Conservation of linear and angular momentum and the interaction of a moving charge with a magnetic dipole

V. Hnizdo
Department of Physics and Schonland Research Centre, University of the Witwatersrand, Johannesburg, 2050 South Africa

(Received 6 March 1991; accepted 4 August 1991)

The operation of the laws of momentum and angular momentum conservation in the interactions between current-carrying bodies and charged particles is analyzed using the correct expression for the force on a magnetic dipole, which takes into account the possible presence of hidden momentum in a current-carrying body. At nonrelativistic velocities, Newton's third law holds for the interactions, and thus the mechanical momentum associated with the motion of current-carrying bodies and charged particles in a closed system is conserved itself in the nonrelativistic limit. There is no conflict with overall linear momentum conservation because the electromagnetic field momentum is equal and opposite to the hidden momentum of the current-carrying bodies. However, the field angular momentum in a system is not compensated by hidden angular momentum, and thus only the sum of mechanical angular momentum, which must include any hidden angular momentum, and field angular momentum is conserved.

I. INTRODUCTION

In classical electrodynamics, the total energy, momentum, and angular momentum of a closed system of charged particles, interacting via their electromagnetic fields, are conserved. It is important, however, to stress that the conserved quantities must include the energy, momentum, and angular momentum of the electromagnetic fields in the system, because the corresponding quantities of the particles alone are not conserved in general. While these laws of microscopic electrodynamics are in principle clear and simple, the operation of the conservation laws in systems consisting of macroscopic current-carrying bodies and moving charged particles can be often surprisingly subtle and involved. Thus it was for a long time commonly held[1] that the momentum associated with the motion of a charged particle and current-carrying body is not conserved in their interaction even at nonrelativistic velocities, or in other words, that here Newton's third law does not hold and that one has to include the momentum of the combined electric field of the charge and the magnetic field of the current-carrying body in order to arrive at an overall momentum balance in the system. This view had to be altered with the realization of more than 20 years ago that a current-carrying body can contain a "hidden" momentum which is not associated with the motion of its center of mass.[2] Closely related to the possible presence of hidden momentum in a macroscopic body is the problem of the correct expression for the force on a magnetic dipole, which has been the subject of an interesting controversy.[3]

In this paper, we consider the interaction of a charged particle of vanishing magnetic moment with a body of zero net charge, but carrying a nonzero current whose distribution can be sufficiently accurately characterized by its magnetic dipole moment, which of course amounts to assuming that the dimensions of the body are sufficiently small compared to the distances at which its interaction with the charged particle is studied. No assumptions or detailed models are made as to the nature and mechanism of the electric currents in the body, nor as to the body itself, apart from assuming that the currents are stationary, or at most quasistationary. Thus, for example, the body can be a conducting solenoid, or two counter-rotating nonconducting disks charged with opposite charges, or a permanent magnet.[4] When the current-carrying body is a conductor, an electric dipole moment and possibly higher-order electric multipole moments will be induced in it by the charged particle, and thus there will be an electrostatic interaction between the body and the charge, apart from the interaction between the magnetic dipole moment of the body and the charge. However, electrostatic interactions automatically satisfy Newton's third law, and so need not be considered in our investigation, which is concerned with momentum and angular momentum conservation. In any case, one may assume that the dimensions of the current-carrying body are so small that already the electric field of any induced electric dipole moment can be neglected at the distances considered.

Using the correct expression for the force on a magnetic dipole, which involves in an essential way the possible presence of hidden momentum in a macroscopic body, we shall show that already the mechanical momentum associated with the motion of the charged particle and the body, assuming that the motion is slow (i.e., nonrelativistic), is conserved. The conservation of overall linear momentum is not affected because the electromagnetic field momentum is compensated by the hidden momentum of the current-carrying body. This will confirm and illustrate in a transparent way the general, but rather more complicated calculations of Furry, made more than 20 years ago.[5] We shall also examine the conservation of angular momentum in the interaction of a charge with a magnetic dipole, paying attention to the possible presence of hidden linear and angular momenta. Here it will turn out that no part of the mechanical angular momentum is conserved alone and that only the sum of the mechanical angular momentum and the field angular momentum is a conserved quantity. The mechanical angular momentum contains in general a hidden angular momentum that does not arise from the motion or rotation of the charged particle and the current-carrying body.

