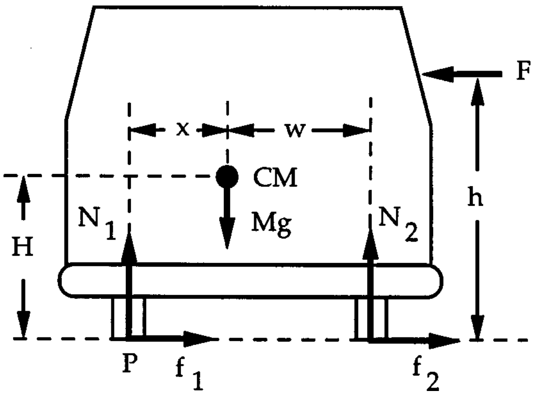

Fig. 1. The forces acting on a vehicle on a horizontal surface (viewed from the rear), when subject to a horizontal force F.

about the CM is then $f_1H-N_1x=M(v^2H/R-gx)$. Consequently, the condition for rolling is that $v^2/R=gx/H$.

The difference between these two sets of results can be reconciled if one changes to a reference frame where the vehicle is at rest. In this frame, the vehicle experiences a horizontal centrifugal force $F=Mv^2/R$ acting through the CM. The situation is then the same as described in the first example where an external force F is applied to the vehicle, provided that h is taken as the height of the CM above the road surface. If $\mu<x/H$, the vehicle will slide out of control when $v^2/R=\mu g$. If $\mu>x/H$, the vehicle will roll over when $v^2/R=gx/H$. For most vehicles, $x/H\sim1.1$ and $\mu<1$. Most vehicles are therefore stable against rolling, but a vehicle that is normally stable can still roll if the load is distributed in such a way that $\mu>x/H$ or if the vehicle slides into a curb and is then tripped by the curb.

An alternative solution of the problem, from an external observer's point of view, is that the torque acting about the point P is not zero since the angular momentum of the vehicle about point P does not remain constant. The velocity changes direction with time and changes vectorially by an amount $\Delta v=v\Delta s/R$ while the vehicle traverses an arc of length Δs. During the same time, the angular momentum about P changes by $(M\Delta v)H=MvH\Delta s/R$. The change in v is directed horizontally toward the center of the circular path, but the *change* in angular momentum and the torque are both parallel to the velocity vector. The rate of change of velocity is v^2/R and the rate of change of the angular momentum is Mv^2H/R. The torque about P is therefore $Mgx-N_2(x+w)=Mv^2H/R$ so $N_2=M(gx-v^2H/R)/(x+w)$, as obtained above. Personally, I prefer the centrifugal force argument since it is simpler, more intuitive, and it preserves the analogy with the rectangular block. Readers who dislike centrifugal forces should read the article by De Jong[3] who presents a case for avoiding the term ''centripetal force'' since it can be interpreted by students as a fictitious additional force.

[1]D. B. Swinson, ''Vehicle Rollover,'' Phys. Teach. **33**, 360–366 (1995).
[2]R. P. Bauman, ''What is centrifugal force?'' Phys. Teach. **18**, 527–529 (1980).
[3]M. L. De Jong, ''What name should be used for the force required to move a mass in a circle?'' Phys. Teach. **26**, 470–471 (1988).
[4]J. M. Goodman and D. S. Chandler, ''A rotating coordinate frame visualizer,'' Am. J. Phys. **39**, 1129–1133 (1971).

Ampère was not the author of ''Ampère's Circuital Law''

Herman Erlichson
Department of Engineering Science and Physics, The College of Staten Island, The City University of New York, Staten Island, New York 10314

(Received 31 August 1998; accepted 12 October 1998)

The original motivation for this note was an interest in finding out exactly what the contribution of Ampère was to the subject of the magnetic fields due to steady currents. At the outset I had the notion that Ampère was responsible for the Maxwell equation commonly labelled ''Ampère's Circuital Law.'' This law is the familiar

$$\oint_C \mathbf{B}\cdot\mathbf{ds}=\mu_0 i, \tag{1}$$

that is, that the line integral of the magnetic induction **B** around a closed path C equals μ_0 times the current crossing the area bounded by C. I had not, for example, reflected on the strange fact that at the time of Ampère, circa 1820, the use of the vector **B** had not yet appeared on the electromagnetic scene, so that it was chronologically impossible that Eq. (1) should really be due to Ampère.

