# "HIDDEN MOMENTUM" IN MAGNETS AND INTERACTION ENERGY

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Coleman and Van Vleck's formula for the mass center associated with the Darwin Lagrangian entails an interesting corollary to an earlier remark by De Broglie and Brillouin.

In a recent paper Coleman and Van Vleck [1] use the Darwin Lagrangian for interacting point charges to analyse, up to order $c^{-2}$ inclusive, the generation of the Penfield-Haus [2] recoil force associated with the velocity dependence of the masses of the particles. A key point in their deduction is their definition of the mass center

$$MX = \sum_a m_a(1+\tfrac{1}{2}\beta_a^2)\,\boldsymbol{r}_a + \tfrac{1}{2}c^{-2}\sum_{a\neq b} e_a e_b r_{ab}^{-1}\,\boldsymbol{r}_a \qquad (1)$$

(which they prove to be the one associated with the Darwin Lagrangian), with as usual

$$M = \sum_a m_a(1+\tfrac{1}{2}\beta_a^2) + \tfrac{1}{2}c^{-2}\sum_{a\neq b} e_a e_b r_{ab}^{-1}\,. \qquad (2)$$

They have thus justified (up to order $c^{-2}$ inclusive) Brillouin's [3] earlier guess that, when conceiving the potential energy-momentum as concentrated in the point charges (which is the definition of "action at a distance" theories), the interaction energy of two particles should be equally divided between them two; any ratio other than 1/1 would be operationally wrong.

It thus turns out that not only the total interaction-energy in eq. (2). but even its distribution in eq. (1), are exactly defined with no arbitrariness in their expressions. And this, of course, is a direct consequence of the relativistic equivalence between energy and mass.

Now, by introducing the electric potential created at $a$ by all the other charges

$$V_{(a)} = \sum_{b\neq a} e_b r_{ab}^{-1}\,, \qquad (3)$$

the typical interaction terms in eqs. (1) and (2) are written

$$\tfrac{1}{2}c^{-2}\sum_a e_a V_{(a)}\,\boldsymbol{r}_a \quad \text{and} \quad \tfrac{1}{2}c^{-2}\sum e_a V_{(a)}\,, \qquad (4)$$

and we thus come to the important conclusion that in the action-at-a distance Darwin interaction theory a particular gauge is, by necessity, selected: namely, the Coulomb potential must be written as $er^{-1}$ + zero. The second part of the argument has already been stated by De Broglie [4] and by Brillouin [3], but the first part of it (though also guessed by Brillouin [3]) can only be proved (at order $c^{-2}$) via the Coleman-Van Vleck deduction.

We have investigated the derivation and formulation of similar statements which hold in the fully covariant, radiationless, Wheeler-Feynman interaction theory, and are submitting our paper for publication.

The remarks are merely presented here as formal, and their detailed interpretation in terms of general gauge invariance remains unclear.

*References*

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3. L. Brillouin. Proc. Natl. Acad. Sci. U.S. 53 (1964) 475 and 1280; Compt. Rend. 259 (1964) 2361. See also R. Lucas. Compt. Rend. 259 (1964) 2359.
4. L. De Broglie. Compt. Rend. 255 (1947) 163; Optique électronique et corpusculaire (Hermann. Paris. 1950) p. 45-49.