II. LINEAR MOMENTUM

The force on a magnetic dipole **m** *at rest* in an external magnetic field **B** and an external electric field **E** is given correctly by[6] (S.I. units will be used here)

$$\mathbf{F}=\nabla(\mathbf{m}\cdot\mathbf{B})-\frac{1}{c^2}\frac{d}{dt}(\mathbf{m}\times\mathbf{E}). \tag{1}$$

This expression takes account of the possible presence of the hidden momentum $(\mathbf{m}\times\mathbf{E})/c^2$ in the body that carries the current that gives rise to the magnetic dipole moment **m**. The rate of change of the total momentum **P** of the body is given by the commonly used expression

$$\frac{d}{dt}\mathbf{P}=\nabla(\mathbf{m}\cdot\mathbf{B}), \tag{2}$$

but this momentum is the sum of the momentum associated with the motion of the center of mass of the body $M\mathbf{V}$ and the hidden momentum,

$$\mathbf{P}=M\mathbf{V}+(1/c^2)\mathbf{m}\times\mathbf{E}, \tag{3}$$

and thus the force on the dipole, as $\mathbf{F}=(d/dt)M\mathbf{V}$, is given by Eq. (1). We stress that Eq. (1) holds only in an inertial frame in which the translational velocity **V** of the center of mass of the body vanishes at the given instant of time. Our conclusions regarding conservation laws will however be valid also in any other inertial frame, as long as the velocities will remain small compared to the speed of light. This follows from the fact that forces are invariant except for terms of order V^2/c^2.

The existence of hidden momentum in a stationary body can be seen as a necessary consequence of the fact that the total momentum (mechanical plus electromagnetic) in a system of any static distribution of charge and current is zero.[7] As the momentum of the electromagnetic fields in such a system does not vanish in general,[8] there must be an equal and opposite momentum "hidden" in the static distribution of charge and current. In the case of the body that carries the currents being a conductor, it turns out that the electromagnetic field momentum actually vanishes due to the fact that the net electric potential inside a conductor is constant.[9] Then the hidden momentum of the conducting body must be zero too, but one can show that the force on a magnetic dipole in a conducting body is still given by Eq. (1), where the term $(-d/dt)(\mathbf{m}\times\mathbf{E})/c^2$ now arises from the interaction between the magnetic dipole moment **m** and the current induced in the conductor by the time-varying external electric field **E**.[10]

After these preliminaries, let us now consider a magnetic dipole **m** momentarily at rest, interacting with a charge q moving with a velocity **v**, whose magnitude is small compared to the speed of light. The moving charge creates a magnetic field $\mathbf{B}_q=\mathbf{v}\times\mathbf{E}_q/c^2$, where $\mathbf{E}_q$ is the electric field of the charge. The magnetic field $\mathbf{B}_q$ and the electric field $\mathbf{E}_q$ of the charge act on the magnetic dipole with a force given by Eq. (1),

$$\begin{aligned}\mathbf{F}_d&=\nabla_d(\mathbf{m}\cdot\mathbf{B}_q)-\frac{1}{c^2}\frac{d}{dt}(\mathbf{m}\times\mathbf{E}_q)\\&=(\mathbf{m}\cdot\nabla_d)\mathbf{B}_q-\frac{1}{c^2}\frac{d\mathbf{m}}{dt}\times\mathbf{E}_q,\end{aligned} \tag{4}$$

where ∇_d is the gradient operator with respect to the radius vector $\mathbf{r}_d$ of the magnetic dipole and where we used the vector identity

