ponsible for the large value of F_c observed. The observations in the [110] specimen can be understood on the similar model of the electron transfer from the light mass ($m^* = 1.0\, m_t$) [1$\bar{1}$1] and [$\bar{1}$11] valleys, to the heavy mass ($m^* = 2.73\, m_t$) [111] and [11$\bar{1}$] vallyes. The absence of the increase of F_c in the [100] specimens is due to the absence of the splitting of the valley energies by the application of the stress in the [100] direction.

The variations of R and ρ with the strength of the [111] oriented electric field were measured. The values of R and ρ were found to decrease rapidly with the increase of the electric field near the breakdown and R/ρ was observed to increase and tend to saturate. The mobility of electrons in the conduction band was estimated from the saturation value of R/ρ. It was found that the application of the stress of 8×10^8 dynes/cm^2 reduced the mobility to one fourth of that observed at $X = 0$. An interesting thing is that the relation, $(R/\rho) \times F_c{}^2 \approx$ const, still holds. This fact may suggest that the increase of F_c with the stress is caused predominantly through the decrease of the electron mobility. It should, however, be noted that the decrease of the mobility observed can not be explained quantitatively by the increase of m^* of electrons in the hottest valleys unless the remarkable increase of the collision time in the neutral impurity scattering with the stress is assumed.

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FORCE ON A CURRENT LOOP

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Two models for a microscopic current loop are examined. In each case the force, which is related to the acceleration through Newton's law, is found to agree with the force on a magnetic-charge dipole of the same magnetic moment.

Tellegen [1] has suggested that the force exerted on a pair of magnetic charges in a time-variant electromagnetic field is different from that on a circulating current loop. His expression for the force exerted on a magnetic-charge dipole model of moment $\boldsymbol{m}$ is

$$F_d = \mu_0 \boldsymbol{m} \cdot \nabla \boldsymbol{H} - \frac{\partial \boldsymbol{m}}{\partial t} \times \frac{\boldsymbol{E}}{c^2}, \qquad (1)$$

whereas his expression for the force on a current loop of the same magnetic moment amounts to

$$\boldsymbol{F}' = \mu_0 \nabla \boldsymbol{H} \cdot \boldsymbol{m} . \qquad (2)$$

We have shown [2], however, that a circulating-current loop in an electric field has a momentum which, when changed, requires a force. This momentum is $-(\boldsymbol{E} \times \boldsymbol{m})/c^2$, so that the force leading toward acceleration of the current loop should be

$$\boldsymbol{F} = \mu_0 \nabla \boldsymbol{H} \cdot \boldsymbol{m} + \frac{\partial}{\partial t} \left[\frac{\boldsymbol{E} \times \boldsymbol{m}}{c^2} \right] . \qquad (3)$$

It is seen from Maxwell's equations that eqs. (1) and (3) are equal.

This problem has also been discussed independently by Costa de Beauregard [3] and Shockley and James [4]. Costa de Beauregard assumed the equivalence of the two models, and chose eq. (1) rather than eq. (2) as the correct force. He has also shown that this choice is compatible with conservation of momentum. Shockley and James deduced the need for the final term in eq. (3) from momentum conservation, and they interpreted the momentum physically.

With regard to this point, these three results [2-4] are in agreement. The final term of eq. (3) has been derived from several points of view: 1) The principle of virtual work [2]; Hamilton's principle *; 3) Equivalence of the models [2,3]; 4) Conservation of momentum [3,4]; 5) Symmetry of the energy-momentum tensor [2,4]; 6) The momentum has been interpreted as kinetic momentum of the moving charges in the current loop [2]. Here, eq. (3) is derived directly by considering the forces on the current-loop model in detail. For simplicity, $\boldsymbol{m}$ is assumed to be constant.

Two models are considered. The first is a superconducting cylinder carrying an azimuthal current density $\boldsymbol{J}$ with dipole moment $\boldsymbol{m}$ parallel to the axis of the cylinder. An applied field $\boldsymbol{E}$ induces charges on opposite sides of the cylinder, and when $\boldsymbol{E}$ is changed the induced charges change, causing an induced current $\boldsymbol{J}_{\mathrm{ind}}$ to flow. The total force is then $\boldsymbol{J}\times \mu_0\boldsymbol{H} + \boldsymbol{J}_{\mathrm{ind}}\times \mu_0\boldsymbol{H}$. The first term, when integrated over the volume of the cylinder, yields (2), and the second term yields the last term in (3). Because this model is so simple, we do not go through the derivation in detail.

The second model has charges traveling at the speed of light (thus the magnetic moment is constant) in a circular loop of constant radius. In order to have no net electric charge, we suppose that an equal number of positive and negative charges travel in opposite directions.

In order to eliminate radiation effects, the limit of a continuum consisting of an infinite number of charges of infinitesimal charge, but finite charge-to-mass ratio, is invoked at the end of the calculation. The particle motion may be considered constrained to the circular path in a frictionless way by a "rigid tube" whose walls have to support the centrifugal force of the rotating particles as well as the Lorentz force perpendicular to the tube. The net force transferred to the tube may be legitimately defined as "the force on the current loop" for the model under consideration.

The force on a typical charge Q is responsible for its time rate of change of momentum

$$Q\boldsymbol{E} + Qc\boldsymbol{i}_{\varphi} \times \mu_0\boldsymbol{H} + \boldsymbol{F}_{\mathrm{a}} = \mathrm{d}(M\boldsymbol{v})/\mathrm{d}t \qquad (4)$$

The first two terms are the Lorentz force ($\boldsymbol{i}_{\varphi}$ is unit vector in the circumferential direction) and $\boldsymbol{F}_{\mathrm{a}}$ is the force exerted on the particle by the "tube". The sum of all $\boldsymbol{F}_{\mathrm{a}}$ is the negative of the "force on the current loop".

The component of (4) perpendicular to $\boldsymbol{i}_{\varphi}$ gives us an expression for $\boldsymbol{F}_{\mathrm{a}}$, and the remainder of eq. (4) gives the rate of change of relativistic mass, $\mathrm{d}M/\mathrm{d}t = QE_{\varphi}/c$. Since the motion of each charge and the field as a function of time, are known, the mass of each particle as a function of time, can be found and substiuted into eq. (4). The result is summed over all charges, and averaged over the arbitrarily small time it takes each particle to make a single revolution. When this is done, we find eq. (3); the details are given elsewhere [5] and are not repeated here.

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* The Lagrangian is given on p. 219 of ref. 2, although the details are omitted.

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