From (6) we get

$$\frac{\mathrm{d}T_\alpha}{\mathrm{d}t} = -\frac{2m_\alpha n_\beta \mu_{\alpha\beta}}{m_\alpha + m_\beta}(T_\alpha - T_\beta) \tag{8}$$

which coincides with Spitzer's formula [4]. Next we calculate the mean electron velocity $\boldsymbol{u}_\mathrm{e}$ under the effect of an electric field $\boldsymbol{E} = \boldsymbol{E}_0 \exp(-\mathrm{i}\omega t)$. Again using eq. (6) we have

$$\frac{\mathrm{d}\boldsymbol{u}_\mathrm{e}}{\mathrm{d}t} = \frac{e\boldsymbol{E}}{m_\mathrm{e}} - n_\mathrm{i}\mu_\mathrm{ei}\eta_\mathrm{ei}(\boldsymbol{u}_\mathrm{e} - \boldsymbol{u}_\mathrm{i})$$

Putting $\eta_\mathrm{ei} \approx 1$ and $\boldsymbol{u}_\mathrm{e} \gg \boldsymbol{u}_\mathrm{i}$ we get

$$\mathrm{e} = \frac{e}{m_\mathrm{e}}\frac{\boldsymbol{E}}{n_\mathrm{i}\mu_\mathrm{ei} - \mathrm{i}\omega}, \qquad \mu_\mathrm{ei} \approx \tfrac{4}{3}\sqrt{2\pi}\frac{e^4 \ln(T_\mathrm{e}/e^2K_\mathrm{D})}{m_\mathrm{e}^{\frac{1}{2}} T_\mathrm{e}^{\frac{3}{2}}} \tag{9}$$

Comparing (9) with the corresponding expression obtained using the correct form (1) of $S_{\alpha\beta}$ [5] we conclude that $n_\mathrm{i}\mu_\mathrm{ei}$ is the collision frequency for momentum exchange for a process varying with a frequency $\omega \gg n_\mathrm{i}\mu_\mathrm{ei}$. This leads to the conclusion that eq. (6) with $\mu_{\alpha\beta}$ from (4) could be applied to small amplitude oscillations with frequency greater than $n_\beta \mu_{\alpha\beta}$.

References

1. P.L.Bhatnagar, E.P.Gross and M.Krook, Phys. Rev. 94 (1954) 511.
2. J.P.Dougherty, Phys. Fluids 7 (1964) 1788.
3. A.Oppenheim, Phys. Fluids 8 (1965) 900.
4. L.Spitzer Jr., Physics of fully ionized gases, (Interscience Publishers, Inc. New York, 1956).
5. V.L.Ginzburg, Propagation of electromagnetic waves in plasma, (Gordon and Breach Science Publishers, Inc. New York, 1961).

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A NEW LAW IN ELECTRODYNAMICS

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As a consequence of the "Laplace" force $R\boldsymbol{E}\times\boldsymbol{\beta}$ applied to a moving magnetic charge R, a varying dipole of moment $\boldsymbol{M}$ undergoes a force $\boldsymbol{E}\times \mathrm{d}\boldsymbol{M}/\mathrm{d}t$, and a slowly varying current of intensity i a force $(\mathrm{d}i/\mathrm{d}t)\oint V\delta \boldsymbol{l}$, where V denotes the scalar potential.

Let us recall first that a straightforward argument of relativistic covariance shows that a magnetic point charge R (e.m.u.), which by definition undergoes, when at rest, the Coulomb force $R\boldsymbol{H}$ must, when in motion with velocity $c\boldsymbol{\beta}$, undergo the "Lorentz" force

$$\boldsymbol{F} = R(\boldsymbol{E}\times\boldsymbol{\beta} + \boldsymbol{H}). \tag{1}$$

It follows then that a magnetic dipole made of two poles $+R$ and $-R$ with separation $\boldsymbol{a}$ and moment $\boldsymbol{M} \equiv R\boldsymbol{a}$, situated at a given point in a static electric field $\boldsymbol{E}$, undergoes when $\boldsymbol{M}$ changes a force

$$\boldsymbol{F} = \boldsymbol{E}\times \mathrm{d}\boldsymbol{M}/\mathrm{d}t. \tag{2}$$

If we assume for simplicity that the source of $\boldsymbol{E}$ is a single point charge Q with separation $\boldsymbol{r}$ from the dipole's center, then one can rewrite eq. (2) in the following form