$$\nabla_d(\mathbf{m}\cdot\mathbf{B}_q)=(\mathbf{m}\cdot\nabla_d)\mathbf{B}_q+\mathbf{m}\times(\nabla_d\times\mathbf{B}_q)$$

and Maxwell's equation

$$\nabla_d\times\mathbf{B}_q=\mu_0\mathbf{J}_q+\frac{1}{c^2}\frac{\partial}{\partial t}\mathbf{E}_q,$$

where $\mathbf{J}_q=0$ at the magnetic dipole. Let us assume first that the magnetic dipole moment is constant in time, i.e., $(d/dt)\mathbf{m}=0$. Then

$$\mathbf{F}_d=(1/c^2)(\mathbf{m}\cdot\nabla_d)(\mathbf{v}\times\mathbf{E}_q), \tag{5}$$

where we expressed the magnetic field $\mathbf{B}_q$ in terms of the electric field of the charge $\mathbf{E}_q$.

The force with which the magnetic dipole moment **m** acts on the moving charge q is, on the other hand, given by

$$\mathbf{F}_q=q\mathbf{v}\times\mathbf{B}_d, \tag{6}$$

where

$$\mathbf{B}_d=-(\mu_0/4\pi)\nabla_q(\mathbf{m}\cdot\mathbf{r}/r^3) \tag{7}$$

is the magnetic field of the magnetic dipole at the charge. Here, ∇_q is the gradient operator with respect to the position $\mathbf{r}_q$ of the charge, and $\mathbf{r}=\mathbf{r}_q-\mathbf{r}_d$ is the displacement of the charge from the magnetic dipole. As the electric field of the charge at the dipole is, in the nonrelativistic limit, given by

$$\mathbf{E}_q=-(q/4\pi\epsilon_0)(\mathbf{r}/r^3), \tag{8}$$

and using that $\nabla_q\times\mathbf{E}_q=0$ in transforming $\nabla_q(\mathbf{m}\cdot\mathbf{E}_q)$ into $(\mathbf{m}\cdot\nabla_q)\mathbf{E}_q$, we can write Eq. (6) as

$$\mathbf{F}_q=(1/c^2)\mathbf{v}\times(\mathbf{m}\cdot\nabla_q)\mathbf{E}_q. \tag{9}$$

Comparing Eqs. (5) and (9), we see that

$$\mathbf{F}_d=-\mathbf{F}_q, \tag{10}$$

as $\nabla_q=-\nabla_d$ for functions of $\mathbf{r}=\mathbf{r}_q-\mathbf{r}_d$. Thus action *does* equal reaction in the interaction of a magnetic dipole with an electric charge at nonrelativistic velocities. Note that if the usual expression $\mathbf{F}'_d=\nabla_d(\mathbf{m}\cdot\mathbf{B}_q)$ was used for the force on the magnetic dipole, we would get

$$\mathbf{F}'_d=-\mathbf{F}_q+\frac{1}{c^2}\mathbf{m}\times\frac{d\mathbf{E}_q}{dt}, \tag{11}$$

and Newton's third law would be violated.

Let us now relax the condition $(d/dt)\mathbf{m}=0$. Then the force on the charge q is

$$\mathbf{F}_q=q\mathbf{v}\times\mathbf{B}_d+q\mathbf{E}_d, \tag{12}$$

where $\mathbf{E}_d$ is the electric field at the charge due to the time variation of the magnetic field of the dipole $\mathbf{B}_d$. This electric field is given by

$$\mathbf{E}_d=-\frac{\partial\mathbf{A}_d}{\partial t}=-\frac{\mu_0}{4\pi}\frac{d\mathbf{m}}{dt}\times\frac{\mathbf{r}}{r^3}, \tag{13}$$

where $\mathbf{A}_d$ is the vector potential of the magnetic dipole at the charge, which is now time dependent. Using Eq. (8), we can write the additional force $q\mathbf{E}_d$ on charge q as

$$q\mathbf{E}_d=\frac{1}{c^2}\frac{d\mathbf{m}}{dt}\times\mathbf{E}_q. \tag{14}$$

