Electromagnetic mass, relativity, and the Kaufmann experiments

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This paper presents the theoretical background for and the detailed analysis of Kaufmann's 1901–1905 experiments to determine the e/m ratio for fast electrons. Far from providing the first experimental confirmation of Einstein's special theory of relativity, as is often claimed in physics textbooks today, these data were initially interpreted as confirming Abraham's classical model of a rigid spherical electron and as providing evidence against special relativity. Only in 1906–1907, upon Planck's subsequent reanalysis of Kaufmann's 1905 data, did these experiments become evidence marginally in favor of relativity over classical models of the electron. This particular issue, of the superiority of special realtivity over classical theory in providing a fit to e/m determinations, was not definitely settled until 1914 with new extensive and accurate data obtained by Neumann. The entire episode provides another example that science does not proceed by a strict falsificationist methodology. It shows rather that a great scientist such as Einstein at times gives more weight to a theory that has a certain beauty and produces equations simple in form than he does to experimental results that apparently conflict with such a theory.

I. INTRODUCTION

The purpose of this paper is mainly a pedagogical one of presenting the background, details, and early interpretations of a set of experiments that are still often cited in textbooks as the first empirical confirmation of the variation of mass with velocity as predicted by Einstein's special theory of relativity. The actual historical facts turn out to be more complicated, and also more interesting and illuminating, than the pat prediction–confirmation scenario indicated in several modern physics texts.

As we shall see, this episode with the Kaufmann experiments proves to be another instance of the following interesting phenomenon. Accurate representations of several major developments in fairly modern physics are known to historians and philosophers of science (see Refs. 3, 4, 8, 11, and 53). Nevertheless, grossly inaccurate, but widely accepted, folklore versions of these great advances are passed from one generation of textbooks to another by the physicists who write them. We illustrate this effect briefly with three examples before developing the main topic of this paper. One of the most famous such pieces of psuedohistory is the standard account of how Maxwell discovered the displacement current term in his field equations. Rudolph Peierls tells us:

> "To me there is still a mystery about the way in which Maxwell obtained his equations and convinced himself of their validity. I shall make a guess about the answer, but I have no evidence that I am right."[1]

He then goes on to present what has become a standard discussion of a rationale for the addition of the displacement current term. Variations of this argument routinely appear, either as a matter of symmetry for Maxwell's four-vector equations or as one of mathematical consistency.[2] The latter approach begins with the observation that Ampère's law in its original form

$$\nabla\times\mathbf{H}=(4\pi/c)\mathbf{J} \tag{1.1}$$

is consistent with the continuity equation

$$\nabla\cdot\mathbf{J}+\frac{\partial\rho}{\partial t}=0, \tag{1.2}$$

only for steady-state situations (i.e., $\partial\rho/\partial t=0$). However, if Ampère's law is modified as

$$\nabla\times\mathbf{H}=\frac{4\pi}{c}\mathbf{J}+\frac{1}{c}\frac{\partial\mathbf{D}}{\partial t}, \tag{1.3}$$

then it becomes consistent with Eq. (1.2) and with Coulomb's law

$$\nabla\cdot\mathbf{D}=4\pi\rho. \tag{1.4}$$

In fact, such an argument is nowhere to be found in Maxwell's published works. He arrived at the concept of a displacement current by first considering a polarizable dielectric medium to which a time-dependent electric field was applied. By analyzing the behavior of a particular model of the ether popular in his day,[3,4] he was then able to extend this concept of a displacement current to what we would today term a vacuum.

Planck is usually credited with having obtained a fit to the empirical curve for the spectrum of blackbody cavity radiation by quantizing a set of harmonic oscillators used to represent this radiation field.[5] Aside from the question of Planck's real position on the quantization issue,[6] the fact is that he initially produced a two-parameter fit to the empirical curve by means of an *ad hoc* thermodynamic argument in which he modified the classical relations involving the entropy of the radiation in order to interpolate between the classically expected (Rayleigh–Jeans) law at low frequencies and Wien's law for high frequencies.[7,8] It was only subsequently that Planck was able to obtain this same law by a statistical-mechanics argument in which he partitioned the total energy of the system in discrete amounts.

Finally, we mention the standard presentation[9] of the semiclassical model of the hydrogen atom in which Bohr is claimed to have selected the allowed stable orbits by introducing the angular-momentum quantum condition

$$l=n(h/2\pi)\quad n=1,2,\ldots. \tag{1.5}$$

Although Eq. (1.5) makes the derivation of the Balmer formula for the hydrogen spectrum direct and immediate, this quantization condition appears to have emerged from nowhere, possibly as an inexplicable act of genius. However,

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reference to Bohr's original article[10,11] shows he felt that some basic quantity of *energy* had to be quantized just as in Planck's theory of radiation. Bohr realized the frequency of a spectral line did not correspond to the classical frequency of revolution of an electron but that light is emitted discontinuously during a transition from one stable orbit to another. Therefore he considered an initially free electron (having zero frequency of revolution) making a transition to an allowed orbit in which it would have the classical frequency (ν_{class}) of revolution. As the frequency to be assigned to the light emitted he took $\frac{1}{2}\nu_{\text{class}}$ [that is, the *average* between the initial frequency (0) and the final one ν_{class})]. Certainly, this particular averaging process, while having simplicity in its favor, is arbitrary. It was this quantity, $\frac{1}{2}\nu_{\text{class}}$, which Bohr quantized as

$$W = nh(\nu_{\text{class}}/2) \quad n = 1,2,\ldots . \tag{1.6}$$

The usual classical stability conditions for a stable circular orbit then lead to the discrete energy levels for a hydrogen atom. In spite of the arbitrariness of Bohr's averaging process, his motivation for this initial procedure is quite comprehensible in terms of a prominent theory of that time (namely, Planck's quantization of *energy*). His procedure does not appear as arbitrary as simply postulating or stating Eq. (1.5) with no perceivable relation to currently accepted theory. It was only subsequently[12] that Bohr pointed out that his original argument produced the same *result* as quantizing the angular momentum via Eq. (1.5).

The reason for mentioning these three episodes in the historical development of physics is to indicate that the actual historical facts are often more interesting and, indeed, comprehensible as a creative process having continuity with the theoretical background of the time than are the pat versions sometimes presented in physics texts. This type of material, much of which already exists in the literature of the history and philosophy of science, really belongs in those journals that will facilitate its being introduced into the physics classroom to present a more accurate and stimulating picture of the growth of new theories.

In the first five years of the present century a series of experiments was performed by Walter Kaufmann (1871–1947) to determine the variation of the electron's mass with velocity. Today physics texts often cite these experiments as confirming Einstein's formula,

$$m = m_0/[1 - (v/c)^2]^{1/2}, \tag{1.7}$$

derived on the basis of the special theory of relativity.[13] In fact, as we show below, Kaufmann's data were taken as evidence *against* the special theory of relativity. As our references will indicate, the essential facts for a proper analysis of this episode already lie scattered throughout the specialist literature of the history and philosophy of science. The purpose of this paper is to put that story in a form suitable for use in physics instruction.

II. ELECTROMAGNETIC MASS OF THE ELECTRON

In an approximately 15-year span centered around 1900, a question of considerable interest to physicists was the possible electromagnetic origin for part or all of the mass of the electron. Although detailed, quantitative arguments are given below, the basic idea is simply that the mass of a body is a measure either of its resistance to acceleration or of the work necessary to set the body into motion. Now classical electrodynamics predicts for a charged body an electromagnetic energy that increases rapidly with speed. Even early experiments to determine the charge to mass ratio e/m for the electron indicated that the experimentally determined mass increased sharply as the speed v of the electron approached the speed of light c. It was then a reasonable question to ask whether or not this empirically observed variation of mass could be accounted for wholly or in part by a model of the electron based on classical electrodynamics.

The main characters in this episode are Walter Kaufmann (1871–1947), Max Abraham (1875–1922), Alfred Bucherer (1863–1927), Hendrik Lorentz (1853–1928), and Max Planck (1858–1947). While Planck and Lorentz are familiar to physicists, Kaufmann, Abraham, and Bucherer are usually not. Walter Kaufmann studied at the Universities of Berlin and Munich, receiving his Ph.D. from Munich in 1894.[14] He was subsequently on the faculties at the Universities of Berlin, Göttingen, Bonn, and Königsberg. It was while at Göttingen and Bonn that Kaufmann performed the experiments (1901–1906) we discuss. At Göttingen Kaufmann became a close associate of Max Abraham, who in 1902 proposed his own model of the electron with its charge uniformly distributed over the surface of a rigid sphere. Abraham had been a doctoral student of Planck's at Berlin and was later on the faculties of the Universities of Göttingen, Illinois, Milan, and Aachen. He was often embroiled in controversy as a result of his openly critical nature, and his death was also a tragic and protracted affair. Although well-known among theorists in his own time, Abraham's name is recognized today mainly by those physicists aware of his *Theorie der Elektrizität* in one of its many editions—often the famous Abraham and Becker, the English translation of which graduate students still consult.

A rival model of the electron was proposed in 1904 by Alfred Bucherer, who had studied at Johns Hopkins and Cornell Universities before completing his graduate work at Strasbourg in 1895 and subsequently joining the faculty at Bonn. In Bucherer's model the electron was deformed as it moved through the ether, but in such a fashion that its volume remained constant. Kaufmann's experiments could not be used to make a definite decision between the models of Abraham and of Bucherer.

Beginning in 1892 Hendrik Lorentz had developed his theory of electrons. By 1904 his theory in its predictions was indistinguishable from Einstein's special theory of relativity. Lorentz's model of the electron at rest pictured it as a uniform spherical surface charge. As the electron was set in motion through the ether, its transverse dimensions remained unaffected but its length in the direction of motion was contracted. The variation of mass with velocity is precisely that given by Eq. (1.7). It was during 1906–1907 that Planck subjected the reduction of Kaufmann's data to a careful logical analysis and showed that they actually favored the Einstein–Lorentz predictions rather than those of Abraham. Finally, by 1908 Bucherer had become sufficiently suspicious of the reliability of Kaufmann's work that he performed a different and more accurate set of measurements as a result of which he agreed with the Einstein–Lorentz theory. By that time he had already abandoned his own model of the electron.

Before we present Abraham's general and detailed analysis for the electromagnetic energy associated with a moving electron, let us give a simple discussion for a charge moving at a velocity small compared to the speed of light.[15,16] The Biot–Savart law for the magnetic field **B** at a position **r** relative to a charge q moving with a velocity **v** is

$$\mathbf{B} = (q/4\pi c)(\mathbf{v}\times\mathbf{r}/r^3). \tag{2.1}$$

The magnetic energy density per unit volume is

$$u_m = \tfrac{1}{2}B^2. \tag{2.2}$$

From Fig. 1 we see that for a spherical electron of radius a with its charge distributed uniformly over its surface the magnetic field is

$$B = ev\sin\theta/4\pi c r^2 \quad r > a.$$

Therefore the total magnetic energy for this field is

$$\begin{aligned} W_m &= \int_{\text{all space}} u\,dv \\ &= \frac{e^2v^2}{32\pi^2c^2}\int_0^{2\pi} d\phi \int_0^{\pi} \sin^3\theta\,d\theta \int_a^{\infty} \frac{dr}{r^2} \\ &= e^2v^2/12\pi c^2 a. \end{aligned} \tag{2.3}$$

If we equate this to the work done on the electron to bring it up to a speed v (i.e., to the kinetic energy), we have

$$W_m = \tfrac{1}{2}mv^2 \tag{2.4}$$

or

$$m = e^2/6\pi c^2 a. \tag{2.5}$$

This expression is valid only for small velocities (really in the limit $v \rightarrow 0$) and represents the electromagnetic mass of the electron under these conditions.