I consulted a recently published biography of Ampère[1] and found therein a statement that the Ampère Circuital Law was not due to Ampère:

> ''Ampère's own achievements should not be confused with a quite different law that is misleadingly named after him. Sometimes referred to as 'Ampère's circuital law' or more simply as 'Ampère's law,' this law depends upon field theoretic concepts and is often stated in the form:
>
> $$\oint \mathbf{B}\cdot\mathbf{ds}=\mu\Sigma I$$
>
> ... Maxwell discussed this law in his *Treatise on Electricity and Magnetism* and he correctly did not attribute it to Ampère.... The contrast between Am-

père's own theorems and 'Ampère's Law' reflects the conceptual gap that separates him from the field theorists of Maxwell's generation.''[2]

It is clear that ''Ampère's Circuital Law'' is not due to Ampère. It expresses a property of the vector **B**.

I found a very interesting book entitled *Early Electrodynamics, The First Law of Circulation*, written by R. A. R. Tricker.[3] In this book, what is known in the U.S.A. as Ampère's Circuital Law is called ''the first law of circulation.'' This confirmed the idea that the title of ''Ampère's Circuital Law'' was a misnomer.

I also investigated two papers in the French journal, *Revue d'Histoire des Sciences et de leurs Applications*. The first, a paper by Hamamdjian,[4] defended the idea that there was some justification for calling the First Law of Circulation by the title of ''Ampère's Circuital Law'' or ''Ampère's theorem.'' Hamamdjian claimed that Ampère's theorem ''expresses, in the best possible way the essence of the thinking of Ampère on magnetism, electromagnetism and electrodynamics, the substance of his deepest intuitions and convictions.''[5] Hamamdjian admits that the theorem was not formulated by Ampère, but rather by Maxwell. However, he said that to to find the theorem Maxwell based himself on Ampère's work, especially on Ampère's concept of the magnetic shell. Hamamdjian also based his claim on an unpublished manuscript by Liouville containing notes on lectures given by Ampère. These notes concerned the topic of the line integrals of forces around closed paths, and specifically the line integral of the magnetic force on an isolated magnetic pole as it moved around a current-carrying wire. J-P. Mathieu subsequently published a paper in the same journal[6] where he at least partly challenged Hamamdjian's paper and attributed the First Law of Circulation to Maxwell. Specifically, Mathieu gave as the first expression of the First Law of Circulation a statement by Maxwell in 1856 (note that this is 30 years after Ampère's death in 1836) as follows:

''the total intensity of magnetizing force in a closed curve passing through and embracing the closed current is constant, and may therefore be made a measure of the quantity of the current.''[7] Mathieu found that the First Law of Circulation (Ampère's Circuital Law) was a consequence of Maxwell's desire to establish a field theory. He argued that what he called the ''local, vector form of Ampère's theorem,'' i.e.,

$$\nabla \times \mathbf{H} = \mathbf{j},$$

owed nothing to Ampère.

We agree with Mathieu's criticism of Hamamdjian's paper. Ampère's Circuital Law concerns the line integral of **B**, a magnetic field vector which is absent from Ampère's work. Ampère developed an action-at-a-distance theory of a Newtonian central force which acted between infinitesimal directed current elements. Ampère's formula is the mathematical expression of this central force and was considered by Ampère as the heart of his *electrodynamics*, a term which he coined.