Comparing Eq. (14) with the term due to $(d/dt)\mathbf{m}$ in Eq. (4), we see that the additional force on the charge that is due to the time variation of **m** is equal and opposite to the corresponding term in the force on the magnetic dipole. Newton's third law, Eq. (10), thus holds also in the general case when the magnetic dipole moment **m** varies with time. We can write Eq. (10) as

$$\frac{d}{dt}(M\mathbf{V} + m\mathbf{v}) = 0, \tag{15}$$

where $M\mathbf{V}$ and $m\mathbf{v}$ are the momenta associated with the motion of the magnetic dipole and the charge, respectively. Equation (15) expresses conservation of the mechanical momentum that is due to the motion of the charge and the magnetic dipole. In this statement no need arises to refer to the momentum of the electromagnetic field in the system, which in general does not vanish but is equal and opposite to the hidden momentum.

III. ANGULAR MOMENTUM

Let us now calculate the torque $\boldsymbol{\tau}_q$ on the charge q with respect to the center of mass of the magnetic dipole $\mathbf{m}$. This torque is given by

$$\boldsymbol{\tau}_q = \mathbf{r}\times\mathbf{F}_q = \mathbf{r}\times q\left(\mathbf{v}\times\mathbf{B}_d - \frac{\mu_0}{4\pi}\frac{d\mathbf{m}}{dt}\times\frac{\mathbf{r}}{r^3}\right). \tag{16}$$

Here, $\mathbf{r}$ is the displacement of the charge from the center of mass of the magnetic dipole and the second term on the right-hand side is due to the force (14) on the charge that arises from a possible time variation of the dipole moment $\mathbf{m}$. For our purpose, it will be advantageous to use in Eq. (16) the usual explicit expression

$$\mathbf{B}_d = \frac{\mu_0}{4\pi r^3}\left(\frac{3\mathbf{m}\cdot\mathbf{r}}{r^2}\mathbf{r} - \mathbf{m}\right) \tag{17}$$

for the magnetic field $\mathbf{B}_d$ of the magnetic dipole rather than that given in Eq. (7). Of interest now is the electromagnetic field angular momentum $\mathbf{M}_f$ in a system of a current-carrying body and an electric charge. For a sufficiently large displacement $\mathbf{r}$ of the charge from the body, so that the magnetic dipole approximation holds, this can be shown to have the value[11]

$$\mathbf{M}_f = \frac{1}{\mu_0 c^2}\int \mathbf{r}'\times(\mathbf{E}_q\times\mathbf{B})\,d^3r' = \frac{q\mu_0}{4\pi}\mathbf{r}\times\frac{\mathbf{m}\times\mathbf{r}}{r^3}, \tag{18}$$

provided that the charge itself does not have a magnetic moment. Here, $\mathbf{B}$ and $\mathbf{m}$ are the magnetic field and the magnetic dipole moment of the current-carrying body, respectively, and the angular momentum is calculated about the center of mass of the body. Using Eqs. (16)–(18), we obtain for the sum of the torque $\boldsymbol{\tau}_q$ on the charge and the rate of change of the field angular momentum $\mathbf{M}_f$ the following expression:

$$\boldsymbol{\tau}_q + \frac{d}{dt}\mathbf{M}_f = \frac{q\mu_0}{4\pi}\mathbf{m}\times\left(\mathbf{v}\times\frac{\mathbf{r}}{r^3}\right) = -\mathbf{m}\times\mathbf{B}_q, \tag{19}$$

after straightforward vector algebra, calculating the time derivative according to the rule $d/dt = \partial/\partial t + (\mathbf{v}\cdot\nabla_q)$. The right-hand side of Eq. (19) is the negative of the well-known expression[12] for the torque $\boldsymbol{\tau}_d$ on a magnetic dipole $\mathbf{m}$ at rest in a magnetic field $\mathbf{B}_q$. Thus Eq. (19) can be written as

$$\boldsymbol{\tau}_q + \boldsymbol{\tau}_d + \frac{d}{dt}\mathbf{M}_f = 0. \tag{20}$$

