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WITH FURTHER REFERENCE TO THE FORCE BETWEEN TWO REVOLVING ELECTRONS AT A GREAT DISTANCE.

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IN a paper[1] published in June, 1917, reasons were given for the conclusion that the current form of electromagnetic equations requires modification in a fundamental manner. Briefly, it appears from a consideration of the mechanical force between two revolving electrons in small circular orbits, the centers being a great distance apart, that the force between two bodies of gross matter at a great distance apart must be immensely greater than the known force between them, namely the gravitational force, assuming, of course, that there was no error in the somewhat involved processes of applying the electromagnetic equations to the case.

In a recent paper[2] G. A. Schott has examined in much detail the investigation in the paper above referred to, and says in his opening paragraph "hence (Crehore) concludes, quite legitimately if his result be correct. that the fundamental equations of the accepted electron theory require substantial modification" and "It is obviously imperative that Crehore's result be either verified or disproved." In the second paragraph he says: "The following investigation is based on Crehore's equations for the electric part of the mechanical force,"[3] "which have been verified, except some obvious misprints,[4] *e. g.*, a_2 for a_1 in the last term of (49)." Above in the same paragraph Schott remarks: "Thus it is desirable to take the magnetic effect into account *ab initio*, although it will be found to be inappreciable on the average for an amorphous medium but not necessarily so for a crystal."

[1] A. C. Crehore, PHYS. REV., Sec. Ser., Vol. IX., p. 445, June, 1917.

[2] G. A. Schott, PHYS. REV., Sec. Ser., Vol. XII., p. 23, July, 1918.

[3] *Loc. cit.*, pp. 453, 454—this referring to the June, 1917, paper above mentioned.

[4] It is hoped that Dr. Schott will publish all of the errors in the expression for the instantaneous force that he alludes to, in order that this important equation may be available in its complete form, having been verified.

In reply to this last criticism it should be stated that the complete instantaneous force, including the magnetic component, had been obtained and averaged in a similar manner to the electric component prior to the time of publishing the June, 1917, paper, but it was not published partly because of its length, and partly because each magnetic term in the final average was affected by a common factor in powers of β higher than the square. So long as terms were being obtained from the electric component having the greater factor β^2, they were large compared with any terms obtained from the magnetic component. The quantity, $\beta_1\beta_2^3$, is a common factor of the equation (48), p. 36 of the Schott paper,[1] which is his final expression for the time average of the magnetic component. The importance of the equation for the instantaneous force with no terms whatever omitted is so great that the equation for the magnetic component, supplementing the electric already published in equations (48), (49) and (50), June, 1917,[2] is given below in an appendix. In the case of coaxial rings of electrons, where the instantaneous and the average forces are identical, this equation has a wider application than to possible long range gravitational forces. As evidence that the complete force had been obtained, equation (92), p. 469, of the June, 1917, paper may be cited. This gives the complete force, including all terms from both the electric and the magnetic parts exerted upon any selected electron in a ring of electrons due to any other electron in the same ring.

The present paper is offered as a preliminary reply to the Schott paper referred to. It is submitted before there has been opportunity to make a careful study of the suggested corrections as to averaging, which at best is a laborious process, and is submitted principally because the deductions that Schott draws from his own formula fall short of the mark. In his concluding remarks, p. 37,[3] Schott states that "from an experimental point of view, however, the deviation is not of much moment, for it is easily seen by expanding the function of β_2 in (50) that the first power which occurs is the fourth, so that, if β_2 be as large as .01, the relative deviation is less than the hundred millionth. For this reason I shall not pursue the matter further here." This statement refers to the previous sentence, in which it is pointed out that the average force of the second electron acting upon the first differs from that of the first acting upon the second, the law of action and reaction not holding true, as Schott states, except to a first approximation.

The only interpretation that I can give to these remarks is that Schott

[1] *Loc. cit.*

[2] *Loc. cit.*

[3] *Loc. cit.*

is making a comparison between the magnitude of the terms containing β, which vary with the value of β, and the constant term, namely the electrostatic force, which does not vary with any change of speed. (See his equation for the average force repoduced in (1) below.) It is manifestly improper to make such a comparison when applying these results to the *atoms* of matter, as was done in the June, 1917, paper referred to. The obtaining of the force between two revolving *electrons* is but one step in the process of finding the force between two *atoms*, supposing these atoms to consist of rings of electrons revolving around a positive nucleus located at the center of the orbits, and having a charge equal to the sum of the charges on the electrons but of opposite sign, as in a neutral atom.