Abraham's basic idea was to provide an electromagnetic foundation for all mechanics.[17] This was essentially a complete reversal from an earlier tendency on the part of theorists, such as Maxwell, to provide a mechanical basis for electromagnetic phenomena via mechanical models of the ether. The beauty of Abraham's arguments[18] is that they are very general and avoid all detailed (and messy) calculations. By analogy with analytical mechanics, in which the Lagrangian is given as the difference between the kinetic energy (or energy of motion) and the potential energy, Abraham writes[19]

$$L = W_m - W_e = -\frac{1}{2}\int (E^2 - B^2)dV. \tag{2.6}$$

This is the usual Lagrangian for the electromagnetic field.[20] Here the magnetic and electric energies are given, respectively, as

$$W_m = \frac{1}{2}\int B^2\,dV, \tag{2.7}$$

$$W_e = \frac{1}{2}\int E^2\,dV. \tag{2.8}$$

The total energy of the electromagnetic field is then

$$W = W_m + W_e = \frac{1}{2}\int (B^2 + E^2)dV. \tag{2.9}$$

The electric and magnetic fields can be expressed in terms of the usual scalar (Φ) and vector (**A**) potentials as

$$E = -\nabla\Phi - \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}, \tag{2.10}$$

$$\mathbf{B} = \nabla\times\mathbf{A}. \tag{2.11}$$

These potentials themselves satisfy the wave equations

$$\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\rho, \tag{2.12}$$

$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{1}{c}\mathbf{J}. \tag{2.13}$$

Abraham was able to use standard vector identities and Gauss's theorem to re-express the Lagrangian as[21]

$$L = -\frac{1}{2}\int \rho\phi\,dV. \tag{2.14}$$

Here ϕ is what Abraham terms the convection potential,

$$\phi = \Phi - (1/c)\mathbf{v}\cdot\mathbf{A}. \tag{2.15}$$

It is also essential for his derivation to assume that the electron's motion be uniform (i.e., $\mathbf{v} = \text{const}$)[22] and to recognize that

$$\mathbf{J} = \rho\mathbf{v}, \tag{2.16}$$

where ρ is the charge density of the electron and **v** its velocity. For uniform motion along the x axis, we can use the fact that

$$\frac{\partial}{\partial t} = -\mathbf{v}\cdot\nabla = -v\frac{\partial}{\partial x}, \tag{2.17}$$

to obtain from Eqs. (2.12) and (2.13),

$$(1-\beta^2)\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = -(1-\beta^2)\rho. \tag{2.18}$$

Here we have written $\beta = v/c$. Abraham next obtains a solution to Eq. (2.18) by a technique due originally to Lorentz.[23] By means of the new variables

$$x' = (1-\beta^2)^{-1/2}x \quad y' = y, \quad z' = z, \tag{2.19}$$

$$\phi = (1-\beta^2)^{1/2}\phi', \tag{2.20}$$

$$\rho' = (1-\beta^2)^{1/2}\rho, \tag{2.21}$$

Eq. (2.18) becomes

$$\nabla'^2\phi' = -\rho', \tag{2.22}$$

which is just Poisson's equation for electrostatics. In this frame the expression for the electrostatic energy becomes

$$W'_e = \frac{1}{2}\int \rho'\phi'\,dV'. \tag{2.23}$$

However, by means of the following observation, Abraham is even able to save himself the trouble of calculating Eq. (2.23) explicitly. Clerk Maxwell (1831–1879) in his *A Treatie on Electricity and Magnetism* had given an expression for the capacitance C of an ellipsoid of revolution as[24]

$$C = (c^2 - b^2)^{1/2}\Big/\ln\left(\frac{c + (c^2 - b^2)^{1/2}}{b}\right). \tag{2.24}$$

Here c is the semimajor axis of revolution and b is radius of the circular cross section ($c > b$). The electrostatic energy contained in a field surrounding an equipotential surface (Φ_0) carrying a charge q is[25]

$$W_e = \frac{1}{2}\int \rho\Phi\,dV = \frac{1}{2}q\Phi_0. \tag{2.25}$$

Since the relation among the charge q, the capacitance C

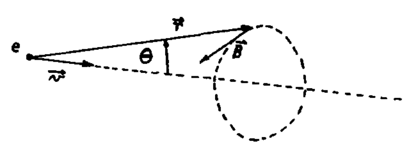

Fig. 1. Magnetic field for a slowly moving charge.

for the conductor, and the potential Φ_0 is

$$q = C\Phi_0, \tag{2.26}$$

the electrostatic energy for the charged equipotential surface is (with $q = e$, the charge on the electron)

$$W_e = e^2/2C. \tag{2.27}$$

From Eqs. (2.14) and (2.19)–(2.21) we have

$$L = -\tfrac{1}{2}\int \rho\phi\, dV = -\tfrac{1}{2}(1-\beta^2)^{1/2}\int \rho'\phi'\, dV'$$
$$= -(1-\beta^2)^{1/2} W_e'. \tag{2.28}$$

It is important to appreciate that W_e' must be evaluated in the *primed* coordinate system. From Eqs. (2.19)–(2.21) we see that W_e', and hence L of Eq. (2.28), will be a function of v. The Hamiltonian (here the total energy W as well) is given in terms of the Lagrangian as[26]

$$W = -L + v\frac{\partial L}{\partial v} = W_e + W_m. \tag{2.29}$$

Therefore, once we have W_e', we easily obtain both W_e and W_m separately. As usual, the canonical momentum for the field is given as

$$\mathbf{G} = \frac{1}{c^2}\int_{\text{all space}} \mathbf{S}\, dV, \tag{2.30}$$

where $\mathbf{S}$ is the Poynting vector

$$\mathbf{S} = c(\mathbf{E}\times\mathbf{B}). \tag{2.31}$$

Abraham shows[27] directly that only G_x is nonvanishing (as can be seen from symmetry considerations) and can be written

$$G_x = \frac{\partial L}{\partial v}, \tag{2.32}$$

as one would expect by analogy with particle mechanics.

Equation (2.32) can be proved as follows. Since $\mathbf{v}$ has only an x component, Eq. (2.13) becomes homogeneous for A_y and A_z and the boundary conditions at infinity requiring the vanishing of $\mathbf{A}$ there imply that

$$A_y = A_z = 0. \tag{2.33}$$

Equation (2.13) for A_x, when compared with Eq. (2.12) for Φ, shows that

$$A_x = \beta\Phi. \tag{2.34}$$

Direct application of Eqs. (2.10) and (2.11) yields

$$B_x = 0, \tag{2.35}$$

$$B_y = \beta\frac{\partial\Phi}{\partial z} = -\beta E_z, \tag{2.36}$$

$$B_z = -\beta\frac{\partial\Phi}{\partial y} = \beta E_y, \tag{2.37}$$

and

$$E_x = -(1-\beta^2)\frac{\partial\Phi}{\partial x}. \tag{2.38}$$

Then Eq. (2.6) becomes

$$L = -\tfrac{1}{2}\int [E_x^2 + (1-\beta^2)(E_y^2 + E_z^2)]dV. \tag{2.39}$$

It follows by use of Eqs. (2.35)–(2.39), as well as of Eq. (2.31), that

$$\frac{\partial L}{\partial v} = \frac{\beta}{c}\int (E_y^2 + E_z^2) dV$$
$$- \int \left[E_x\frac{\partial E_x}{\partial v} + (1-\beta^2)\left(E_y\frac{\partial E_y}{\partial v} + E_z\frac{\partial E_z}{\partial v}\right)\right]dV$$
$$= \frac{1}{c}\int (E_yB_z - E_zB_y)\, dV - \int (1-\beta^2)\nabla\Phi\cdot\frac{\partial}{\partial v}\mathbf{E}\, dV$$
$$= \frac{1}{c^2}\int S_x\, dV - (1-\beta^2)$$
$$\times \int \left[\nabla\cdot\left(\Phi\frac{\partial\mathbf{E}}{\partial v}\right) - \Phi\frac{\partial}{\partial v}(\nabla\cdot\mathbf{E})\right] dV. \tag{2.40}$$

The first term in the second integral vanishes by an application of Gauss's theorem on a surface at infinity. The second term in that integrand becomes (from the Maxwell equation $\nabla\cdot\mathbf{E} = \rho$)

$$\Phi\frac{\partial\rho}{\partial v} = 0, \tag{2.41}$$

since Abraham assumes the charge density of the electron to be independent of the velocity. What remains from Eq. (2.40) is just Eq. (2.32).

Once an expression for G has been obtained, the dependence of the electromagnetic mass upon velocity is readily found.[28] If, as in Fig. 2, a particle initially moving in a straight line in the direction $\hat{v}$ is subjected to a force $\mathbf{F}$ at P, then its instantaneous motion can be considered as circular motion (about 0) with both tangential and normal accelerations. From Newton's second law,

$$\frac{d}{dt}(G\hat{v}) = \mathbf{F}, \tag{2.42}$$

we have

$$\frac{dG}{dt}\hat{v} + G\frac{d\hat{v}}{dt} = F_\parallel\hat{v} + F_\perp\hat{r}, \tag{2.43}$$

where parallel and perpendicular refer to directions along and at right angles to $\hat{v}$, respectively. Since

$$\frac{d\hat{v}}{dt} = \lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t}\hat{r} = \frac{v}{r}\hat{r}, \tag{2.44}$$

we have

$$\frac{dG}{dt} \equiv \frac{dG}{dv}\frac{dv}{dt} = \frac{dG}{dv}a_\parallel = F_\parallel, \tag{2.45}$$

$$G\frac{v}{r} \equiv \left(\frac{G}{v}\right)\frac{v^2}{r} = \left(\frac{G}{v}\right)a_\perp = F_\perp. \tag{2.46}$$

Therefore, if we take $F = ma$ in each case, we arrive at an expression for the *longitudinal mass* m_s as

$$m_s = \frac{dG}{dv}, \tag{2.47}$$

and for the *transverse mass* m_r as

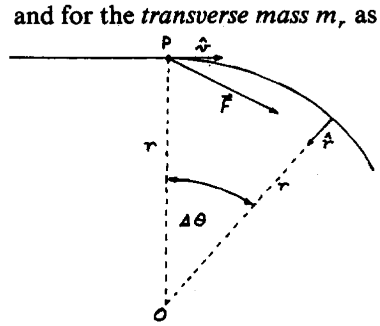

Fig. 2. Components of tangential and radial acceleration.

$$m_r = G/v. \tag{2.48}$$

Notice that we can also obtain m_s by equating the rate at which work is done on the system ($\mathbf{F}\cdot\mathbf{v}$) to the rate of change of energy of the system as

$$\mathbf{F}\cdot\mathbf{v} = F_{\parallel} v = \frac{dW}{dt} = \frac{dW}{dv}\frac{dv}{dt} = \left(\frac{1}{v}\frac{dW}{dv}\right)a_{\parallel} v = (m_s a_{\parallel})v. \tag{2.49}$$

This allows us to write alternatively for m_s:

$$m_s = \frac{1}{v}\frac{dW}{dv}. \tag{2.50}$$

Of course, the reason that both a longitudinal and a transverse mass appear in the older literature is that Newton's law has been written

$$\mathbf{F} \equiv \mathbf{F}_{\parallel} + \mathbf{F}_{\perp} = m_s \mathbf{a}_{\parallel} + m_r \mathbf{a}_{\perp}$$

rather than as

$$\frac{d\mathbf{p}}{dt} = \mathbf{F}.$$

In fact, if we use this latter form and write

$$\mathbf{G} = G\hat{v} = \left(\frac{G}{v}\right)\mathbf{v} \equiv m\mathbf{v},$$

then Newton's law is just

$$\frac{d\mathbf{G}}{dt} = \frac{d}{dt}(m\mathbf{v}) = \mathbf{F}. \tag{2.51}$$

In this formulation of the second law, *the* mass of the particle becomes

$$m = G/v, \tag{2.52}$$

which is identical with the transverse mass of Eq. (2.48). In our subsequent discussion of the Kaufmann experiments, it is the transverse mass that is the relevant quantity.

Finally, if we were to assume that the electron has both a mechanical mass M and electromagnetic masses m_s and m_r, then Eq. (2.42) would become

$$\frac{d}{dt}(M\mathbf{v} + G\hat{v}) = \mathbf{F}. \tag{2.53}$$

This would result in

$$m_{\text{total}}(\text{longitudinal}) = M + m_s, \tag{2.54}$$

$$m_{\text{total}}(\text{transverse}) = M + m_r. \tag{2.55}$$

We can now easily obtain Abraham's, Bucherer's, and Lorentz's expressions for the electromagnetic mass of the electron. We need only calculate W'_e [Eq. (2.23)] for these three cases. As we have mentioned, Abraham took his electron to be a rigid spherical shell that maintained its spherical shape once set in motion.[29] However, we must realize that the W'_e of Eq. (2.23) is to be evaluated in the *primed* coordinate system of Eq. (2.19). A sphere (of radius a) in the unprimed coordinate system becomes, in the primed system, an ellipsoid of revolution with

$$c = a/(1-\beta^2)^{1/2} \quad b = a. \tag{2.56}$$

From Eq. (2.24) we have

$$\frac{1}{C} = \frac{(1-\beta^2)^{1/2}}{8\pi\beta a}\ln\left(\frac{1+\beta}{1-\beta}\right) \tag{2.57}$$

and from Eq. (2.27) (in this primed system)[30]

$$W'_e(\text{Abraham}) = \frac{e^2(1-\beta^2)^{1/2}}{16\pi a\beta}\ln\left(\frac{1+\beta}{1-\beta}\right). \tag{2.58}$$

From Eq. (2.28) we obtain

$$L(\text{Abraham}) = \frac{-e^2}{16\pi a}\frac{(1-\beta^2)}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right). \tag{2.59}$$

Equation (2.32) then implies that

$$G(\text{Abraham}) = \frac{\partial L}{\partial v} = \frac{e^2}{8\pi ac}\frac{1}{\beta}\left[\left(\frac{1+\beta^2}{2\beta}\right)\ln\left(\frac{1+\beta}{1-\beta}\right) - 1\right]. \tag{2.60}$$

In the limit $\beta \ll 1$, this reduces to

$$G(\text{Abraham}) \underset{\beta\to 0}{\to} \frac{e^2}{6\pi ac^2} v \equiv m_0 v. \tag{2.61}$$

Notice that this value of m_0 agrees with Eq. (2.5), as it must.[31] We can express G as

$$G(\text{Abraham}) = \tfrac{3}{4} m_0 \frac{c}{\beta}\left[\left(\frac{1+\beta^2}{2\beta}\right)\ln\left(\frac{1+\beta}{1-\beta}\right) - 1\right]. \tag{2.62}$$

From Eqs. (2.47) and (2.48), respectively, we find[32]

$$m_s(\text{Abraham}) = \tfrac{3}{4} m_0 \frac{1}{\beta^2}\left[-\frac{1}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) + \frac{2}{1-\beta^2}\right] = m_0(1 + \tfrac{6}{5}\beta^2 + \tfrac{9}{7}\beta^4 + \cdots), \tag{2.63}$$

$$m_r(\text{Abraham}) = \tfrac{3}{4} m_0 \frac{1}{\beta^2}\left[\left(\frac{1+\beta^2}{2\beta}\right)\ln\left(\frac{1+\beta}{1-\beta}\right) - 1\right] = m_0(1 + \tfrac{2}{5}\beta^2 + \tfrac{9}{35}\beta^4 + \cdots), \tag{2.64}$$

where the power series represent an expansion for $\beta \ll 1$.