Equation (20) can be cast in a form that expresses the conservation of angular momentum. The torque on the charge is the rate of change of the angular momentum $\mathbf{M}_q$ of the motion of the charge,

$$\boldsymbol{\tau}_q = \frac{d}{dt}(\mathbf{r}\times m\mathbf{v}) = \frac{d}{dt}\mathbf{M}_q \tag{21}$$

and the torque on the magnetic dipole $\boldsymbol{\tau}_d$ must equal the rate of change of the *intrinsic* angular momentum $\mathbf{M}_d$ of the magnetic dipole,

$$\boldsymbol{\tau}_d = \frac{d}{dt}\mathbf{M}_d, \tag{22}$$

as $\mathbf{M}_d$ is here defined with respect to the center of mass of the dipole. Using Eqs. (21) and (22), we can write Eq. (20) as

$$\frac{d}{dt}(\mathbf{M}_q + \mathbf{M}_d + \mathbf{M}_f) = 0, \tag{23}$$

which expresses conservation of the overall angular momentum of a closed system consisting of a magnetic dipole and an electric charge. Note that the field angular momentum $\mathbf{M}_f$ has to be included with the mechanical angular momenta $\mathbf{M}_q$ and $\mathbf{M}_d$ of the charge and the dipole, respectively, in order to obtain a conserved quantity.

The intrinsic angular momentum $\mathbf{M}_d$ of the magnetic dipole must contain any angular momentum that is "hidden" in the sense that it would not appear in the overt rotational motion of the body that carries the currents that give rise to the magnetic dipole moment. Such hidden angular momentum may be for example due to relativistic effects in the motion of the charge carriers in the body,[13] or it can be simply the angular momentum of the charge carriers that is not compensated by the angular momentum of charge carriers of opposite sign, which would move in opposite direction. Hidden intrinsic angular momentum thus depends on details of the mechanism of charge transport in the body and it does not necessarily vanish in the limit of a pointlike body; the ratio of angular momentum of the charge carriers in the body to the magnetic dipole moment of the body can be finite even for a vanishing body size. There is no simple relation between hidden intrinsic angular momentum and the field angular momentum in the system, apart from the conservation requirement of Eq. (23), where hidden intrinsic angular momentum must be included in the intrinsic angular momentum $\mathbf{M}_d$ of the body that carries the magnetic dipole moment. Unlike hidden linear momentum, hidden intrinsic angular momentum is not equal and opposite to the field angular momentum in the system.

It is instructive to examine the effect of displacing the origin with respect to which the torques and angular momenta are calculated in our problem. Let us consider a new origin, displaced by a vector $\mathbf{a}$ from the center of mass of the magnetic dipole, thus

$$\mathbf{r} = \mathbf{r}' + \mathbf{a}, \tag{24}$$

where $\mathbf{r}'$ is the radius vector of the charge with respect to the new origin. The angular momentum $\mathbf{M}_q$ of the charge can be written as

$$\mathbf{M}_q = \mathbf{M}_q' + \mathbf{a}\times m\mathbf{v}, \tag{25}$$

where $\mathbf{M}_q'$ is the angular momentum of the charge calculated about the new origin and $m\mathbf{v}$ is the momentum of the charge. A similar equation should hold for the angular momenta of the magnetic dipole with respect to the old and new origins. However, we cannot use here only the momentum $M\mathbf{V}$ of the motion of the center of mass of the dipole, which is anyway assumed to vanish at the given instant of time, but must use instead the total momentum of the dipole that includes its possible hidden momentum, given by Eq. (3). Thus

$$\mathbf{M}_d = \mathbf{M}'_d + \mathbf{a}\times[M\mathbf{V} + (1/c^2)\mathbf{m}\times\mathbf{E}_q]. \qquad (26)$$