To place the equation under discussion before us, Schott's result, his equation (50) p. 37,[1] is reproduced here, after making an obvious correction in the denominator of the last term under the logarithm, making β^2 read β_2, namely

$$(\mathbf{F}\cdot\mathbf{r})/r = -\frac{e_1e_2}{r^2}\left\{1 + \beta_2^2\left(\frac{1}{1-\beta_2^2} - \frac{1}{2\beta_2}\log\frac{1+\beta_2}{1-\beta_2}\right)\right\}. \quad (1)$$

Adopting Schott's suggestion, and expanding this in powers of β_2, we find

$$\frac{1}{2\beta_2}\log\frac{1+\beta_2}{1-\beta_2} = 1 + \frac{1}{3}\beta_2^2 + \frac{1}{5}\beta_2^4 + \frac{1}{7}\beta_2^6 + \cdots \quad (2)$$

and

$$\frac{1}{1-\beta_2^2} = 1 + \beta_2^2 + \beta_2^4 + \beta_2^6 + \cdots, \quad (3)$$

and, finally, the complete force equivalent to (1) is

$$(\mathbf{F}\cdot\mathbf{r})/r = -\frac{e_1e_2}{r^2}\left(1 + \frac{2}{3}\beta_2^4 + \frac{4}{5}\beta_2^6 + \frac{6}{7}\beta_2^8 + \cdots\right). \quad (4)$$

When the effect of the positive nuclei are taken into the account in summing up the forces between the various component parts of two *atoms*, all the electrostatic terms cancel out, and require no further consideration, the whole force at great distances between such atoms being due solely to the terms involving the speed of the revolving electrons, the β-terms in (1) or (4) above.

Referring again to the paper of June, 1917, top of page 456, it is stated that the electrostatic terms were omitted in taking the average force for the reason that they contribute nothing to the inverse square of the distance terms under discussion, when the effect of the positive charge of the atoms is included so as to get the force between atoms instead of

[1] *Loc. cit.*

electrons only. Schott, however, has carried these electrostatic terms through the averaging process from the beginning, and they give rise to the constant term, $-e_1e_2r^{-2}$, in (1) above, which should now be omitted in making any deductions about atoms or gross matter. Making this omission, and also making a sufficiently close approximation to the value of the force by omitting all powers of β higher than the fourth, we simplify Schott's result, as applied to a pair of electrons in neutral atoms, from (4) to be

$$(\mathbf{F}\cdot\mathbf{r})/r = -\tfrac{2}{3}e^2\beta_2{}^4r^{-2}. \tag{5}$$

Comparing this with the result obtained in June, 1917, the difference is principally in the sign and in the substitution of $\beta_2{}^4$ for $\beta_2{}^2$. As far as the law of action and reaction is concerned, the difference is greater with $\beta_2{}^4$ than with $\beta_2{}^2$. If we substitute β_1 for β_2, and take the ratio of the forces, it is evidently equal to $(\beta_1/\beta_2)^4$. If β_1 is nearly twice as large as β_2, the ratio is nearly 16 to 1, and the forces of Action and Reaction are not only not equal and opposite to a first approximation, as Schott states, but they are not even of the same order of magnitude. The ratio between them is greater than it would be if β^2 appeared in the formula instead of β^4.

An important criticism of the formula, however, is connected with the magnitude of the force between two pieces of matter at great distances. If we had β^2 instead of β^4, as formerly, the magnitude of the force at great distances exceeds the known force of gravitation in an immense ratio, about 10^{31}. The change from β^2 to β^4, assuming β^2 to be of the order of 10^{-4}, only reduces this ratio from 10^{31} to about 10^{27}, the two figures being so nearly alike that there is little choice between them. Whether the formula calls for a repulsive force or an attraction makes no difference in this argument as to the magnitude. We know that no such force exists, and the argument for a revision of the fundamental electromagnetic equations remains valid, taking Schott's results as to averaging to be correct.

This matter of the magnitude of the force may, perhaps, best be shown by a special example. In order not to raise the question of synchronism between the two electrons, let us select two atoms, the one having a single ring of two electrons and the other of four. If the two atoms were supposed to be alike, it is reasonable to take the speeds the same for each, but Schott does not claim that his formula applies to two synchronous electrons. It may be remarked, however, on this point I have found that, when complete rings of equally spaced electrons are concerned, the formula for synchronous electrons reduces to precisely the same form as for incommensurable velocities, since all the terms involving the phase angles disappear when rings are taken, and we obtain the same result as without synchronism. Not to raise the question, however, let us calculate the force on atom A, a two electron single ring atom, due to atom B, a four electron single ring atom, the speed of each electron in A being β_1, and in B, β_2. According to (5) the force of the four electrons in B upon one electron in A is 4 times the expression in (5). Similarly, the force of the four upon the second electron in A is 4 times (5), making a total of 8 times (5).

