}$$

The line integral of the induced electric field intensity along a circuit gives the total electromotive force induced in the circuit, which is

$$\mathcal{E} = \int_a^b v \times B \cdot dl - \int_a^b \frac{\partial A}{\partial t} \cdot dl \tag{52}$$

This is the general formula for the induced voltage in a circuit. It is when this general form is put into special forms that paradoxes arise. These are caused by ignorance or violation of the limiting restrictions imposed by the transformation into the special form. If the second integral in equation 52 is put in terms of the magnetic field, the equation is immediately restricted to closed loop circuits and becomes

$$\mathcal{E} = \oint v \times B \cdot dl - \int \int \frac{\partial B}{\partial t} \cdot dS \tag{53}$$

Although we have one formula here, it is essentially no simpler than using the individual formulas for $\mathcal{E}_M$ and $\mathcal{E}_T$. To obtain a simpler expression, both terms must be combined under one integral sign.

Transforming the first term to an integral over the same surface as the second term by means of Stoke's theorem gives

$$\mathcal{E} = -\int \int \left[\frac{\partial B}{\partial t} - \nabla \times (v \times B) \right] \cdot dS \tag{54}$$

The right-hand side of the following vector integral theorem for the time rate of change of the flux through a moving area

$$\frac{d}{dt} \int \int B \cdot ds = \int \int \left[\frac{\partial B}{\partial t} + v \nabla \cdot B - \nabla \times (v \times B) \right] \cdot dS \tag{55}$$

reduces (except for the minus sign) to the right-hand side of equation 54 since

$$\nabla \cdot B = 0 \tag{56}$$

Substituting equation 55 into equation 54 gives

$$\mathcal{E} = -\frac{d}{dt} \int \int B \cdot dS \tag{57}$$

which is F. E. Neumann's law of induction.

The magnetic flux linking the circuit is given by the following equation

$$\phi = \int \int B \cdot dS \tag{58}$$

Substituting this into Neumann's formula gives

$$\mathcal{E} = -\frac{d\phi}{dt} \tag{1}$$

Strangely enough, this is the form in which Faraday's flux-linking law is normally presented. It is of significance to note that Maxwell's formulation of Faraday's law, equation 15, does not include motional induction.

In order to obtain the limitations on this combined formula, equation 1, let us examine in detail the meaning of the first integral in equation 52.

Rearranging the integral gives

$$\mathcal{E}_M = \int_a^b B \cdot v \times dl \tag{59}$$

As shown in Figure 5, $v \times dl$ is the differential area swept out per unit time by an element of the circuit. This transformation states that all motion resulting in the generation
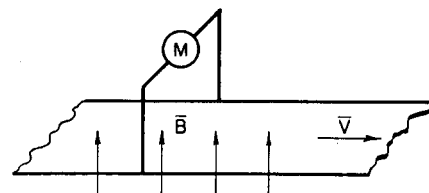
Figure 8. Example of motional induction not covered by the combined formula

of electromotive force changes the path of integration. The path of integration must be taken as moving with the material substance composing it, otherwise the electric fields experienced by the charged particles of the material will not be the same as the electric fields taken in the integration. All fields generated in reference frames other than the reference frame of the material may just as well be fictional, because there is nothing to observe their effect. Only the electric fields experienced by the charged particles produce an observable effect, and in order to calculate *these* fields, the

motion of the paths of integration must be identical with the motion of the material comprising this path at the instant considered.

This point must be kept in mind because it is the basis of the misapplication of the flux-linking (or combined) induction law. This combined induction law is applicable to all closed circuits of constant or changing shape, moving in any way through a constant or changing magnetic field, with sliding contacts or without, provided that the motion of every element of the paths of integration is identical with the motion of the matter comprising that path at the instant considered. To illustrate this limitation, consider Faraday's disk, shown in Figure 1: The radius of the disk from the center to the brush on the rim is the path of integration and it does not move with the material comprising the path. The flux-linking law, therefore, is not applicable to this case.

Examples Fostering Confusion. In the majority of elementary text books all induction is implied to be merely one type of phenomena, even though this is not explicitly stated. The two formulas

$$\varepsilon = -\frac{d\phi}{dt} \tag{1}$$

$$\varepsilon = Bvl \sin \theta \tag{60}$$

are presented as two different ways of expressing the same thing. This implication is usually substantiated by deriving one from the other and by solving problems which give the same result from both formulas.

One example usually given is the rotating coil in a constant uniform magnetic field shown in Figure 6.

Equation 60 gives directly

$$\varepsilon = 2Bvl \sin \theta \tag{61}$$

Equation 1 gives

$$\varepsilon = -\frac{d}{dt}[2lrB \cos \theta] \tag{62}$$

$$\varepsilon = 2 Br\theta l \sin \theta \tag{63}$$

$$\varepsilon = 2Bvl \sin \theta \tag{64}$$

which is identical to equation 61.

Another example usually given is the wire moving through a constant uniform magnetic field with its ends in sliding contact with two long parallel wires as shown in Figure 7.

Equation 60 gives directly

$$\varepsilon = - Bvl \tag{65}$$

Equation 1 gives

$$\varepsilon = -\frac{d}{dt}(Blx) \tag{66}$$

$$\varepsilon = - Blv \tag{67}$$

which is identical to equation 65.

It is unfortunate that the equivalence of these two formulas is stressed by the application of the foregoing technique because it leads to the formation of two schools of thought (both wrong!). One group believes all induction

phenomena can be handled with the flux-cutting law, while another believes that the flux-linking law is all-inclusive. Adherents of these two schools go to extremes in the invention of *ad hoc* explanations to bring all induction phenomena within the scope of their equation.[5]

Examples Avoiding Confusion. It would be a good principle if problems bringing out the difference between the two for-
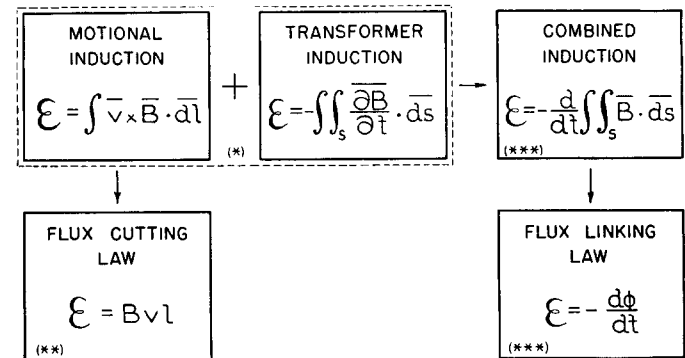

Figure 9. Relation between the induction laws

* *Completely general when combined*

** *Flux must be uniform, and B, v, and l must be mutually orthogonal*

*** *Motion of material and path of integration must be identical*

mulas were given instead of special problems emphasizing similarity. If in the foregoing two problems, a time varying magnetic field were given instead of a constant one, the flux-cutting law no longer would give the correct answer but the flux-linking law would. With constant magnetic fields both examples are pure cases of motional induction but with variable fields they have both motional and transformer induction. The second problem, if altered slightly by substituting a long conducting strip moving with a velocity v in place of the long parallel wires and cross wire as shown in Figure 8, provides a simple example in which the flux-linking law is no longer applicable and in which the correct answer is provided by the flux-cutting law.

SUMMARY

The relationship between the various induction laws is summarized in Figure 9. Only the combined use of both motional and transformer induction (both inside the large dotted rectangle) will guarantee validity of results in all cases of induction. When using any of the restricted laws external to the dotted rectangle, care must be used to avoid violation of restrictions incident to such law (as noted in parenthesis).

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