In Bucherer's model of the electron, the charge was again a surface charge, but this time on an ellipsoid whose *volume* remained constant during deformation as it moved.[33] The electron was spherical (radius a) only when at rest relative to the ether. When in motion with a speed v, its longitudinal semiaxis became contracted to

$$c = a(1-\beta^2)^{1/3}, \tag{2.65}$$

while its transverse one dilated to

$$b = a(1-\beta^2)^{-1/6}. \tag{2.66}$$

However, in the primed coordinate system of Eq. (2.19) its dimensions are

$$c' = c(1-\beta^2)^{-1/2} = a(1-\beta^2)^{-1/6}, \tag{2.67}$$

$$b' = b = a(1-\beta^2)^{-1/6}, \tag{2.68}$$

so that we have a sphere of radius $a(1-\beta^2)^{-1/6}$.

Bucherer's basic motivation[33] for the choice Eqs. (2.65)–(2.66) was to ask whether or not it was possible for the charge density of the electron to remain constant as its shape was deformed in its motion through the ether. If the total charge is conserved, then the volume must remain constant. However, there is a further constraint. Since the charge configuration must be such as to render the potential energy of the electron a minimum during its motion, the ratio of the longitudinal to transverse dimensions of the ellipsoid must be[34]

$$(1-\beta^2)^{1/2}{:}1{:}1.$$

If, as above, we denote by c the longitudinal semi-major axis and by b the transverse ones, then we have

$$b/c = (1-\beta^2)^{-1/2}.$$

The requirement of constant volume becomes

$$cb^2 = (1-\beta^2)^{-1}c^3 = a^3,$$

which yields Eqs. (2.65)–(2.66).

In the limit $c = b$ in Eq. (2.24) we obtain the well-known capacitance of a sphere

$$C_{\text{sphere}} = 4\pi b. \tag{2.69}$$

A calculation similar to that for the Abraham electron yields

$$L\,(\text{Bucherer}) = \tfrac{3}{4}m_0c^2(1-\beta^2)^{2/3}, \tag{2.70}$$

$$G\,(\text{Bucherer}) = m_0c\beta(1-\beta^2)^{-1/3}, \tag{2.71}$$

$$m_s(\text{Bucherer}) = m_0(1-\tfrac{1}{3}\beta^2)(1-\beta^2)^{-4/3} = m_0[1+\beta^2+(10/9)\beta^4+\cdots], \tag{2.72}$$

$$m_r(\text{Bucherer}) = m_0(1-\beta^2)^{-1/3} = m_0[1+(1/3)\beta^2+(2/9)\beta^4+\cdots]. \tag{2.73}$$

Finally, for the Lorentz electron we have a spherical electron (when at rest) which becomes flattened into an ellipsoid (the so-called "Heaviside" ellipsoid in the older literature[34]) with

$$c = (1-\beta^2)^{1/2}a, \tag{2.74}$$

$$b = a. \tag{2.75}$$

In the primed coordinate system of Eq. (2.19) this becomes a sphere of radius a. We then find[35]

$$L\,(\text{Lorentz}) = -m_0c^2(1-\beta^2)^{1/2}, \tag{2.76}$$

$$G\,(\text{Lorentz}) = m_0c(1-\beta^2)^{-1/2}, \tag{2.77}$$

$$m_s(\text{Lorentz}) = m_0(1-\beta^2)^{-3/2} = m_0[1+(3/2)\beta^2+(15/8)\beta^4+\cdots], \tag{2.78}$$

$$m_r(\text{Lorentz}) = m_0(1-\beta^2)^{-1/2} = m_0[1+(1/2)\beta^2+(3/8)\beta^4+\cdots]. \tag{2.79}$$

III. KAUFMANN'S EXPERIMENTS

During the period 1901–1906 Walter Kaufmann published the results of a series of experiments designed to measure the variation of the charge to mass ratio of the electron. Within the framework of classical electrodynamics there was no reason to expect the charge e of the electron to vary with v, whereas we have just seen that the electromagnetic mass of the electron should be highly velocity dependent. Therefore the observed decrease in the measured ratio e/m was interpreted in terms of an increase in m.

The apparatus which Kaufmann employed for these measurements was essentially always of the same design, although the precise dimensions appear to have varied slightly over the years. Figure 3 is a full-scale diagram of this apparatus as given by Kaufmann in one of his later publications.[36] The exterior cylindrical container is a vacuum chamber within which is a pair of vertical condenser plates (P_1,P_2) separated by quartz insulators (Q). At 0 a few grains of radium chloride (supplied by Pierre and Marie Curie) provided a source of high-speed β rays. [In previous experiments the identity of Becquerel (or β) rays and cathonde rays (or electrons) had been established.[37]] A horizontal electric field was maintained between the plates (by means of a potential difference V across the condenser). The entire apparatus was surrounded by a stack of permanent magnets that provided a uniform horizontal magnetic field (parallel to the **E** field) everywhere in the interior of the chamber. Those electrons which had the proper velocity to pass from the source 0 up between the plates emerged through a tiny (0.2-mm) diaphragm D into an evacuated region having only the magnetic field. They eventually hit the horizontal photographic plate at the top of the chamber and produced a series of exposures at points on the plate. Figure 4 gives a large schematic of this apparatus.

If there were no **E** field, then the **B** field above would produce the circular trajectory indicated in the x–z plane of Fig. 4 and the electron would impinge on the photographic plate at point $(x_2,0,\bar{z})$. When both the **E** and **B** fields are on, the electron also receives a horizontal acceleration during the time it travels between the condenser plates so that it finally hits the photographic plate at a point $(x_2,\bar{y},\bar{z})$. We now develop expressions for e/m in terms of the measured coordinates $(\bar{y},\bar{z})$. We derive approximate expressions based on classical electrodynamics with the simplifying assumption that v, the speed of the electron, remains essentially constant throughout its motion.[38] We later give exact expressions during our discussion of Planck's subsequent analysis of Kaufmann's work. The simplifying approximations used here are fully justified for the dimensions of the experimental apparatus and were not the source of Kaufmann's trouble. Since v remains nearly constant for a given electron along its trajectory, the value of the mass (*whatever* its dependence on v) is also constant. We shall always use the *transverse* mass of the electron since the electron's motion is nearly uniform and almost completely circular (caused by the action of the **B** field). Therefore we write simply m rather than m_r below.

The Lorentz force provides the centripetal force for motion on a circle of radius ρ as

$$mv^2/\rho = (e/c)vB, \tag{3.1}$$

so that

$$\rho = mcv/eB. \tag{3.2}$$

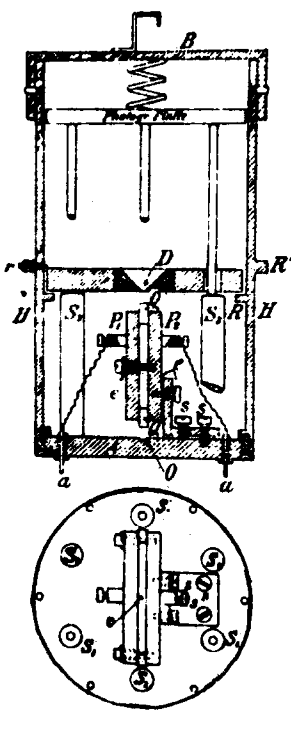

Fig. 3. Full-scale diagram of Kaufmann's apparatus.[36]

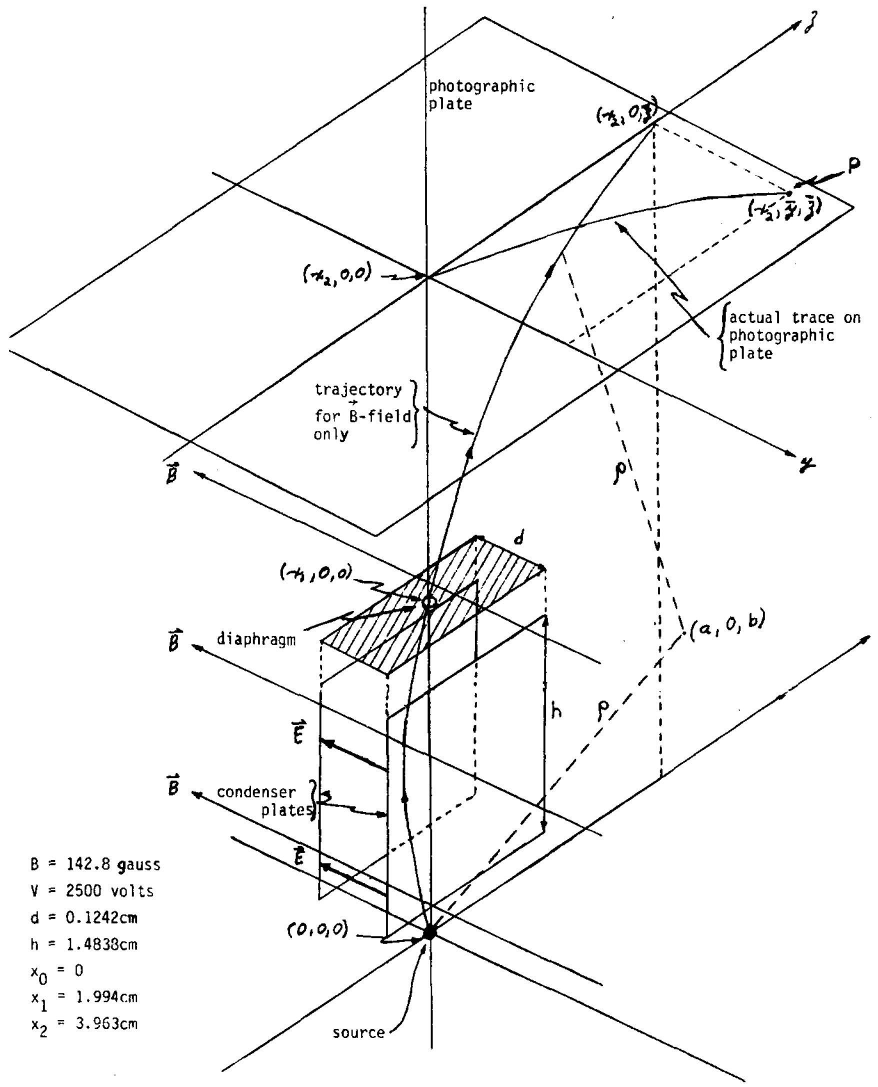

Fig. 4. Enlarged schematic of Kaufmann's apparatus.

The acceleration produced by the E field is

$$\frac{d^2y}{dt^2} = \frac{e}{m}E. \tag{3.3}$$

In Fig. 5 we show the motion of the electron projected onto the x–y plane. Both the electric deflection ($\bar{y}$) and the magnetic deflection ($\bar{z}$) are small. This is due, ultimately, to the dimensions of the apparatus (see Fig. 7), especially the fact that the plate height was about 1.5 cm whereas the plate separation was only 0.12 cm. Hence the velocity of the electron remains almost completely parallel to the x axis so that

$$\dot{x} \simeq v = \text{const.}$$

Therefore we can write

$$\frac{dy}{dt} = \frac{dy}{dx}\dot{x} = v\frac{dy}{dx}. \tag{3.4}$$

Since the trajectory must pass through the points (0,0) (the source) and (x_1,0) (the diaphragm), we can integrate Eq. (3.3) to obtain

$$y = (eE/2mv^2)x(x - x_1) \quad 0 \leqslant x \leqslant x_1. \tag{3.5}$$

For $x_1 \leqslant x \leqslant x_2$, the electric field vanishes and the trajectory is a straight line of slope

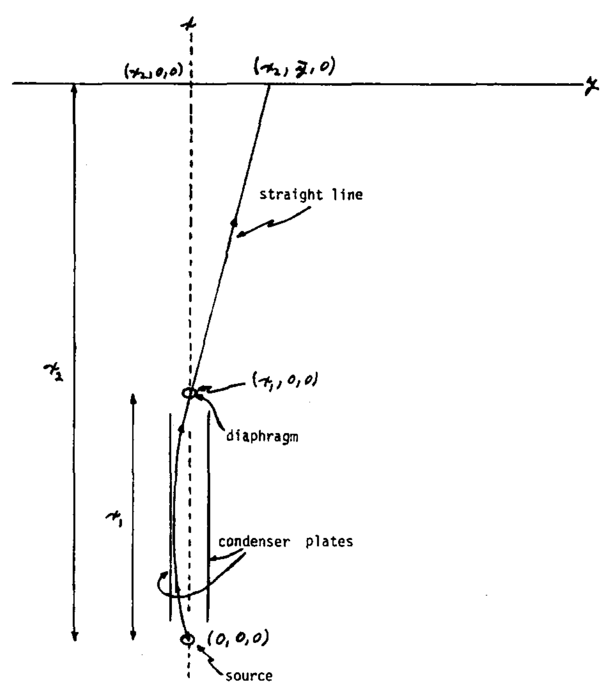

Fig. 5. Projected motion in $x-y$ plane due to E field.