Equation (26) expresses the fact that the angular momentum $\mathbf{M}'_d$ of the magnetic dipole, taken about an origin that is displaced from the center of the mass of the dipole, can contain a hidden angular momentum $-\mathbf{a}\times(\mathbf{m}\times\mathbf{E}_q)/c^2$ arising from the possible hidden linear momentum of the dipole $\mathbf{m}\times\mathbf{E}_q/c^2$. We may call it a hidden orbital angular momentum, even though it is not due to an orbital motion of the dipole, to contrast it with the hidden intrinsic angular momentum discussed above.

The field angular momentum transforms as follows:

$$\mathbf{M}_f = \mathbf{M}'_f + \mathbf{a}\times\mathbf{P}_f. \qquad (27)$$

Here, $\mathbf{M}'_f$ is the field angular momentum calculated with respect to the new origin and $\mathbf{P}_f$ is the field linear momentum in the system. This field momentum can be shown to have the value[14]

$$\mathbf{P}_f = \frac{1}{\mu_0 c^2}\int \mathbf{E}_q\times\mathbf{B}\,d^3r' = \frac{q\mu_0}{4\pi}\mathbf{m}\times\frac{\mathbf{r}}{r^3}, \qquad (28)$$

which is equal and opposite to the hidden linear momentum $(\mathbf{m}\times\mathbf{E}_q)/c^2$ in the system. Substituting Eqs. (25)–(27) into Eq. (23), we obtain

$$\frac{d}{dt}(\mathbf{M}'_q + \mathbf{M}'_d + \mathbf{M}'_f) + \mathbf{a}\times\frac{d}{dt}\left(m\mathbf{v} + M\mathbf{V} + \frac{1}{c^2}\,m\times\mathbf{E}_q + \mathbf{P}_f\right) = 0. \qquad (29)$$

Conservation of angular momentum must not depend on the choice of the origin with respect to which angular momenta are calculated, as long as the origin is at rest in an inertial system. Thus we must have that

$$\frac{d}{dt}\left(m\mathbf{v} + M\mathbf{V} + \frac{1}{c^2}\,\mathbf{m}\times\mathbf{E}_q + \mathbf{P}_f\right) = 0, \qquad (30)$$

which is indeed true on the strength of Eqs. (15) and (28). Note that the hidden orbital angular momentum $-\mathbf{a}\times(\mathbf{m}\times\mathbf{E}_q)/c^2$ was needed to establish the conservation of the overall angular momentum taken about an origin that is displaced from the center of mass of the dipole.

IV. CONCLUDING REMARKS

Using the magnetic dipole approximation for the force on a current-carrying body, we showed that the forces acting between an electric charge and a current-carrying body satisfy Newton's third law, with the electromagnetic field momentum and the hidden momentum of the current-carrying body balancing each other. Similarly, the conservation of angular momentum in the system of the current and current-carrying body was examined here relying on the magnetic dipole approximation. But this approximation was used only because the simple expression for the force on a magnetic dipole enabled us to perform the calculations in a simple and transparent way. The conservation statements arrived at here hold in general for any nonrelativistic motion of the bodies, without the restriction to distances at which the magnetic dipole approximation applies. This was shown by Furry, for linear momentum in a detailed and complete manner, already more than 20 years ago.[15]

Examples of systems of current-carrying bodies interacting with charged particles are often analyzed with a view to illustrating the physical reality of the linear and angular momenta of static or quasistatic electromagnetic fields. As the mechanical linear momentum of the motion of the material bodies in such systems is conserved in the nonrelativistic limit, such examples are flawed as far as linear momentum is concerned because the existence of linear momentum of static electromagnetic field can be detected only through violations of Newton's third law. However, angular momentum is different in this respect, because field angular momentum is not compensated by hidden angular momentum and has to be included with the mechanical angular momentum of the motion of the bodies in the system and with hidden angular momentum in order to obtain a conserved quantity. Thus examples such as the famous and much discussed Feynman's disk "paradox"[16] are valid illustrations of the physical reality of the angular momentum of a static electromagnetic field. But a detailed analysis of such examples would have to take account of the hidden angular momentum in the systems under consideration.