The incorrect assignment of the name Ampère's Circuital Law was probably due to the desire to associate an important law of magnetism with the name of the historical father of electrodynamics. We must remember that Ampère's formula, the formula for the force between two infinitesimal current elements, on which Ampere placed so much importance as the foundational law of electrodynamics, has received hardly any attention at all in our century (Tricker's book is the exception). Indeed, this was already largely the case in the second half of the nineteenth century. Maxwell himself, who agreed with Ampère that his formula was truly foundational, nevertheless did not use this formula but instead used the concept of the electromagnetic field as foundational. Contrast Maxwell's praise for Ampère's formula in his *Treatise*:

> ''The experimental investigation by which Ampère established the laws of the mechanical action between electric currents is one of the most brilliant achievements in science.
>
> The whole, theory and experiment, seems as if it had leaped, full grown and fully armed, from the brain of the 'Newton of electricity.' It is perfect in form, and unassailable in accuracy, and it is summed up in a formula from which all the phenomena may be deduced, and which must always remain the cardinal formula of electro-dynamics,''[8]

with, in the same *Treatise*, his considering Ampère's action-at-a-distance formula as only worth an ''outline'' before continuing with his discussion of the Faraday–Maxwell field theory:

> ''We have considered in the last chapter the nature of the magnetic field produced by an electric current, and the mechanical action on a conductor carrying an electric current placed in a magnetic field. From this we went on to consider the action of one electric circuit upon another, by determining the action of the first due to the magnetic field produced by the second. But the action of one circuit upon another was originally investigated in a direct manner by Ampère almost immediately after the publication of Oersted's discovery. We shall therefore give an outline of Ampère's method, resuming the method of this treatise in the next chapter.''[9]

This ambiguity of Maxwell concerning Ampère was not lost on Heaviside, who in 1888 remarked:

> ''It has been stated, on no less authority than that of the great Maxwell, that Ampère's law of force between a pair of current elements is the cardinal formula of electrodynamics. If so, should we not be always using it? Do we *ever* use it? Did Maxwell in his Treatise? Surely there is some mistake. I do not in the least mean to rob Ampère of the credit of being the father of electrodynamics: I would only transfer the name of cardinal formula to another due to him, expressing the mechanical force on an element of conductor supporting current in any magnetic field—the vector product of current and induction. There is something real about it; it is not like his force between a pair of unclosed elements; it is fundamental; and, as everybody knows, it is in continual use, either actually or virtually (through electromotive force), both by theorists and practicians.''[10]

Of course, it is clear that Heaviside's ''the vector product of current and induction'' should, strictly speaking, not be attributed to Ampère, since the ''vector product'' and the magnetic ''induction'' were foreign to Ampère. Nevertheless, in the spirit of the modern Biot-Savart law, it would indeed make sense to call this Ampère's Law. This was precisely the title given to it in a well-known American physics text of the 1930s by Haussman and Slack. In that text one reads:

> "For a straight current-carrying wire of length l that is perpendicular to the flux of constant density, the force in mks units is
>
> $$F=Bil$$
>
> where F is in newtons, B in webers per square meters, i in amperes, and l in meters... . The foregoing expression is a mathematical statement of Ampère's Law and is the key equation of electromagnetism; it can be applied to all forms of circuits and is the operating principle of electric motors and electromagnetic devices."[11]

This statement by Haussman and Slack, in a widely used text, surely indicates the persistence of the desire to associate the name of Ampère with some important law of electromagnetism.

As closely as we can tell it was around the period of the first edition of the Haussman and Slack text that the myth of "Ampère's Circuital Law" became part of the established repertoire of American texts. The first instance we have been able to find was in the influential 1940 text by N. H. Frank of M.I.T. wherein it is stated "There is a a more general relation, than Eq. (24) between the magnetic intensity H and the steady current i which produces it, and this relation is known as the *circuital law*."[12]

The law which is currently called "Ampère's Law," or "Ampère's Circuital Law" is a misnomer and should not be attributed to Ampère. The law which has a proper historical basis to be called Ampère's Law is the force law for the magnetic force on a current element, which expresses the fact that this force is perpendicular to the element.

[a]Electronic mail: erlichson@postbox.csi.cuny.edu

[1]James R. Hoffman, *André-Marie Ampère* (Cambridge U. P., New York, 1996), first published (Blackwell, Oxford, 1995).

[2]*Ibid.*, p. 349.

[3]R. A. R. Tricker, *Early Electrodynamics, The First Law of Circulation* (Pergamon, Oxford, 1965).

