$$a_{\ell m} = -\frac{1}{4\pi}\sqrt{\frac{\mu}{\epsilon}}\frac{(-1)^m}{\ell(\ell+1)}\int_{V_0}\mathbf{J}(\mathbf{r}')\cdot\nabla'\times\nabla'$$

$$\times(\mathbf{r}' j_\ell(kr)Y_\ell^{-m})dV',$$

$$b_{\ell m} = \frac{\imath k}{4\pi}\frac{(-1)^m}{\ell(\ell+1)}\int_{V_0}\mathbf{J}(\mathbf{r}')\cdot\nabla'\times(\mathbf{r}' j_\ell(kr)Y_\ell^{-m})dV'.$$

For arbitrary current distributions one can show[4] from these expressions that an isotropic radiation pattern is impossible. However in the long wavelength limit, $ka \ll 1$, for atomic radiators where the multipole coefficients (properly calculated using quantum mechanics) are independent of m,[5] the radiation is isotropic. Once again, quantum mechanics breaks the ''classical'' rules, a situation not unlike the violation of the classical Bohr–van Leeuven theorem of diamagnetism.

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Reply to ''Comments on 'The problem of spherically symmetric electromagnetic radiation,' '' by E. Comay [Am. J. Phys. 70 (7), 715–716 (2002)]

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The comments[1,2] by Rosenthal and Choy give much insight into the nature of the restrictions imposed on vector fields. They correctly show that a transverse vector field cannot have a uniform size over an entire spherical shell. [The claim holds also for a nonvanishing vector field, because such a field can be cast into a unit vector field by means of a multiplication factor $f(\theta,\varphi)$.] Thus, the use of Maxwell's equations looks redundant.

However, we may use Maxwell's equations for the transverse electromagnetic fields and take an additional step. The proof given in Ref. 3 holds for the spherical region R defined at the equator between latitudes $\pm d$. The proof requires that the electric field at a point P on the equator is tangent to it. This requirement can be satisfied by applying a duality transformation to the fields,[4] $\mathbf{E}' = \mathbf{E}\cos\alpha + \mathbf{B}\sin\alpha$ and $\mathbf{B}' = -\mathbf{E}\sin\alpha + \mathbf{B}\cos\alpha$, and fixing the value of α.

If a path C departs from the equator, then the proof in Ref. 3 holds [see case (1), a few lines after Eq. (2) therein]. If, on the other hand, the path C coincides with the equator, then one makes a closed path using the following sections: a section S of a longitude between angles $0 \leq \theta \leq d'' < d$, and a full circle C'' on the latitude d''. The integral on the closed path $C-S-C''-S$ can be used for completing the proof.

Thus, electromagnetic fields cannot take a uniform magnitude in the spherical region R defined above. This property is not satisfied by a general transverse vector field, as shown by the following example: $\mathbf{v}(\theta,\varphi) = \mathbf{u}_\varphi$ $(-d \leq \theta \leq d, 0 \leq \varphi < 2\pi)$, where $\mathbf{u}_\varphi$ is the ordinary unit vector in spherical coordinates.

As correctly pointed out by Choy,[2] the discussion in Ref. 3 is restricted to classical physics. Indeed, the probability function of a photon emitted from an atom whose state is m-fold degenerate is uniform on a sphere.

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