ACKNOWLEDGMENT

The author wishes to thank E. C. H. Silk for his helpful comments on the manuscript.

[1] See, e.g., G. T. Trammel, "Aharonov-Bohm paradox," Phys. Rev. B **134**, 1183–1184 (1964); M. G. Calkin, "Linear momentum of quasistatic electromagnetic fields," Am. J. Phys. **34**, 921–925 (1966).

[2] W. Shockley and R. P. James, "'Try simplest cases' discovery of 'hidden momentum' forces on 'magnetic currents,'" Phys Rev. Lett. **18**, 876–879 (1967); H. A. Haus and P. Penfield, "Force on a current loop," Phys. Lett. **26A**, 412–413 (1968).

[3] A recent discussion of this controversy is by L. Vaidman, "Torque and force on a magnetic dipole," Am. J. Phys. **58**, 978–983 (1990).

[4] The magnetic moment of a permanent magnet can be modeled by electric current loops. Indeed, the intrinsic magnetic moments of electrons and nuclei can be modeled accurately by circulating electric currents; for a recent discussion see T. H. Boyer, "The force on a magnetic dipole," Am. J. Phys. **56**, 688–692 (1988).

[5] W. E. Furry, "Examples of momentum distributions in the electromagnetic field and in matter," Am. J. Phys. **37**, 621–636 (1969).

[6] This formula was first given by Haus and Penfield, Ref. 2; its validity for various models of current loops has been discussed recently by Vaidman, Ref. 3.

[7] M. G. Calkin, "Linear momentum of the source of a static electromagnetic field," Am. J. Phys. **39**, 513–516 (1971); Y. Aharanov, P. Pearle, and L. Vaidman, "Comment on 'Proposed Aharonov-Casher effect: another example of an Aharonov-Bohm effect arising from a classical lag,'" Phys. Rev. A **37**, 4052–4055 (1988).

[8] General proofs of this are, e.g., in Furry, Ref. 5; Calkin, Ref. 7; Aharanov, Pearle, and Vaidman, Ref. 7.

[9] This is shown by Furry, Ref. 5 and Calkin, Ref. 7.

[10] V. Hnizdo, "Comment on 'Torque and force on a magnetic dipole,' by L. Vaidman," Am. J. Phys. **60**, 242–246 (1992).

[11] A general formula for the electromagnetic field angular momentum of a system of a charge and a stationary current distribution in terms of the vector potential of the currents was derived by Trammel, Ref. 1. Our Eq. (18) is Trammel's formula with the vector potential of a magnetic dipole. Note that the magnetic field does not have to be exactly that of a point magnetic dipole, i.e., given as in our Eq. (17), down to small distances. In the nonrelativistic treatment, the contributions of the magnetic field $\mathbf{B}_q$ of the slowly moving charge q and of the electric field $\mathbf{E}_d$ of the slowly changing magnetic dipole moment $\mathbf{m}$, to the field angular momentum $\mathbf{M}_f$ are neglected.

[12] Unlike with the force on a magnetic dipole, there has been no controversy as to the correct expression for the torque on a magnetic dipole, see Vaidman, Ref. 3.

