A DC Voltage is Equivalent to Two Traveling Waves on a Lossless, Nonuniform Transmission Line

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***Abstract*— A static dc voltage can be treated as two traveling waves propagating in opposite directions of a lossless, nonuniform transmission line. The amplitudes of these two traveling waves are a function of the characteristic impedance of signal line. The concept of two traveling waves is applied to a time-domain-scattering-parameters analysis in a lossless, nonuniform transmission line terminated with nonlinear loads.**

I. Introduction

When a LOSSLESS transmission line is connected to a dc voltage[1] source at one end and an appropriate resistive load at the other end, a steady state dc voltage will finally be reached in the lossless signal line. Such a dc voltage is often treated as a static signal. In this paper, we treat such a static dc signal as two traveling waves propagating in opposite directions of the lossless signal line. In general, these two traveling waves have different signal amplitudes; the summation of these two traveling waves is equal to the static dc voltage. Such an approach will give us physical insights regarding the interaction between the transmission line and associated terminations in time domain analysis [1]–[4].

II. Traveling Wave Solutions

The time-space domain solutions of a lossless, uniform transmission line are [1]

$$V(t,x) = V_+\left(t - \frac{x}{u}\right) + V_-\left(t + \frac{x}{u}\right), \tag{1a}$$

$$I(t,x) = \frac{1}{Z}\left[V_+\left(t - \frac{x}{u}\right) - V_-\left(t + \frac{x}{u}\right)\right], \tag{1b}$$

where V represents voltage, I is the current, $Z = (L/C)^{1/2}$ is the characteristic impedance, $u = (LC)^{-1/2}$ is the wave velocity, L and C are inductance and capacitance of the signal line per unit length, t is the time and x is the space variable. Note that $V_+(t - x/u)$ and $V_-(t + x/u)$ represent the waves traveling in the $+x$ (forward) and $-x$ (backward) directions, respectively.

We assume that a uniform transmission line having a characteristic impedance Z is loaded with a dc voltage V_S, source resistance R_S at the left-hand side and a dc voltage V_L, resistor load R_L at the right-hand side. For such a configuration, we have a steady state voltage on the signal line,

$$V(t,x) = V_{SS} = \frac{R_L}{R_S + R_L} V_S + \frac{R_S}{R_S + R_L} V_L. \tag{2}$$

The current on the signal line is given by

$$I(t,x) = \frac{V_S - V_L}{R_S + R_L}. \tag{3}$$

By substituting (2) and (3) into (1), we obtain two traveling voltage waves

$$V_{+,-}(t,x) = \frac{1}{2} V_{SS} \pm \frac{1}{2}\left[\frac{Z}{R_S + R_L}\right](V_S - V_L), \tag{4}$$

where V_{SS} is given in (2) and the minus sign on the right-hand side of (4) accounts for the backward traveling wave $V_-(t,x)$. Notice that both traveling waves $V_+(t,x)$ and $V_-(t,x)$ are really independent of t and x. Nevertheless, we may actually state that two traveling waves exist along this transmission line.

We now consider a configuration of multiple-section line, i.e., the transmission line is not uniform. As shown in Fig. 1, two traveling waves exist in each of the two uniform lines. The boundary conditions at the junction connecting two uniform lines are

$$V_{1-} = \frac{Z_2 - Z_1}{Z_2 + Z_1} V_{1+} + \frac{2Z_1}{Z_2 + Z_1} V_{2-}, \tag{5a}$$

and

$$V_{2+} = \frac{2Z_2}{Z_2 + Z_1} V_{1+} + \frac{Z_1 - Z_2}{Z_1 + Z_2} V_{2-}. \tag{5b}$$