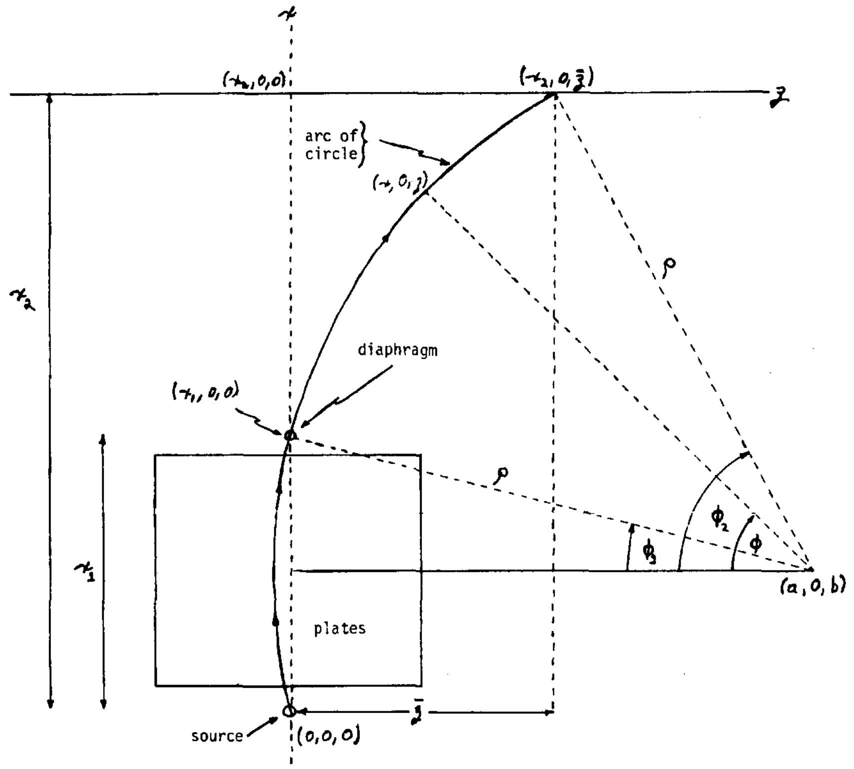

Fig. 6. Projected motion in x–z plane due to **B** field.

$$\frac{dy}{dx}=\left(\frac{dy}{dx}\right)_{x_1}=\frac{eE}{2mv^2}x_1.$$

The coordinate $\bar{y}$ at which the electron impinges on the photographic plate is

$$\bar{y}=\left(\frac{dy}{dx}\right)_{x_1}(x_2-x_1)=\frac{eE}{2mv^2}x_1(x_2-x_1). \quad (3.6)$$

Similarly, the projected motion in the x–z plane due to the action of the magnetic field is shown in Fig. 6. Since the motion is uniform circular, we can write

$$(x-a)^2+(z-b)^2=\rho^2. \quad (3.7)$$

If we require that this circle pass through the points (0,0), $(x_1,0)$,$(x_2,\bar{z})$, a little algebra yields

$$a=\tfrac{1}{2}x_1, \quad (3.8)$$

$$b=[x_2(x_2-x_1)+\bar{z}^2]/2\bar{z}\simeq x_2(x_2-x_1)/2\bar{z}, \quad (3.9)$$

$$\rho=(a^2+b^2)\simeq b\simeq x_2(x_2-x_1)/2\bar{z}. \quad (3.10)$$

The approximate forms of Eqs. (3.9) and (3.10) are valid when $\bar{z}$ is much smaller than x_1, x_2, and (x_2-x_1), which is always the case for Kaufmann's apparatus. Since

$$\bar{z}=x_2(x_2-x_1)/2\rho, \quad (3.11)$$

we can use Eq. (3.2) to write

$$\bar{z}=(eB/2mcv)x_2(x_2-x_1). \quad (3.12)$$

If Eqs. (3.6) and (3.12) are used to eliminate v, an expression for e/m is found as[39]

$$e/m=[2Ec^2x_1/B^2x_2^2(x_2-x_1)]\bar{z}^2/\bar{y}. \quad (3.13)$$

The quantity in square brackets represents a constant, characteristic of Kaufmann's apparatus, while $\bar{z}$ and $\bar{y}$ are the coordinates gotten directly from the photographic plate. The result of Eq. (3.13) is valid for *any* dependence of m on v (relativistic or otherwise) *to the extent* that v remains nearly constant during the motion. Essentially, this means that **E** cannot be so strong as to produce such large accelerations that $\Delta v/v \ll 1$ ceases to be valid. The reason that even relativistic kinematics will not change this result (under the stated approximation $v\simeq$const) is that the relativistic version of Newton's second law (as originally stated by Planck[40]) is

$$\frac{d}{dt}\left(\frac{m_0}{(1-\beta^2)^{1/2}}\mathbf{v}\right)=\mathbf{F}_{\text{Lorentz}}, \quad (3.14)$$

which is of the form of Eq. (2.51).

Kaufmann's first experimental results on the e/m values for high-speed β rays $(0.787\leqslant\beta\leqslant0.945)$ were published in a 1901 paper.[41] It is a curious paper to study carefully. In his analysis of his five data points, Kaufmann tries to decide how much of the electron's mass is mechanical [cf. the M of Eqs. (2.53)–(2.55) above; Kaufmann refers to this at the "true" mass] and how much is electromagnetic (his term is "apparent" mass). He writes

$$m_{\text{total}}=M_{\text{true}}+m_{\text{apparent}}. \quad (3.15)$$

For m_{apparent} (that is, the electromagnetic mass) he refers to an older work by Searle[34] in which the total energy (that is, W_e+W_m) of a moving spherical electron is given as

$$W=(e^2/8\pi a)\{(1/\beta)\ln[(1+\beta)/(1-\beta)]-1\}. \quad (3.16)$$

This is just what one would get for the Abraham model of the electron by use of Eqs. (2.59) and (2.29), as it must be. The interesting point is that Kaufmann then obtains an expression for m_{apparent} as

$$m_{\text{apparent}}=\frac{1}{v}\frac{dW}{dv},$$

which yields the *longitudinal* mass [recall Eq. (2.50)] exactly as given in Eq. (2.63). As we have already discussed, it is the transverse mass that is appropriate for use in the present case of deflection by a magnetic field.

From his data points and Eqs. (3.13) and (3.12) Kaufmann was able to obtain values for e/m_{total} and $\beta=v/c$.

Table I. Kaufmann's original (1901) data.[41]

Data point(i)	$\bar{z}$	$\bar{y}$	$\beta = v/c$	$\frac{e}{m_{total}} \times 10^{-7}$(exp.)	$\frac{(e/m)i}{(e/m)5}$ (exp.)	$\frac{e}{m_{total}} \times 10^{-7}$(calc)	$\frac{(e/m)i}{(e/m)5}$ (calc)	$[(1-\beta_i^2)/(1-\beta_5^2)]^{1/2}$
1	0.271	0.0621	0.945	0.63	0.481	0.524	0.403	0.539
2	0.348	0.0839	0.907	0.77	0.588	0.775	0.597	0.683
3	0.461	0.1175	0.864	0.975	0.744	1.010	0.778	0.819
4	0.576	0.1565	0.827	1.17	0.893	1.163	0.895	0.911
5	0.688	0.198	0.787	1.31	1.000	1.299	1.000	1.000

These are given in the third and fourth columns of Table I. Using Eqs. (3.15) and (2.63) he then did a least-squares fit to these data to obtain the best values for e/M and e/m_0. [Here m_0 is the "rest" mass appearing in Eq. (2.63).] He found that the best fit is given by (in units of g/emu)

$$M/e = 0.39 \times 10^{-7}, \quad (3.17)$$

$$m_0/e = 0.122 \times 10^{-7}. \quad (3.18)$$

In the limit $\beta \to 0$ he had

$$\left.\frac{e}{m_{\text{total}}}\right|_{\beta=0} = \frac{e}{M + m_0} = 1.95 \times 10^7 \text{ emu/g}, \quad (3.19)$$

which agreed quite well with the then-known value of 1.865×10^7 from cathode-ray determinations. The sixth column in Table I gives the values calculated by Kaufmann using the values of Eqs. (3.17) and (3.18). He concluded that

$$m_0/M = 0.313, \quad (3.20)$$

or that the "true" mass was about three times the "apparent" mass as $\beta \to 0$. It should be evident, however, that Kaufmann's analysis is quite incorrect since he used the longitudinal rather than the transverse mass.

Also in the last column of Table I we have given those values expected by special relativity [Eq. (1.7)]. Comparison of columns five and eight of this table shows that these data do not give any convincing support for relativity theory.[42]

In the next year (1902) Kaufmann published[43] more data and analyzed them in terms of Abraham's *transverse* mass,[44] of which he had become aware. This time he handled his data in the following manner. He discovered an algebraic error in a formula of his 1901 paper,[41] which he corrected in subsequent analysis. This made the major contribution to the difference between the results stated in his 1901 and 1902 papers. Kaufmann also made geometrical corrections for the dimensions of his apparatus and eliminated the E and B fields from his formulas by introducing "reduced" electric and magnetic deflections, η and ζ, respectively, gotten from an empirical curve relating $\bar{y}$ and $\bar{z}$. These η and ζ are proportional to the actual experimental deflections $\bar{y}$ and $\bar{z}$ and differ very little from them, so that we continue to use $\bar{y}$ and $\bar{z}$ for simplicity in the following discussion. Kaufmann combined Eqs. (3.6) and (3.12) as

$$\beta = k_1(\bar{z}/\bar{y}), \quad (3.21)$$

where k_1 is taken to be an apparatus constant whose determination will be discussed shortly. Equation (3.13) he wrote as

$$e/m = k_2(e/m_0)(\bar{z}^2/\bar{y}), \quad (3.22)$$

with k_2 being another constant to be determined. With Eq. (2.64) expressed as

$$m = m_0\psi(\beta), \quad (3.23)$$

Kaufmann combined Eqs. (3.21)–(3.23) to obtain

$$\bar{y}/\bar{z}^2\psi\left(k_1\frac{\bar{z}}{\bar{y}}\right) = k_2. \quad (3.24)$$

Using the experimental values of $\bar{y}$ and $\bar{z}$, he then adjusted the value of k_1 in Eq. (3.24) to obtain a nearly constant value of k_2 (in the sense that the mean square deviation of k_2 was minimized for a given set of data). Although Kaufmann presented much more data in this 1902 work than he had in 1901, we list in Table II only the results for the same data used in Table I. (Kaufmann had dropped the first entry in Table I as being unreliable, presumably because his corrected reduction formulas yield $\beta = 1.006$ for it.) Notice that the corresponding values of β for Tables I and II differ since those of Table I were determined directly from the data [by Eqs. (3.12) and (3.13)] while those of Table II were gotten by a least-squares fit (after correcting for the error mentioned above). The second method yields larger values for β. From the mean value of k_2, Kaufmann obtained

$$e/m_0 = 1.84 \times 10^7 \text{ emu/g}, \quad (3.25)$$

by comparing the constant coefficients of Eqs. (3.13) and (3.22). In spite of the fact that he realized that a small experimental error in determining β would cause a large uncertainty in m because of the rapid variation in $\psi(\beta)$ [Eq. (3.23)] for β close to unity, Kaufmann nevertheless declared:

"The mass of the Becquerel ray constituting the electron depends upon the speed; the dependence can be accurately represented by Abraham's formula. Therefore, the mass of the electron is purely electromagnetic in nature."[45]

In 1903 Kaufmann published further data[46] and in 1905[47] attempted to decide definitively among the theories of Abraham, Bucherer, and Lorentz. This work was finally summarized in a massive review article of 1906.[36] We discuss the 1905 paper before moving on to alternative interpretations of Kaufmann's data. It is here that the famous set of nine data points appears, which Planck would later scrutinize.[48] All these data are new and independent of Kaufmann's previous measurements. In reducing the data

Table II. Kaufmann's reinterpretation (1902) with Abraham's formula[43] $k_1 = 0.532$ and $\bar{k}_2 = 2.173$).