[13] G. E. Stedman, "Observability of static electromagnetic angular mo-

mentum," Phys. Lett. A **81**, 15–16 (1981).
[14] See, e.g, Trammel, Ref. 1, who gives a general formula for the electromagnetic field momentum of a system of a charge and a stationary current distribution in terms of the vector potential of the currents. Similar comments to those of Ref. 11 apply here too.
[15] Furry, Ref. 5.
[16] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA, 1964), Vol. II, pp. 17-5–17-6; an example most recently analyzed in this Journal is by A. S. de Castro, "Electromagnetic angular momentum for a rotating charged shell," Am. J. Phys. **59**, 180–181 (1991), where references to other papers on this topic are given.

A laboratory course in computer interfacing and instrumentation

Carl A. Kocher
Department of Physics, Oregon State University, Corvallis, Oregon 97331

(Received 18 March 1991; accepted 13 July 1991)

The rationale and content are described for an electronics-oriented course in computer-aided measurements. Performed on a prototyping board connected to an IBM PC-AT, the experiments emphasize digital-to-analog and analog-to-digital conversion at the chip level. Techniques are presented for parallel and serial communication; waveform generation; the acquisition, display, and analysis of data; the use of graphics; control and feedback; mechanical positioning with stepper motors; and signal averaging. Students have undertaken a wide range of original projects.

I. COURSE CONCEPT

Advances in integrated-circuit electronics have revolutionized the possibilities for laboratory applications of small computers. Not only can an interfaced computer save time by accurately performing repetitive measurements, but its speed and flexibility make new experiments possible. To meet the need for instruction in this new field, Oregon State University has developed a 10-week laboratory course devoted to the uses of computers as scientific instruments. The class meets in two 3-h laboratory sessions per week and is open to students who have completed a prerequisite course in digital and analog electronics.

In a research experiment, the operation of a fundamental measuring device is of critical importance and must be well understood. The electronics for data acquisition, while also important, may come to be regarded as a "black box" that can be trusted to perform its function in a predictable way. In the establishment of courses dealing with computerized instrumentation, an important question of philosophy must be settled near the outset: What will be the boundary of the "black box"?

Computer structure and architecture are discussed only briefly in our course, allowing time for the students to explore the hardware, software, and methodology for linking a digital computer to the analog world of the laboratory. Interfacing circuits are therefore kept "outside the box," and the experiments are set up on a prototyping breadboard, where the role and properties of digital-to-analog (D/A) and analog-to-digital (A/D) converters can be studied. This approach strikes a balance between two extremes: (1) a preoccupation with first principles, using primitive single-board computers and discrete interfacing circuits that may give poor results; and (2) specialized (and expensive) packaged data acquisition systems with factory-designed multifunction interfaces and commercial software.[1] The latter can be of real value in research, but in a course their use might obscure the comprehension of interfacing techniques. From the standpoint of pedagogy, the use of a breadboard offers the advantage of full access to the circuit, which is essential if experimentation and innovation are to be encouraged.

The IBM PC-AT was adopted as the computer for the course because it has become a *de facto* standard and because an abundance of inexpensive software is available for it. Various manufacturers[2] offer "clones" of the AT for less than $600. As noted in Sec. II, the standard IBM serial/parallel input/output (I/O) board, with minor modifications, is satisfactory for use in an instructional laboratory.

During the first week the students become acquainted with the instruments on their table, including the computer and its graphics capability. Next the parallel and serial interfaces are studied, followed by consideration of the function and uses of D/A and A/D converters. Applications, selected for their relevance to research in physics and related fields, include on-line data acquisition, the display and analysis of data, signal averaging, and several control and feedback situations. The demonstrations and exercises are given in Microsoft QuickBASIC, which is convenient and easy to learn; programs can be run in both interpreted and compiled modes.

Working in pairs, the students keep notebooks showing programs, circuits, and results, together with suggestions for future development or improvement. Toward the end of the term, each member of the class selects and develops an original interfacing project, demonstrates it to the class, and writes a descriptive paper. This combination of directed and independent study provides balance, stimulates interest, and helps students discover ways to cope with unfamiliar situations in a research context.

Since its inception the course has been very popular. It has been taught 25 times, by five members of the faculty,