On the other hand, the force of the two electrons in A upon one in B is two times (5), writing β_1 instead of β_2. The total force of the two in A upon the four electrons in B is then eight times (5) writing β_1 instead of β_2. We have then

$$\text{Total force of } B \text{ on } A = -\tfrac{16}{3} e^2\beta_2^4 r^{-2}. \tag{6}$$

$$\text{Total force of } A \text{ on } B = -\tfrac{16}{3} e^2\beta_1^4 r^{-2}. \tag{7}$$

If it is permitted, in making an approximation to the speeds of the electrons, to adopt the approximate formula (3), page 14, Physical Review, July, 1918, for the speed of any ring of electrons, and a moderate error in the estimated speed cannot affect our argument at all, namely

$$\beta^2 = p\frac{\pi^2e^4}{c^2h^2}, \tag{8}$$

we have for the fourth power of the speed of each electron in atom A

$$\beta_1^4 = 4\frac{\pi^4e^8}{c^4h^4}, \tag{9}$$

and in atom B

$$\beta_2^4 = 16\frac{\pi^4e^8}{c^4h^4}. \tag{10}$$

whence (6) and (7) become

$$F_{BA} = -\frac{256}{3}\frac{\pi^4e^{10}}{c^4h^4}r^{-2}, \tag{11}$$

$$F_{AB} = -\frac{64}{3}\frac{\pi^4e^{10}}{c^4h^4}r^{-2}. \tag{12}$$

We shall also make a further assumption that the two-electron atom, A, represents hydrogen, and the four electron atom, B, helium. This assumption also will not affect our argument where such a large ratio as 10^{27} is concerned. According to the law of gravitation the average force between one atom of hydrogen and one of helium is

$$F = km_Hm_{He}r^{-2}. \tag{13}$$

The mass of the helium atom is approximately four times that of the hydrogen atom, so that, in terms of m_H, (13) becomes

$$F = 4km_H^2 r^{-2}. \quad (14)$$

Taking the smaller of the two forces, namely that in (12), for the purpose of comparison with the magnitude of the gravitational force, and taking the ratio, we have

Ratio of the calculated force to the actual gravitational force

$$= \frac{16}{3} \frac{\pi^4 e^{10}}{km_H^2 c^4 h^4}. \quad (15)$$

It was pointed out in the paper above referred to, equation (9), p. 15, July, 1918, that a very exact numerical value of the gravitational constant, k, is given by the expression,

$$k = \frac{p^4}{3K} \frac{m_0}{m_4^2} \frac{\pi^4 e^{10}}{c^4 h^4}, \quad (16)$$

in which $K = 1$, is the specific inductive capacity, and $p = 2$, the number of electrons in the hydrogen atom. m_0 is the mass of the negative electron at slow velocities. With these values, we have

$$k = \frac{16}{3} \frac{m_0}{m_H^2} \frac{\pi^4 e^{10}}{c^4 h^4}. \quad (17)$$

Upon substitution of this correct numerical value of the Newtonian constant, k, in (15), the expression reduces to

Ratio of calculated force to the gravitational force

$$= 1/m_0 = 1/.90 \times 10^{-27} = 1.11 \times 10^{27}. \quad (18)$$

The ratio would be four times greater than this if we had used the larger of the two forces as given in (11) above. This completes the special example, and shows that the force given by the Schott formula is greater than the gravitational force by a factor of about 10^{27}, as was pointed out before considering the example.

In conclusion, it has, we believe, been clearly shown that the application which Schott has made of his final result, equation (50) of his paper, or (1) above, to gross matter is incorrect. The positive nuclei of the atoms must be taken into the account as well as the revolving electrons. Whether or not the matter has been properly treated above in the simple omission of the constant, which is independent of the speed remains to be seen. It seems that any system of electromagnetic equations that does not make the mechanical force of one revolving electron acting upon a second exactly equal and opposite to that of the other on the one, thus strictly conforming to the law of action and reaction is pretty certain to lead to insuperable difficulties when applied to the accepted forms of atoms. It is difficult to believe that the average mechanical force on the first electron due to the second is entirely independent of the state of motion of the first electron. It seems as if its motion must in some way alter the average force upon it due to the second electron independent of any change that may be taking place in the motion of the second electron, and yet this is contrary to the Larmor-Lorentz form of theory.

Let us admit for the moment that a form of theory is possible that makes the average mechanical force between the two electrons a function of the product of the two speeds instead of the speed of one of the charges only. Then, when either of the two speeds vanishes the product vanishes, and the force between a revolving negative electron and a stationary positive charge is the same whether we consider the force of the one on the other or the other on the one, and is equal simply to the electrostatic terms. In the case of two revolving electrons the force of the one on the other would be the same as the other on the one, the Law of Action and Reaction being preserved way down to the ultimate constituent parts of the atoms and of gross matter, namely the electrons.

It seems as if the finding of a simple expression for the Newtonian constant given in a former paper, equation (9),[1] has some bearing upon this question, especially because all of the laws of gravitation have been shown to follow from the premises assumed. It is very evident that the least change in the fundamental equations of electromagnetic theory may easily switch the resulting force at great distances about from a repulsion to an attraction, the force being so small that the residual may take either a positive or a negative sign. The chief point that has now been established is that there are existing forces at great distances that vary inversely as the square of the distance, the gravitational law, as demanded by the present unmodified form of theory. Not much attention has been given to these forces in the past, and it seems as if it was not realized that they existed, as demanded by the accepted theory.

[1] *Loc. cit.*