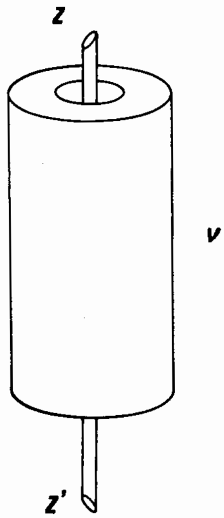

Fig. 1.

$$\boldsymbol{F} = Qr^{-3}\boldsymbol{r} \times \mathrm{d}\boldsymbol{M}/\mathrm{d}t. \tag{3}$$

But, as

$$\boldsymbol{A} = r^{-3}\boldsymbol{r} \times \boldsymbol{M} \tag{4}$$

is the vector potential of the dipole, then $-\boldsymbol{F}$ ($\boldsymbol{F}$ having the expression (3)) is the force imparted to the electric charge Q by the induced electric field when $\boldsymbol{M}$ is slowly varying (questions of relativistic covariance and wave retardation are neglected here). It is thus clear that the preceding formulas satisfy the principle of conservation of linear momentum.

Now we consider a magnetic shell of strength i and surface element ds. Its contour being at rest in a static electric field

$$\boldsymbol{E} \equiv \mathrm{grad}\, V, \tag{5}$$

it undergoes, when i varies, a force

$$\boldsymbol{F} = (\mathrm{d}i/\mathrm{d}t) \iint \boldsymbol{E} \times \delta \boldsymbol{s} \equiv (\mathrm{d}i/\mathrm{d}t) \oint V \delta \boldsymbol{l}. \tag{6}$$

Assuming the usual physical equivalence between a magnetic shell and a closed current, the latter expression for $\boldsymbol{F}$ (which is potential dependent) applies also to a closed current. An alternative proof of formula (6) uses the principle of equality of action and reaction between a point charge and a closed current with slowly varying intensity.

The usefulness of the concepts of a magnetic dipole and a magnetic shell should incidentally be noted. It would be possible, of course, but more difficult, to derive formulae (2) and (6) through an argument of relativistic covariance using only the concept of an Amperian current.

Also, one may legitimately inquire through which mechanism the closed current is driven. An alternative version of the magnetic shell is a network with the intensity i running round each mesh. In such a case each mesh undergoes the force $\delta \boldsymbol{F} = (\mathrm{d}i/\mathrm{d}t)\, \boldsymbol{E} \times \delta \boldsymbol{s}$, the resultant of which is eq. (6).

Finally we show that from the law (6) we can deduce reasonable consequences. Consider a toroidal magnet or solenoid V of axis z interacting with a static electric charge uniformly distributed, with a linear density q, along z; it can be shown [1] that, when the magnetic flux Φ through a meridian section of V is changed, the charged line z receives, in the z direction, a force $-\boldsymbol{F}$ of value

$$-F = -q\,\mathrm{d}\Phi/\mathrm{d}t, \tag{7}$$

while the reaction $+F$ is applied to V.

The origin of the reaction force $\boldsymbol{F}$ is clear in the preceding context. If $E = 2q/r$ denotes the (radial) field of the charged line and $B = \delta\Phi/\delta s$ the (tangential) magnetic induction inside V, then (7) can be rewritten as

$$\boldsymbol{F} = \frac{1}{4\pi} \iiint_V \boldsymbol{E} \times \frac{\mathrm{d}\boldsymbol{B}}{\mathrm{d}t}\, \delta v, \tag{8}$$

which is the volume form of eq. (2). In the magnet case,

$$\boldsymbol{k} \equiv \partial \boldsymbol{B}/4\pi\, c\, \delta t \tag{9}$$

is the magnetic polarization current, so (8) assumes the "Laplace" form

$$\boldsymbol{F} = \iiint_V \boldsymbol{E} \times \boldsymbol{j}\, \delta v \tag{10}$$

(compare with eq. (1)).

In the solenoid case, according to Ampère's theorem and I denoting the total intensity, $B = I/2\pi r$, so that eq. (7) is rewritten as

$$\boldsymbol{F} = \frac{\mathrm{d}I}{\mathrm{d}t} \iint \boldsymbol{E} \times \delta \boldsymbol{s} = \frac{\mathrm{d}I}{\mathrm{d}t} \oint V \delta \boldsymbol{l} \tag{11}$$

as in (6).

This is a new version of the theory [1] of the only conceivable artifact in Goillot's [2] experiments; it is found to be numerically smaller than the measured effect.

References

1. O. Costa de Beauregard, Compt. Rend. 261 (1965) 4637; Physics Letters 21 (1966) 410; Nuovo Cimento, to be published.
2. Ch. Goillot, Compt. Rend. 260 (1965) 861 and 2181; Phys. Letters 21 (1966) 408.