We know that the total voltage on the signal line is constant, i.e.,

$$V_{1+} + V_{1-} = V_{2+} + V_{2-} = V_{SS}. \tag{6}$$

The amplitudes of the traveling waves in the signal line are

$$V_{1+,-} = \frac{1}{2} V_{SS} \pm \frac{1}{2}\left[\frac{Z_1}{R_S + R_L}\right](V_S - V_L), \tag{7a}$$

for signal on uniform line 1 and

$$V_{2+,-} = \frac{1}{2} V_{SS} \pm \frac{1}{2}\left[\frac{Z_2}{R_S + R_L}\right](V_S - V_L), \tag{7b}$$

for signal on uniform line 2. The traveling wave solutions on a continuous, nonuniform line can be expressed as

$$V_{x+,-}(t,x) = \frac{1}{2} V_{SS} \pm \frac{1}{2}\left[\frac{Z(x)}{R_S + R_L}\right](V_S - V_L), \tag{8}$$

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[1] The terminology "dc voltage" here and in the title might indicate not a continuous dc voltage but a long-period pulse.

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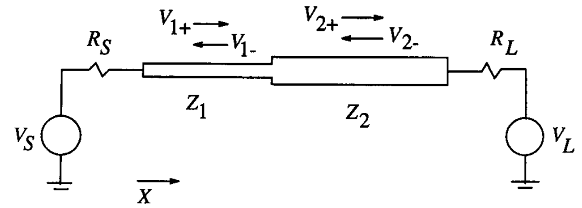

Fig. 1. Two traveling waves on a two-section line.

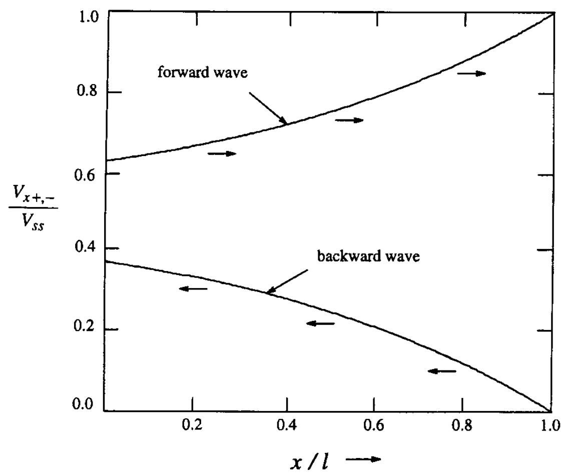

Fig. 2. Forward and backward waves on an exponential line.

where $Z(x)$ represents the characteristic impedance of the nonuniform line at x. The magnitudes of the traveling waves are determined by the characteristic impedance of the transmission line, source/load resistances and voltage sources. As an example, Fig. 2 shows the magnitudes of the traveling waves for an exponential line having a characteristic impedance given by

$$Z(x) = R_S e^{\frac{\ln(R_L/R_S)}{l}x}, \tag{9}$$

where l is the length of the exponential line, $R_L/R_S = 4$ and $V_L = 0$. As can be inferred from (8), the normalized voltages $V_{x+}(t,x)$ and $V_{x-}(t,x)$ are symmetric with respect to the horizontal line $V_{\pm}(t,x) = 1/2$. When the load is open, i.e., $R_L = \infty$, we have $V_{x+}(t,x) = V_{x-}(t,x) = 1/2V_S$ regardless of the characteristic impedance of the transmission line.

III. Applications

As shown in Fig. 3, a signal line can be represented by a set of time-domain scattering parameters $S_{11}(t), S_{12}(t), S_{21}(t)$ and $S_{22}(t)$. The time-domain scattering parameters relate two reflected waves and two incident waves as follows:

$$b_1(t) = S_{11}(t) * a_1(t) + S_{12}(t) * a_2(t), \tag{10a}$$

$$b_2(t) = S_{21}(t) * a_1(t) + S_{22}(t) * a_2(t), \tag{10b}$$

where $*$ denotes a convolution in the time domain, $a_1(t), b_1(t), a_2(t), b_2(t)$ are the incident and reflected waves for ports 1 and 2, respectively. The numerical formulations