$\bar{z}$	$\bar{y}$	β	$\psi(\beta)$	k_2	$\delta = (k_2 - \bar{k}_2)/\bar{k}_2(\%)$
0.348	0.0839	0.957	3.08	2.16	− 0.6
0.461	0.1175	0.907	2.49	2.165	− 0.4
0.576	0.1565	0.847	2.13	2.20	+ 1.2
0.688	0.198	0.799	1.96	2.165	− 0.4

Table III. Kaufmann's 1905 data and analysis.[47]

$\bar{z}$	$\bar{y}_{exp}$	Weight	$\bar{y}_{theo}$ (1)	(2)	(3)	$\delta=(\bar{y}_{exp}-y_{theo})\times 10^4$ (1)	(2)	(3)	β (1)	(2)	(3)
0.1350	0.0246	1/2	0.0251	0.0246	0.0254	−5	0	−8	0.974	0.924	0.971
0.1919	0.0376	1	0.0377	0.0375	0.0379	−1	+1	−3	0.922	0.875	0.919
0.2400	0.0502	1	0.0502	0.0502	0.0502	0	0	0	0.867	0.823	0.864
0.2890	0.0545	1	0.0649	0.0651	0.0647	−4	−6	−2	0.807	0.765	0.805
0.3359	0.0811	1	0.0811	0.0813	0.0808	0	−2	+3	0.752	0.713	0.750
0.3832	0.1001	1	0.0995	0.0997	0.0992	+6	+4	+9	0.697	0.661	0.695
0.4305	0.1205	1	0.1201	0.1202	0.1200	+4	+3	+5	0.649	0.616	0.647
0.4735	0.1404	1/4	0.1408	0.1405	0.1409	−4	−1	−5	0.610	0.579	0.608
0.5252	0.1666	1/4	0.1682	0.1678	0.1687	−16	−12	−21	0.566	0.527	0.564

he wrote what are essentially Eqs. (3.6) and (3.12) in the form

$$\bar{y}=(e/m)(A/v^2), \tag{3.26}$$

$$\bar{z}=(e/m)(A'/v). \tag{3.27}$$

Here, as usual, A and A' are constants characterizing the apparatus for a particular run. Recall, also, that just as in Eqs. (3.21)–(3.22), $\bar{y}$ and $\bar{z}$ are geometrically corrected coordinates that differ slightly (typically a few percent) from the raw experimental numbers taken from the photographic plate.[49] With

$$m=m_0\psi(\beta),$$

as in Eqs. (3.23), the previous equations can be written

$$\bar{z}=(e/m_0)(A/c)[1/\beta\psi(\beta)], \tag{3.28}$$

$$\bar{y}=(e/m_0)(A'/c^2)[1/\beta^2\psi(\beta)]. \tag{3.29}$$

The three theories are distinguished by the choices for $\psi(\beta)$. From Eqs. (2.64), (2.79), and (2.73) we have

$$\text{Abraham:}\quad \psi(\beta)=\frac{3}{4}\frac{1}{\beta^2}\left[\left(\frac{1+\beta^2}{2\beta}\right)\ln\left(\frac{1+\beta}{1-\beta}\right)-1\right], \tag{3.30}$$

$$\text{Lorentz:}\quad \psi(\beta)=(1-\beta^2)^{-1/2}, \tag{3.31}$$

$$\text{Bucherer:}\quad \psi(\beta)=(1-\beta^2)^{-1/3}. \tag{3.32}$$

Kaufmann refers to these as theories 1, 2, and 3, respectively. Just as in his 1902 paper he uses a least-squares fit. Let

$$(e/m_0)(A/c)=D, \tag{3.33}$$

$$(e/m_0)(A'/c^2)=D', \tag{3.34}$$

$$1/\beta\psi(\beta)=r, \tag{3.35}$$

$$1/\beta^2\psi(\beta)=s=f(r), \tag{3.36}$$

so that

$$\bar{y}=D's=D'f(\bar{z}/D). \tag{3.37}$$

The nine data points of Table III are used to find least-squares values for the constants D and D' from Eq. (3.37). Equation (3.28) then yields a value of β for each theory for every data point $\bar{z}$. With this, calculated values of $\bar{y}$ can be gotten from Eq. (3.29). Also, since A and A' are independently known, Eq. (3.33) or (3.34) can be used to calculate e/m_0. The results of Kaufmann's analysis are given in Table III. The third column gives the weight that he assigned to the various data as reliable experimental points. Abraham's theory (1) does do a somewhat better job at predicting $\bar{y}_{theo}$ than do the other two. However, Kaufmann attaches the most weight to the values of e/m_0 extracted and compared to the then-accepted value from cathode-ray determinations extrapolated to $v=0(e/m_0=1.885\times 10^7$ emu/g). His values are

$$\text{Abraham:}\quad e/m_0=1.823\times 10^7,$$

$$\text{Lorentz:}\quad e/m_0=1.660\times 10^7,$$

$$\text{Bucherer:}\quad e/m_0=1.808\times 10^7.$$

He ends his 1905 paper with some far-reaching conclusions:

"The results above speak against the correctness of Lorentz's, and also consequently of Einstein's, fundamental hypothesis. If one considers this hypothesis as thereby refuted, then the attempt to base the whole of physics, including electrodynamics and optics, upon the principle of relative motion is also a failure."

"A decision between the theories of Abraham and of Bucherer is meanwhile impossible and appears not attainable by observations of the type described above because of the general numerical agreement of the values of $\psi(\beta)$ (for these two theories). Whether the Bucherer formula for the optics of a moving medium is within the realm of feasible observations, as is the case for Lorentz's (theory), remains to be seen."[50]

IV. PLANCK'S ANALYSIS OF KAUFMANN'S WORK

As early as 1904 Lorentz[51] questioned whether or not Kaufmann's analysis of his 1902 data[43] really gave conclusive support to Abraham's model over Lorentz's own model. Quite simply, what Lorentz did was to use Kaufmann's method of analysis [Eqs. (3.21)–(3.24) above] but substituting for the function $\psi(\beta)$ that corresponding to his own theory [Eq. (3.31)] rather to Abraham's [Eq. (3.30)]. He found a fit nearly as good as Kaufmann's.[52] Lorentz concluded that either theory provided an acceptable fit to the data.

The gist of Lorentz's procedure was the following. He replaced Eqs. (3.21) and (3.24) (Kaufmann's equations for Abraham's theory) with

Table IV. Lorentz analysis of Kaufmann's 1902 data[51] ($\bar{k}'_2=2.745$).

$\bar{z}$	$\bar{y}$	β'	$s=0.952$ k'_2	$=(k'_2-\bar{k}'_2)(\%)/\bar{k}'_2$
0.348	0.0839	0.911	2.74	−0.2
0.461	0.1175	0.863	2.72	−0.9
0.576	0.1565	0.806	2.77	+0.9
0.688	0.198	0.761	2.75	+0.2

$$\beta' = sk_1(\bar{z}/\bar{y}) = s\beta, \tag{4.1}$$

$$\psi' = (\beta') \equiv (1-\beta'^2)^{-1/2} = \bar{y}/k_2'\bar{z}^2. \tag{4.2}$$

That is, he used the ψ for his own theory and the corresponding values of β (i.e., β'). He combined Eqs. (4.1) and (4.2) with Eq. (3.24) as

$$k_2' = \frac{1}{\psi'(\beta')}\frac{\bar{y}}{\bar{z}^2} = \frac{k_2\psi(\beta)}{\psi'(\beta')} = [k_2\psi(\beta)](1-s^2\beta^2)^{1/2}. \tag{4.3}$$

The first term in square brackets on the right can be gotten from Kaufmann's analysis (see Table II) and s is chosen by the best least-squares fit for k_2' for the data for a given run. Although Lorentz actually used a large number of Kaufmann's 1902 and 1903 data, we illustrate a fit produced by Lorentz's analysis to the data of Table II. The results are given in Table IV. It is evident that k_2' is as nearly constant here as in Kaufmann's analysis using Abraham's theory. Lorentz also went on to calculate from Eq. (4.2) a $\bar{y}_{\text{theo}}$ for each $\bar{z}$ (as Kaufmann had done in Table III) and obtained nearly as good results as Kaufmann had.

However, it was Max Planck who decisively reversed the interpretation of Kaufmann's data from disconfirmation of relativity to confirmation of that theory. Planck's was a classically beautiful application of strict logic to a rather confused situation.[53] In an important 1906 paper[40] he used what are essentially form invariance arguments to obtain a modification of Newton's second law of motion for a charged particle in external **E** and **B** fields as

$$\frac{d}{dt}\left(\frac{m_0\mathbf{v}}{(1-\beta^2)^{1/2}}\right) = e\left(\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B}\right). \tag{4.4}$$

This is the relativistically correct form of the law still used today. He also took the Lagrangian to be

$$L = -m_0c^2[(1-\beta^2)^{1/2} - 1]. \tag{4.5}$$

This differs from Eq. (2.76) only by the additive constant m_0c^2. Equation (4.5) represents the kinetic energy of a free particle and has the expected limiting value $\frac{1}{2}m_0v^2$ as $\beta \to 0$. The equations of motion can be written as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = F_x, \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) = F_y, \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) = F_z. \tag{4.6}$$

Although emphasizing the great value and beauty of the work of Lorentz and of Einstein on the relativity principle and the importance of investigating its consequences, Planck acknowledges Kaufmann's (1905) experiments.

> "To be sure, this question (of the acceptability of the relativity principle) appears to be already settled by the recent important measurements of Kaufmann, and in the negative, so that further investigation remains to be done."[54]

Later in 1906[55] Planck returned to the problem posed by Kaufmann's results. Basically what he did was to develop expressions that were valid *independent* of the specific form of the Lagrangian and then to compute the expected deflections on the basis of the Abraham and of the Lorentz Lagrangians. He was forced to conclude, as had Kaufmann, that Abraham's theory did provide a better fit than the Lorentz–Einstein one. However, he did point out the discomforting fact that, if one used the empirical numbers as provided by Kaufmann, then for *either* Lagrangian, one of the data points gave $\beta > 1$, which was inconsistent for both theories. Before we discuss Planck's final paper on this subject, in which he found the source of the discrepancy and resolved the entire issue, we give some details of Planck's dynamical analysis.[56]

As reference to Fig. 4 shows, Eqs. (4.6) become

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = -\frac{e}{c}B\dot{z}, \tag{4.7}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) = eE, \tag{4.8}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) = \frac{e}{c}B\dot{x}. \tag{4.9}$$

The Lagrangian for Lorentz's theory is given by Eq. (4.5) and that for Abraham's theory Planck took to be

$$L\,(\text{Abraham}) = -\tfrac{3}{4}m_0c^2\left[\left(\frac{1-\beta^2}{2\beta}\right)\ln\left(\frac{1+\beta}{1-\beta}\right) - 1\right]. \tag{4.10}$$

[Notice that this differs from Eq. (2.59) by an additive constant that cannot affect the resulting equations of motion. Equation (4.10) has the correct limiting value for the usual classical kinetic energy as $\beta \to 0$.] With

$$v = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2} \tag{4.11}$$

and

$$p = \frac{\partial L}{\partial v}, \tag{4.12}$$

the equations of motion become

$$\frac{d}{dt}\left(p\frac{\dot{x}}{v}\right) = -\frac{e}{c}B\dot{z}, \tag{4.13}$$

$$\frac{d}{dt}\left(p\frac{\dot{y}}{v}\right) = eE, \tag{4.14}$$

$$\frac{d}{dt}\left(p\frac{\dot{z}}{v}\right) = \frac{e}{c}B\dot{x}. \tag{4.15}$$

Before these equations can be integrated, E and B must be known as functions of the positions.