[4]P-G. Hamamdjian, "Contribution d'Ampère au 'théorème d'Ampère,'" Revue d'Histoire des Sciences et de leurs Applications **31**, 249–268 (1978).

[5]*Ibid.*, p. 250.

[6]J-P. Mathieu, "Sur le théorème d'Ampère," Revue d'Histoire des Sciences et de leurs Applications **43**, 333–338 (1990).

[7]*Ibid.*, p. 335 (Maxwell quotation from his paper entitled "On Faraday's lines of force").

[8]J. C. Maxwell, *A Treatise on Electricity and Magnetism* (Clarendon, Oxford, 1891; Dover, New York, 1954), Vol. 2, p. 175.

[9]*Ibid.*, Vol. 2, p. 158.

[10]As quoted by E. Whittaker in his *A History of the Theories of Aether and Electricity* (two volumes, originally published by Nelson and Sons, London, 1910) (revised and enlarged in 1951, reprinted as a Harper Torchbook in 1960), p. 88. Whittaker gave the source of the Heaviside quotation as *Electrician* (28 Dec. 1888); O. Heaviside's, *Electrical Papers*, ii, p. 500.

[11]E. Hausman and E. P. Slack, *Physics* (van Nostrand, Princeton, NJ, 1935; 2nd ed. 1939; 3rd ed. 1948, 4th ed. 1957).

[12]N. H. Frank, *Introduction to Electricity and Optics*, 1st ed. (McGraw–Hill, New York, 1940), p. 103.

Comment on "Specific Heat Revisited," by C. A. Pizarro, C. A. Condat, P. W. Lamberti, and D. P. Prato [Am. J. Phys 64 (6), 736–744 (1996)]

V. Granados[a] and N. Aquino
Departamento de Física, Universidad Autónoma Metropolitana-Iztapalapa, Apartado Postal 55-534, México D.F. 09340, México

(Received 3 September 1998; accepted 17 November 1998)

We want to comment on some of the results of Pizarro *et al.*,[1] in particular the problem of the specific heat of a particle in a box.

As is well known, the eigenfunctions and energy eigenvalues of the problem are obtained straightforwardly in many textbooks.[2] The analytical calculation of the partition function Z [their Eq. (39)] and the expression for the specific heat C_v [their Eq. (40)] are neither so common nor easy. To have a rough idea of how lower states contribute to the specific heat, we compute C_v including only the ground state and the first excited state in the partition function Z as function of KT/E_1; the result of this calculation is in curve (a) of Fig. 1. We repeated the calculation of C_v including the second excited state, too, obtaining curve (b); finally, we made the calculation with the necessary terms to obtain a convergent series with a precision of 1×10^{-5}, and the result is shown in curve (c).

It is easily seen that our results differ from those obtained

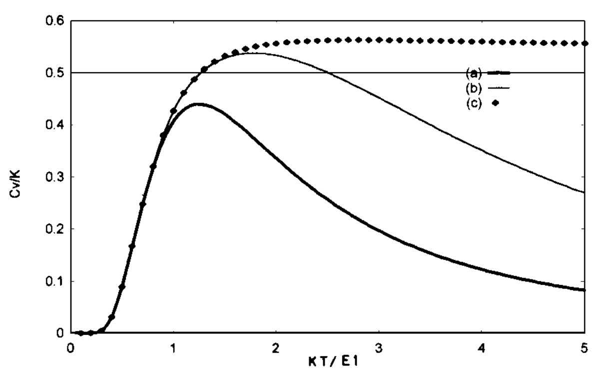

Fig. 1. Specific heat as function of KT/E_1, where $E_1=\pi^2\hbar^2/2ma^2$ is the ground state energy of a particle in a box of length a. Curve (a) was computed including only energies for the ground and first excited state in the partition function Z, whereas, curve (b) was computed including second excited state too. Curve (c) is the exact result. The curves do not show peaks, and curve (c) increases above 0.5 C_v/K and then decreases asymptotically to the classical limit 0.5 C_v/K.