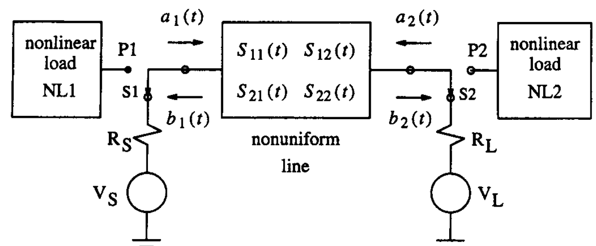

Fig. 3. Time-domain analysis and initial condition considerations.

for solving $a_j(t)$ and $b_j(t)(j = 1,2)$ can be found in [5]. We assume that the scattering parameters $S_{ij}(t)(i,j = 1,2)$ are given values and the objective is to evaluate $a_j(t), b_j(t)$ when the circuits at both ends of the signal line are specified.

When the circuit is in steady state, the conditions that describe the behaviors of the signal line at both ends are

$$a_1(t) = V_{x+}(t,0), \quad b_1(t) = V_{x-}(t,0), \tag{11a}$$

$$a_2(t) = V_{x-}(t,l), \quad b_2(t) = V_{x+}(t,l), \tag{11b}$$

where l is the length of the signal line, V_{x+} and V_{x-} are two traveling waves on the signal line. The incident and reflected waves $a_j(t)$, $b_j(t)$ in the time-domain-scattering-parameters analysis are equal to the two traveling waves at both ends of the transmission line in the steady state condition. For a uniform line, the change of circuit condition at input end, for example, will not immediately affect $b_1(t)$. As just shown, the backward wave V_{x-} is determined by the boundary condition at the load end. The change at the input end does not affect, and cannot possibly affect the condition at the load end until $\tau = l/u$ after the change. The condition on V_{x-}, therefore, will sustain for a time interval 2τ after the change of circuit condition at the input (left) end. For a nonuniform transmission line, due to internal reflection-transmission processes, the total backward wave $b_1(t)$ at the source (left) end could immediately vary once we change the condition at the input end. Nevertheless, the backward traveling wave $V_{x-}(t,0)$ still exists and contributes to the total reflected and incident waves $b_1(t)$ and $a_1(t)$, respectively. The two traveling waves, V_{x+} and V_{x-} in the steady state, therefore serve as the initial conditions to the new circuit configurations.

When the switch S1 is thrown to P1 at t_s, for example, the total incident and reflected waves $a_1(t)$, $b_1(t)(t > t_s)$ at the source (left) end are determined by the nonlinear load NL1, backward traveling wave $V_{x-}(t,0)$ and the scattering parameters $S_{ij}(t)$. In other words, $V_{x-}(t,0)$ can be viewed as an additional signal source to the new circuit. However, the contribution of backward traveling wave $V_{x-}(t,0)$ to the new circuit will last only for a limited time interval. If the circuit configuration at the load (right) end remains unchanged, backward traveling wave $V_{x-}(t,0)$ contribution to the circuit at the source (left) end vanishes after a round-trip time $2\tau = 21/u$. If the circuit at the load end changes its condition at $t_l(t_l > t_s)$, for example, S2 is thrown to P2 and assuming $t_l - t_s < \tau$, the traveling wave $V_{x-}(t,0)$ contribution to $a_1(t), b_1(t)$ at the source (left) end will extend over a time range $t_s < t < t_l + \tau$. The argument just described is applicable to the circuit configuration at the load (right) end. For the circuit previously stated, the traveling wave $V_{x+}(t, l)$ makes a contribution to the total incident and reflected waves $a_2(t)$,, $b_2(t)$ at the load (left) end and the existence of $V_{x+}(t, l)$ extends over the time interval $t_l < t < \tau + t_s$. Notice that the summation of existing time of traveling waves $V_{x+,-}$ at both ends of the line is 2τ.

IV. Conclusion

We decomposed a dc voltage on a lossless, nonuniform transmission line into two traveling waves propagating in opposite directions of the signal line. The amplitudes of two traveling waves are symmetric with respect to a horizontal line representing half of the steady state voltage. This approach provides us physical insights regarding the interaction between transmission lines and associated loads in time-domain considerations.

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