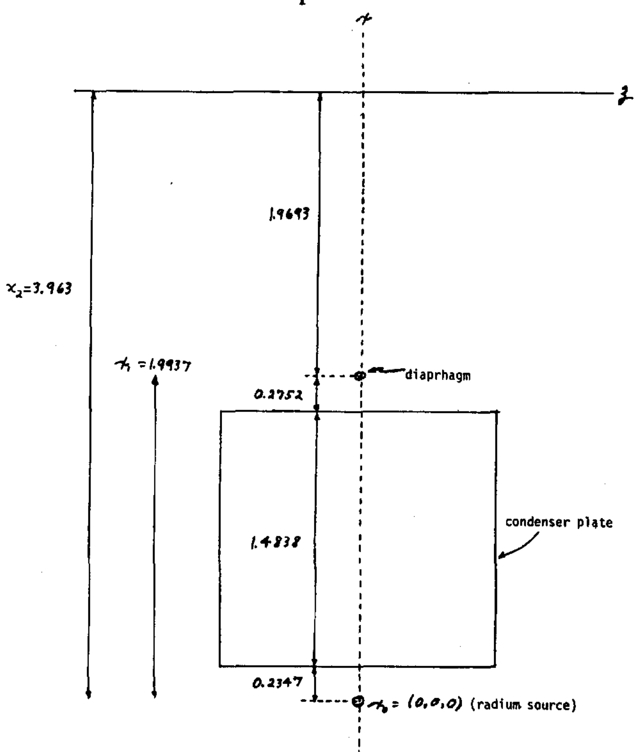

Fig. 7. Dimensions of Kaufmann's (1905) apparatus.[36]

The relevant dimensions of Kaufmann's apparatus are shown in Fig. 7.[57] Now the E field is not exactly a constant over the entire region between the source (x_0) and the diaphragm (x_1). Kaufmann had made an independent measurement of the variation of E with x.[58] Planck represented the electric field analytically as[59]

$$E = E_m F(x). \qquad (4.16)$$

Here E_m, the (uniform) E field between the condenser plates, is given in terms of the potential difference V across the plates and the plate separation as

$$E_m = v/d. \qquad (4.17)$$

The $F(x)$ in Eq. (4.16) which represents Kaufmann's measurements is taken by Planck to be

$$F(x) = \left\{ \begin{array}{ll} ax = 2.475x & 0 \leqslant x \leqslant \xi_1 \equiv 0.404 \\ 1 & 0.404 \leqslant x \leqslant \xi_2 \equiv 1.590 \\ b - ax = 4.935 - 2.475x & 1.590 \leqslant x \leqslant 1.994 \\ 0 & x \geqslant x_1 = 1.994 \end{array} \right\}. \qquad (4.18)$$

This function is graphed in Fig. 8. Kaufmann's parameters, as used by Planck, are

$$\begin{aligned} x_0 &= 0 \text{ (source)}, \\ x_1 &= 1.994 \text{ cm (diaphragm)}, \\ x_2 &= 3.963 \text{ cm (photographic plate)}, \\ v &= 2500 \text{ V}, \\ d &= 0.1242 \text{ cm}. \end{aligned} \qquad (4.19)$$

The **B** field was measured by Kaufmann and found to be essentially constant throughout the region $0 \leqslant x \leqslant x_2$ with the value

$$B = 142.8 \text{ G}. \qquad (4.20)$$

The problem is now to solve Eqs. (4.13)–(4.15) for the electron's trajectory subject to the conditions that it pass through the points (0,0,0) (the source), $(x_1,0,0)$ (the diaphragm), and $(x_2, \bar{y}, \bar{z})$ (the point exposed on the photographic plate). Since B is a constant, Eqs. (4.13) and (4.15) can each be integrated with respect to time. If the resultant equations are divided by each other, a perfect differential is obtained which can be integrated to yield the relation between x and z. As is well known, this turns out to be a circle. That is, the projection of the electron's trajectory into the x–z plane is a circle. From Fig. 6 we can write for any point (x,z) on this circle of constant radius ρ

$$x = (x_1/2) + \rho \sin\phi, \qquad (4.21)$$

$$z = (x_1/2) \cot\phi_1 - \rho \sin\phi, \qquad (4.22)$$

with

$$\rho = x_1/2 \sin\phi_1. \qquad (4.23)$$

The values of ϕ for which $x = x_1$ and $x = x_2$ can be found from the geometry of Fig. 6 as [recall Eq. (3.9)]

$$\tan\phi_1 = \tfrac{1}{2}x_1/b = x_1\bar{z}/[(x_2 - x_1)x_2 + \bar{z}^2], \qquad (4.24)$$

$$\sin\phi_2 = \frac{(x_2 - \frac{1}{2}x_1)}{\rho} = \frac{(2x_2 - x_1)}{x_1}(\tfrac{1}{2}x_1/\rho) = \frac{(2x_2 - x_1)}{x_1}\sin\phi_1. \qquad (4.25)$$

The importance of these for later discussion is that apparatus constants (x_1 and x_2) and a coordinate from the photographic plate ($\bar{z}$) determine ϕ_1 and ϕ_2.

Use of Eqs. (4.21) and (4.22) in Eq. (4.13) shows (since $\sin\phi$ and $\cos\phi$ are linearly independent functions) that

$$(p/v)\dot{\phi} = (e/c)B. \qquad (4.26)$$

One numerical fact makes the rest of the solution quite simple. Kaufmann argued[60] that $\dot{y}$ could be neglected compared to v with an error of less than 3%. However, even very simple geometrical considerations justify this. From Fig. 5 we see that

$$\bar{y}/(x_2 - x_1) \simeq \dot{y}/\dot{x}, \qquad (4.27)$$

since $\dot{y}$ remains constant outside the condenser plates and $\dot{x}$ decreases (slightly) as the electron moves upward (because the motion in the x–z plane is along a circle). From Kaufmann's data (see even Tables I–III), all the $\bar{y}$ were less than 0.2 cm so that

$$\dot{y}/\dot{x} \simeq 0.2/1.969 \simeq 0.1 \qquad (4.28)$$

and

$$v(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2} \simeq (\dot{x}^2 + \dot{z}^2)^{1/2}. \qquad (4.29)$$

In most cases y is even smaller and hence so is the error made in neglecting $\dot{y}$ compared to $\dot{x}$. Therefore we can write

$$v^2 = \rho^2\dot{\phi}^2 + \dot{y}^2 \simeq \rho^2\dot{\phi}^2. \qquad (4.30)$$

Equation (4.26) becomes

$$p = (e/c)B\rho = eBx_1/2c \sin\phi_1, \qquad (4.31)$$

which is a constant.

The remaining equation of motion, Eq. (4.14), becomes

$$\frac{p}{v}\frac{d^2y}{d\phi^2}\dot{\phi}^2 = eE$$

or

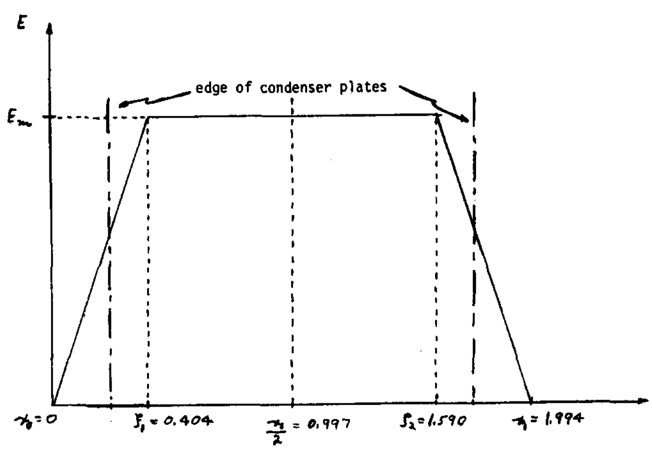

Fig. 8. Planck's representation of the E field.[55]

Table V. Planck's predictions from Kaufmann's 1905 data.[55]

$\bar{z}$	ϕ_1	u	$\bar{y}$	Abraham β	Abraham $\bar{y}_{\text{theo}}$	Lorentz β	Lorentz $\bar{y}_{\text{theo}}$
0.1354	1.977°	0.3871	0.0247	0.9747	0.0262	0.9326	0.0273
0.1930	2.810	0.5502	0.0378	0.9238	0.0394	0.8762	0.0415
0.2423	3.517	0.6883	0.0506	0.8689	0.0526	0.8237	0.0555
0.2930	4.237	0.8290	0.0653	0.8096	0.0682	0.7699	0.0717
0.3423	4.925	0.9634	0.0825	0.7542	0.0853	0.7202	0.0893
0.3930	5.623	1.100	0.1025	0.7013	0.1054	0.6728	0.1099
0.4446	6.325	1.236	0.1242	0.6526	0.1280	0.6289	0.1328
0.4926	6.962	1.360	0.1457	0.6124	0.1511	0.5924	0.1562
0.5522	7.735	1.510	0.1746	0.5685	0.1823	0.5521	0.1878

$$\frac{d^2y}{d\phi^2} = \frac{\rho c}{v}\frac{E}{B} = \frac{\rho c}{vB}E(\phi). \tag{4.32}$$

We must keep in mind that E is not a constant but is given by Eqs. (4.16) and (4.18). It is, however, symmetric about $\phi = 0$ [see Eq. (4.21) and Fig. 6]. Since $E = 0$ for $\phi > \phi_1$, we can use the symmetry of $y(\phi)$ about $\phi = 0$ to integrate Eq. (4.32) as

$$\left(\frac{dy}{d\phi}\right)_{\phi_1} = \frac{\rho c}{vB}\int_0^{\phi_1} E(\phi)d\phi. \tag{4.33}$$

If we denote by ϕ' that angle for which $x = \xi_2$, then we find

$$\left(\frac{dy}{d\phi}\right)_{\phi_1} = \frac{\rho c E_m}{vB}\left[\phi' + \left(b - \frac{ax_1}{2}\right)(\phi_1 - \phi') - ap(\cos\phi' - \cos\phi)\right]. \tag{4.34}$$

Since the trajectory $y = y(\phi)$ is a straight line for $\phi > \phi_1$, we have

$$\bar{y} = \left(\frac{dy}{d\phi}\right)_{\phi_1}(\phi_2 - \phi_1). \tag{4.35}$$

The logic of Planck's argument is now very easy to outline. Kaufmann had a set of nine data points (Table III). From the value of $\bar{z}$ one uses Eqs. (4.24) and (4.25) to find the corresponding values of ϕ_1 and ϕ_2. From Eq. (4.31) we can obtain the value of a quantity u which Planck introduces,

$$u \equiv m_0 c/p = (m_0/e)(2c^2 \sin\phi_1/x_1 B). \tag{4.36}$$

Notice that the value of u is given *directly* from the data and does not depend upon the specific form of the Lagrangian. Planck, like Kaufmann,[61] used for e/m_0 the value

$$e/m_0 = 1.878\times 10^7. \tag{4.37}$$

By use of the Lagrangians (4.5) and (4.10) Planck calculates p explicitly as

$$p(\text{Lorentz}) = m_0 v/(1-\beta^2)^{1/2}, \tag{4.38}$$

$$p(\text{Abraham}) = \frac{3}{4}\frac{m_0c^2}{v}\left[\left(\frac{1+\beta^2}{2\beta}\right)\ln\left(\frac{1+\beta}{1-\beta}\right) - 1\right]. \tag{4.39}$$

From these and Eq. (4.36) he extracts values of β for each theory. It is important to appreciate that the experimental value of $\bar{y}$ has not yet been used. Once β is known, Eqs. (4.34) and (4.35) can be employed to predict theoretical values of $\bar{y}$ for each theory. The results are given in Table V.

Planck (somewhat unhappily one senses) admitted that Abraham's theory yielded a better fit than did Lorentz's, although neither theory fit the data exactly. He concluded his paper with some general observations, one of which was extremely disconcerting. Equations (4.23)–(4.25), (4.34), and (4.35) taken together are independent of the form of the Lagrangian (and therefore of a choice between Abraham's and Lorentz's theory) and can be used, given the experimental values ($\bar{y}$,$\bar{z}$), to find β. For the first entry in Table V this yields $\beta = 1.033$, a value greater than unity. Since β had to be less than one in either theory, a strict falsificationist interpretation of Kaufmann's data would be that *both* theories are incorrect. (Incidentally, Kaufmann might have had suspicions of his own since one of his 1903 data points yielded $\beta = 1.04$ as listed in his table[46] and we earlier pointed out that the 1901 data point he discarded gave $\beta > 1$.)

Of course, Planck realized that there were auxiliary assumptions made in this analysis. In a 1907 paper[62] he focused on one of these. It appeared that there was reason to question how good a vacuum Kaufmann had been able to maintain during his runs. The β rays could ionize residual

Table VI. Planck's reanalysis upon correcting for the E field.[62]

$\bar{z}$	ϕ_1	u	$\bar{y}$	Abraham β	Abraham α	Lorentz β	Lorentz α
0.1354	1.977°	0.4226	0.0247	0.9655	18 840	0.9211	17 970
0.1930	2.810	0.6006	0.0378	0.9045	18 920	0.8572	17 930
0.2423	3.517	0.7515	0.0506	0.8424	18 770	0.7994	17 810
0.2930	4.237	0.9050	0.0653	0.7779	18 520	0.7414	17 650
0.3423	4.925	1.052	0.0825	0.7194	18 580	0.6891	17 800
0.3930	5.623	1.200	0.1025	0.6650	18 560	0.6400	17 860
0.4446	6.325	1.350	0.1242	0.6157	18 430	0.5953	17 820
0.4926	6.962	1.485	0.1457	0.5756	18 240	0.5586	17 710
0.5522	7.735	1.649	0.1746	0.5321	18 040	0.5186	17 590

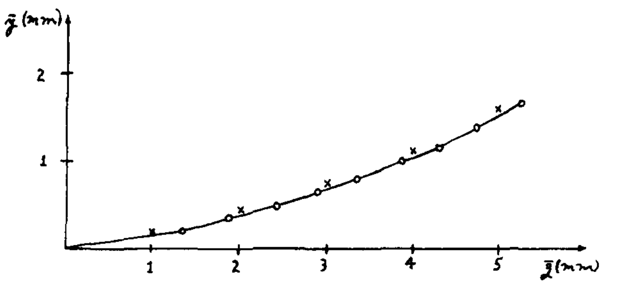

Fig. 9. Einstein's fit to Kaufmann's data.[65]

air molecules thus reducing the electric field strength E_m of Eqs. (4.16) and (4.17). Planck suggested reanalyzing Kaufmann's data, this time replacing Eq. (4.16) with

$$E = \alpha F(x), \tag{4.40}$$

where α is a free parameter to be determined to produce agreement for each data point $(\bar{y},\bar{z})$. In the previous analysis α had the value $E_m = 20\,130$ V/cm. He also used a more recently determined[63] value of e/m_0,

$$e/m_0 = 1.72\times 10^7. \tag{4.41}$$

The results are given in Table VI. It is apparent that for either theory $\alpha < E_m$, thus lending some credence to the ionization mechanism as being responsible for producing a field less than the idealized value Kaufmann had assumed. Since α represents the electric field strength between the plates, its value should be a characteristic of the apparatus and hence the same value for all the data taken. Therefore the fact that α for the Lorentz theory has less variation (2%) than for the Abraham theory (5%) allows us to conclude that Kaufmann's data favor the Lorentz–Einstein relativity theory. This is Planck's line of reasoning.[64] However, in any event it is a close call (2% vs 5%) and scarcely overwhelming evidence in favor of relativity. The important outcome of Planck's analysis was that Kaufmann's experiments no longer presented a stumbling block to the acceptance of relativity theory.

In a 1907 review article on relativity,[65] Einstein presented a graph (see Fig. 9) of $\bar{y}$ vs $\bar{z}$ for Kaufmann's data and compared it to those for relativity (with $e/m_0 = 1.878\times 10^7$). In this graph the circles represent Kaufmann's nine data points.[36] The solid curve appears to consist of straight-line segments joining these points. The crosses are Einstein's predictions for $\bar{y}$ when $\bar{z} = 1,2,...,5$ mm. Einstein's evaluation of this comparison was

"Considering the difficulty of the investigation, one would be inclined to take the agreement as satisfactory. The existing discrepancies are nevertheless systematic and beyond the limits of error of Kaufmann's experiment. That Kaufmann's calculations are error-free follows from the fact that Planck, using another method of calculation, was led to results which completely agree with those of Kaufmann."[66]

Table VII. Bestelmeyer's (1907) results.[63]

β	$(e/m)_{\text{exp.}}$	Abraham $e/m_0 =$ 1.720	Lorentz $e/m_0 =$ 1.733	Bucherer $e/m_0 =$ 1.713
0.195	1.697	1.694	1.700	1.690
0.247	1.678	1.678	1.679	1.677
0.322	1.643	1.647	1.640	1.651

Table VIII. Bucherer's (1909) results.[68]

β	e/m_0 Lorentz	e/m_0 Abraham
0.3173	1.752	1.726
0.3787	1.761	1.733
0.4281	1.760	1.723
0.5154	1.763	1.706
0.6870	1.767	1.642

He then acknowledges that the theories of Abraham[18] and of Bucherer[33] provide a better fit but offers his opinion that they have a small probability of being correct since they produce complicated expressions for the mass of a moving electron.

V. SUBSEQUENT DETERMINATIONS OF e/m_0

We have seen that even after Planck's reanalysis of Kaufmann's data, the issue was still not clearly decided in favor of either the Abraham or Lorentz theory. For completeness, we now indicate the results of e/m_0 determinations in the years following Kaufmann's experiments.[67] In 1907 Bestelmeyer[63] used the secondary cathode rays ejected from a metal by incident x rays. He measured e/m for these electrons that were first sent through crossed **E** and **B** fields acting as a velocity selector and then deflected by the magnetic field alone. His results are given in Table VII. (In this and subsequent tables all values of e/m have been multiplied by 10^{-7} for convenience.) The value of e/m_0 has been adjusted to give the best fit for each of the three theories. It is clear that for these rather small values of β no theory is definitively favored over the other two.

In 1909 Bucherer[68] used β rays from a radium flouride source and subjected them to crossed electric and magnetic fields, much as Bestelmeyer had done. In reducing his data, Bucherer extracted the value of e/m_0 for each measurement and listed these. That theory was to be preferred which yielded nearly constant values for e/m_0 as β varied. This is the criterion that subsequent experiments also used to judge one theory over another. At the beginning of his paper Bucherer states that while both classical electron theory (Abraham's) and relativity (Lorentz's) have had other empirical successes, his own theory of a constant-volume electron is contradicted by dispersion phenomena. It remains only to decide between Abraham's theory and Lorentz's. Table VIII shows that e/m_0 is more nearly constant for Lorentz's theory than for Abraham's. Unfortunately, there was a protracted controversy in the literature about the validity of Bucherer's conclusions because of fringing effects on his condenser plates.[69]

It was Neumann[70] in 1914 who definitively settled the question of which theory yielded the most nearly constant values of e/m_0. Using a refinement of Bucherer's method, he obtained 26 data points for $0.39152 \leqslant \beta \leqslant 0.80730$. The superiority of Lorentz's theory over Abraham's is manifest from Fig. 10 which Neumann published. These results were corroborated by Guye and Lavanchy[71] in 1915.

VI. CONCLUSIONS

Aside from the rather obvious conclusion that Kaufmann's 1905 data were not at first taken as providing sup-

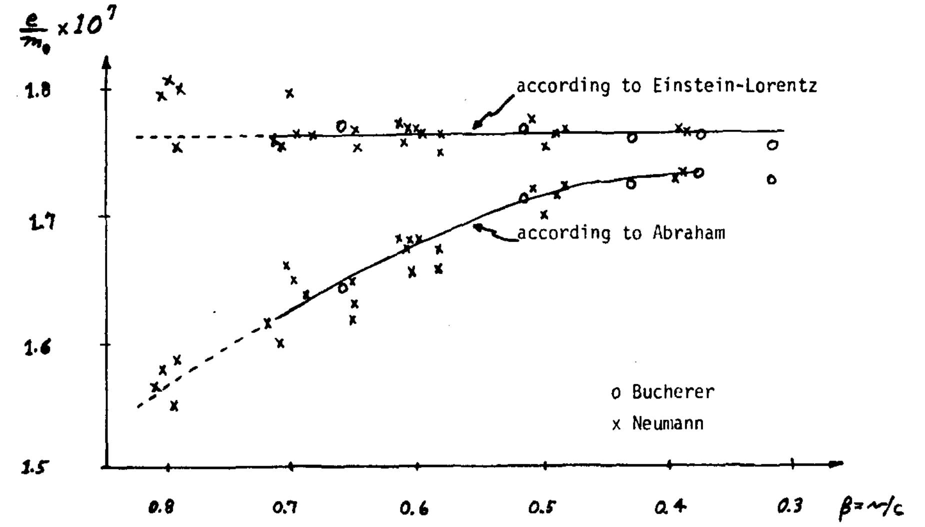

Fig. 10. Neumann's (1914) values for e/m_0.[70]

portive evidence for the special theory of relativity as opposed to classical theories, this case study illustrates several interesting features of the development and acceptance of scientific theory. The Kaufmann experiment, even though it initially appeared to refute special relativity, did not act as a crucial experiment in the sense that a strict falsificationist view of science would lead one to expect.[72] That is, science does not operate according to the simple scheme[73]

$$\text{hypotheses} \to \text{prediction} \to \text{refutation} \to \text{rejection of hypothesis.}$$

In fact, Planck, aware of Kaufmann's results, still developed a covariant formulation of Newton's second law because of the great potential and generality of relativity. Even after he had himself verified Kaufmann's calculations he did not reject that theory. Einstein went still further in that he took Kaufmann's data as *agreeing* with relativity. On the grounds that the formulas they produced were too complicated in appearance, he discounted the likelihood of the two classical models being correct in spite of their admittedly better agreement with that experiment.[74] By the time that Neumann's observations conclusively settled the e/m question in 1914, the special theory of relativity had already become firmly established through its many other successes. The lack of conclusive support for relativity in this one area no longer had any aspect of a crucial experiment. This question had been reduced to the status of a minor skirmish or niggling detail that would be, and in fact was, taken care of by subsequent measurements.

ACKNOWLEDGMENTS

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Note added in proof: I came across Arthur I. Miller's article "On Some Other Approaches to Electrodynamics in 1905" in *Some Strangness in the Proportion* (Addison-Wesley, Reading, MA, 1980). Readers of this paper will want to consult this article as well as Miller's new book *Albert Einstein's Special Theory of Relativity* (Addison-Wesley, Reading, MA, 1981).

[1]R. E. Peierls, "Field Theory since Maxwell," in *Clerk Maxwell and Modern Science*, edited by C. Domb (Athlone, London, 1963), p. 28. See also the subsequent discussion on pp. 28–30.

[2]See, for example, J. D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1975), pp. 217–218; J. S. Trefil, *Physics as a Liberal Art* (Pergamon, New York, 1978), pp. 184–186.

[3]A. M. Bork, Am. J. Phys. **31**, 854–859 (1963).

[4]J. Bromberg, Arch. Hist. Exact Sci. **4**, 218–234 (1967).

[5]See, for example, R. M. Eisberg, *Fundamentals of Modern Physics* (Wiley, New York, 1961), pp. 63–64; H. Semat and J. R. Albright, *Introduction to Atomic and Nuclear Physics* (Holt, Rinehart and Winston, New York, 1972), p. 88.

[6]T. S. Kuhn, *Black-Body Theory and the Quantum Discontinuity: 1894–1912* (Clarendon, Oxford, 1978).

[7]M. Jammer, *The Conceptual Development of Quantum Mechanics* (McGraw-Hill, New York, 1966), pp. 10–22.

[8]M. J. Klein, Arch. Hist. Exact Sci. **1**, 459–479 (1962). Actually, Planck seems to have been unaware of the Rayleigh–Jeans formula, although he did know about the experimental data for low-frequency cavity radiation.

[9]See, for example, P. J. Brancazio, *The Nature of Physics* (Macmillan, New York, 1975), p. 529; Trefil, Ref. 2, pp. 290–291.

[10]N. Bohr, Philos. Mag. **26**, 1–25, especially pp. 4–5 (1913).

[11]J. L. Heilbron and T. S. Kuhn, "The Genesis of the Bohr Atom," in *Historical Studies in the Physical Sciences* (University of Pennsylvania, Philadelphia, 1969), Vol. 1, pp. 211–270, especially pp. 268–270. Even this energy quantization argument may have been *post hoc* since Bohr "knew" the form required for E_n from the available spectral data.

[12]Ref. 10, p. 15.

[13]See, for example, Semat and Albright, Ref. 5, p. 44; R. D. Rush, *Introduction to Atomic and Nuclear Physics* (Appleton–Century–Crofts, New York, 1958), p. 143.

[14]The biographical material for this section is taken from *Dictionary of Scientific Biography*, edited by C. C. Gillispie (Scribner's, New York, 1970).

[15]In order to facilitate comparison with the original papers, we write all electrodynamics expressions in Heaviside–Lorentz units. For a comparison of this with the mks and other systems see Jackson, Ref. 2, p. 818.

[16]A very readable elementary account of the electromagnetic mass question and a summary of early e/m measurements can be found in J. D. Stranathan, *The "Particles" of Modern Physics* (Blakiston, Philadelphia, 1942), pp. 131–133; 110–119; 138–141; see also, H. A. Lorentz, *Lectures on Theoretical Physics*, translated by L. Silberstein and A. P. Trivelli (Macmillian, London, 1931), Vol. III, Chap. VII.

[17]S. Goldberg, Arch. Hist. Exact Sci. 7, 7–25, especially p. 15. (1970–71).

[18]M. Abraham, Ann. Phys. **10**, 105–179 (1903).

[19]Ref. 18, p. 143. We need consider only the case in which the electron is moving through a vacuum (or the "ether") so that **D** = **E** and **H** = **B**.

[20]Jackson, Ref. 2, pp. 596, 550.

[21]The necessary manipulations are similar to those ordinarily used to obtain expressions for the electrostatic and magnetostatic energies in terms of ρ, Φ, **J**, and **A**. See Jackson, Ref. 2, pp. 158–159, 215–216.

[22]What is actually necessary for the validity of the discussion is that changes in the motion be slow enough that variations taking place in a time a/c can be neglected, where a is the radius of the electron. See H. A. Lorentz, *The Theory of Electrons* (Teubner, Leipzig, 1909); reprinted (Dover, New York, 1952), p. 37.

[23]H. A. Lorentz, *Versuch Einer Theorie der Electrischen und Optischen Erscheinungen in Bewegten Körpern* (Brill, Leiden, 1895), reprinted in Lorentz's *Collected Papers* (Nijhoff, The Hague, 1937), Vol. V, pp. 1–137, especially pp. 35–38; see also Ref. 22, pp. 35–36. The origin of what eventually became known as the Lorentz transformations is evident in Eq. (2.19) of the text.

[24]J. C. Maxwell, *A Treatise on Electricity and Magnetism, Vol. I*, 3rd ed. (Clarendon, Oxford, 1892); reprinted (Dover, New York, 1954), article 152, pp. 239–240; see also W. R. Smythe, *Static and Dynamic Electricity*, 2nd ed. (McGraw-Hill, New York, 1950), p. 113 (with $b = c$).

[25]Jackson, Ref. 2, pp. 46–48.

[26]Jackson, Ref. 2, p. 575.

[27]Reference 18, p. 144.

[28]Reference 18, pp. 150–151; Ref. 22, pp. 37–38.

[29]Reference 18, p. 146.

[30]Incidentally, the expression for W_e is essentially the same if the electron's charge is assumed to be uniformly distributed throughout its volume rather than over its surface. One need only multiply W_e for a spherical shell by a factor of 6/5 to obtain that for a spherical volume of uniform charge. This is very easily seen for a sphere by integrating Eq. (2.25) directly to obtain $W(\text{solid sphere}) = (6/5)(e^2/2_a)$. See also, Ref. 18, p. 146, footnote 2.

[31]Once again, just as in the previous footnote, if the calculation of Eq. (2.3) is done for a uniform spherical volume distribution of charge, the final result for the m of Eq. (2.5) [as for m_0 of Eq. (2.61)] is simply multiplied by a factor of 6/5. However, in either case we absorb the factor with e^2, a, and c into the definition of m_0 so that, as far as the functional dependence of m on β is concerned, it really makes no difference whether we assume a surface or volume charge distribution.

[32]Ref. 18, p. 152.

[33]A. H. Bucherer, *Mathematische Einführung in die Elektronentheorie* (Teubner, Leipzig, 1904), pp. 57–58; see also, Ref. 22, pp. 219–220.

[34]G. F. Searle, Philos. Mag. **44**, 329–341, especially p. 333 (1897).

[35]Ref. 22, pp. 210–213.

[36]W. Kaufmann, Ann. Phys. **19**, 487–553, especially p. 496 (1906).

[37]E. Whittaker, *A History of the Theories of Aether and Electricity, Vol. II* (Philosophical Library, New York, 1951); reprinted (Humanities, New York, 1973), p. 3.

[38]A detailed discussion is given by Lorentz, Ref. 16, pp. 273–274.

[39]In obtaining Eq. (3.13) we have neglected the small gap between the source and the bottom edge of the condenser plate and that between its top edge and the diaphragm (see Figs. 3–6). This will be treated more carefully in Planck's analysis. Kaufmann himself, however, did make the necessary geometrical corrections in his paper (Ref. 41, below).

[40]M. Planck, Verh. Dtsch. Phys. Ges. **8**, 136–141 (1906).

[41]W. Kaufmann, Nachr. K. Ges. Wiss. Goettingen (hereafter to be referred to as Goettingen Nachr.) **2**, 143–145 (1901); most of this paper also appears in English translation, "Magnetic and Electric Deflectability of the Becquerel Rays and the Apparent Mass of the Electron," in *The World of the Atom, Vol. I*, edited by H. A. Boorse and L. Motz (Basic, New York, 1966), pp. 506–512.

[42]In a rather curious move, H. A. Boorse and L. Motz, Ref. 41, pp. 505, 511, present a graph in which the curve for $m = m_0(1 - \beta^2)^{-1/2}$ is compared to the "data" points generated by Kaufmann's theoretical interpretation (essentially column six of Table I) of the experimental data rather than with the actual data (column four) themselves.

[43]W. Kaufmann, Goettingen Nachr. **3**, 291–296 (1902); Phys. Z. **4**, 54–57 (1902).

[44]M. Abraham, Phys. Z. **4**, 57–62 (1902).

[45]Ref. 43 (Phys. Z.), p. 56. My translation. Kaufmann's words are "Die Masse der die Becquerelstrahlen bildenden Elektronen ist von der Geschwindigkeit abhängig; die Abhangigkeit ist genau darstellbar durch die Abrahamasche Formel. Es ist demnach die Masse der Elektronen rein electromagnetischer Natur."

[46]W. Kaufmann, Goettingen Nachr. **4**, 90–103 (1903).

[47]W. Kaufmann, Sitzungsber. K. Preuss. Akad. Wiss. **2**, 949–956 (1905).

[48]Actually, these nine data points represent grouped averages of nearly fifty individual measurements. (See Ref. 36, pp. 519–522, 524.)

[49]Kaufmann gives a detailed discussion of this reduction procedure in Ref. 36, pp. 524–530.

[50]Ref. 47, p. 956. My translation. Kaufmann's words are "Die vorstehenden Ergebnisse sprechen entschieden gegen die Richtigkeit der Lorentzschen und somit auch der Einsteinschen Grundannahme. Erachtet man diese Grundannahme als hierdurch widerlegt, so wüde der Versuch, die ganze Physik, einschliesslich der Elektrodynamik und der Optik auf das Prinzip der Relativbewegung zu gründen, einstweilen als missglückt zu bezeichnen sein. Eine Entscheidung zwischen den Theorien von Abraham und Bucherer ist einstweilen unmöglich und scheint durch Beobachtungen der oben beschriebenen Art wegen der weitgehenden numerischen Übereinstimmung der Werte von $\psi(\beta)$ überhaupt nicht erreichbar. Ob die BuchererscheFormel für die Optik bewegter Körper im Bereiche der möglichen Beobachtungen dasselbe leistet, wie die Lorentzsche, muss erst ermittelt werden."

[51]H. A. Lorentz, Proc. R. Acad. Sci. Amsterdam **6**, 809–836 (1904); also appears in H. A. Lorentz, A. Einstein, H. Minkowski, and H. Weyl, *The Principle of Relativity* (Dover, New York), pp. 11–34.

[52]See the tables at the end of Ref. 51.

[53]The logical and philosophical implications of Planck's reappraisal has been carefully and elegantly discussed by E. Zahar, " 'Crucial' Experiments: A Case Study," in *Progress and Rationality in Science*, edited by G. Radnitzky and G. Andersson (Reidel, Dordrecht, 1978), pp. 71–97. It was Zahar's paper which initiated my own interest in the Kaufmann experiments.

[54]Ref. 40, p. 136. My translation. Planck's words are "Freilich scheint diese Frage durch die neusten wichtigen Messungen von W. Kaufmann bereits erledigt zu sein, und zwar in negativem Sinne, so dass sich jede weitere Untersuchung erübrigen würde."

[55]M. Planck, Verh. Dtschen. Phys. Ges. **8**, 418–432 (1906).

[56]In Ref. 53, pp. 81–94, Zahar gives a clear and detailed presentation of Planck's dynamical development.

[57]Ref. 36, p. 503, Fig. 4. Notice also the correction given for x_0 as 0.0111 cm on p. 507. This has already been incorporated in our choice $x_0 = (0,0,0)$.

[58]Ref. 36, p. 515, Fig. 9.

[59]Ref. 55, pp. 420–421. We have not introduced Planck's auxiliary variables here.

[60]Ref. 36, p. 527.

[61]Ref. 36, p. 551.

[62]M. Planck, Verh. Dtsch. Phys. Ges. **9**, 301–305 (1907).

[63]A. Bestelmeyer, Ann. Phys. **22**, 429–447 (1907).

[64]Ref. 62, p. 304.

[65]A. Einstein, Jahrb. Radioakt. Elektron. **4**, 411–462 (1907).

[66]Ref. 65, p. 439. My translation. Einstein's words are "In Anbetracht der Schwierigkeit der Untersuchung möchte man geneigt sein, die Übereinstimmung als eine genügende anzusehen. Die vorhandenen Abweichungen sind jedoch systematisch und erheblich ausserhalb der Fehlergrenze der Kaufmannschen Untersuchung. Dass die Berechnungen von Herrn Kaufmann fehlerfrei sind, geht daraus hervor, dass Herr Planck bei Benutzung einer anderen Berechnungsmethode zu Resultaten geführt wurde, die mit denen von Herrn Kaufmann durchaus übereinstimmen."

[67]A table listing values of e/m_0 determined between 1908 and 1939 is given in Stranathan, Ref. 16, p. 130. A detailed discussion of the measurements and experimental procedures of Bestelmeyer *et al.* and Lavanchy is presented in Lorentz, Ref. 16, pp. 274–288.

[68]A. H. Bucherer, Ann. Phys. **28**, 513–536 (1909).

[69]See Lorentz, Ref. 16, pp. 279–280.

[70]G. Neumann, Ann. Phys. **45**, 529–579 (1914).

[71]C. E. Guye and C. Lavanchy, Compt. Rend. **161**, 52–55 (1915).

[72]See Ref. 53, especially pp. 71–73.

[73]For a discussion of a model of science which goes beyond Popper's falsificationism, see I. Lakatos, "Falsification and the Methodology of Scientific Research Programs," in *Criticism and the Growth of Knowledge*, edited by I. Lakatos and A. Musgrave (Cambridge University, London, 1970), pp. 91–196; I Lakatos, "History of Science and its Rational Reconstructions," in *Method and Appraisal in the Physical Sciences*, edited by C. Howson (Cambridge University, Cambridge, 1976), pp. 1–39.

[74]Lakatos, Ref. 73, *Criticism and the Growth of Knowledge*, p. 137, has said about the autonomy of a developing theory, "Which problems scientists working in powerful research programmes rationally choose, is determined by the positive heuristic of the programme rather than by psychologically worrying (or technologically urgent) anomalies. The anomalies are listed but shoved aside in the hope that they will turn, in due course, into corroborations of the programme."

High-sensitivity microwave optics

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Microwave optics experiments operating at a 3.33-cm wavelength (9 GHz) are described that have an overall signal gain of 58 dB, which is achieved by using a 1000-Hz square-wave modulated reflex klystron and a high-gain, narrowband amplifier following a point-contact detector. A metal-plate electromagnetic lens is employed that has a gain of 22.5 dB relative to an isotropic radiator, produces a beam collimated to within 12 deg between half-power points, and has a 3.7% bandwidth around a center frequency of 9 GHz. This basic system was used to developed microwave versions of the Michelson interferometer, Bragg reflection, Brewter's law and total internal reflection, and Young's interference experiment. Measured radiation intensities for four signal sources are shown to lie well within Federal performance regulations established in 1971.

INTRODUCTION

The use of microwaves to illustrate many beautiful properties of physical optics has been discussed in these pages for more than three decades.[1–4] An advantage of microwaves is that their long wavelengths compared to light render them unsusceptible to disturbances caused by small mechanical vibrations. Therefore, microwave experiments can be conveniently performed on ordinary laboratory tables; whereas certain optics experiments (e.g., those employing interferometry and holographic techniques) require vibration-isolation tables.

A bulk-effect diode operating as a continuous-wave microwave signal source has recently been described[4] with a point-contact detector connected to a microammeter, affording extreme simplicity and a minimum of electronics. At the 1-cm wavelength (30 GHz) utilized, the microwave components are especially convenient because of their small size and high gain. A parabolic reflector ten wavelengths in diameter is only 10 cm and has a gain of 1000 (30 dB) relative to an isotropic radiator. However, while Andrews[4] correctly states that the gain for the combined transmitter and receiver reflectors is one million (60 dB), the overall signal gain is less than this value, as will be explained in the next section. Furthermore, at this short wavelength an uncertainty in a spatial setting as small as 1 mm amounts to a 10% error, in terms of wavelength. Careful mechanical settings are therefore important with 1-cm microwaves.

This paper describes a 3.33-cm wavelength (9-GHz) microwave system, shown in block form in Fig. 1, that achieves a high overall signal sensitivity and a well-collimated beam with moderate-size equipment. An overall signal gain of 58 dB is obtained by employing square-wave

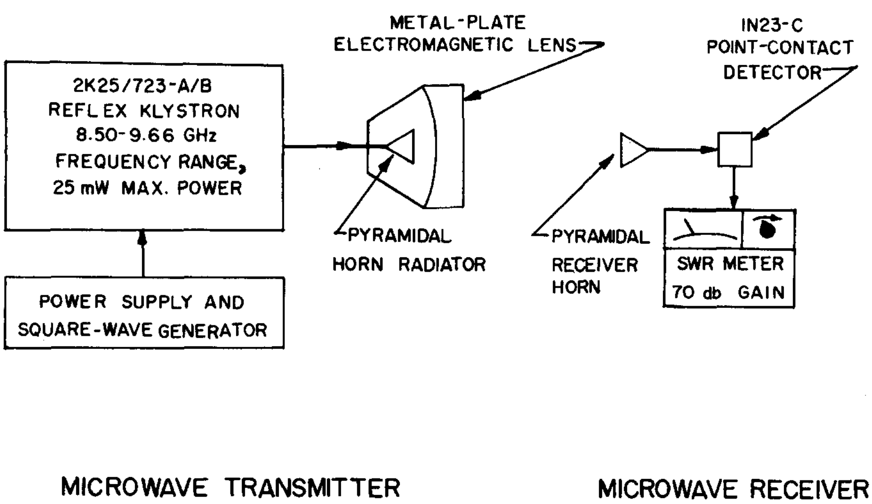

Fig. 1. Block diagram of the microwave system.

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