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Comment on "The easiest way to the Heaviside ellipsoid," by Valery P. Dmitriyev [Am. J. Phys. 70 (7), 717–718 (2002)]

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In a recent paper, Dmitriyev[1] described a simple and elegant method for deriving the electric and magnetic fields of a charged particle moving at constant velocity. The main simplification lies in the author's method for obtaining the scalar and vector potentials, φ and $\mathbf{A}$, in the Lorentz gauge for a constant-velocity particle.

In this comment, I describe a possibly even simpler method that utilizes elementary properties of Fourier transforms and linear, second-order ordinary differential equations, and the well-known (and easily derivable) fact that $1/k^2$ and $1/r$ are three-dimensional Fourier transform pairs:

$$\int \frac{d\mathbf{k}}{(2\pi)^3}\frac{1}{k^2}\exp(i\mathbf{k}\cdot\mathbf{r})=\frac{1}{4\pi r}. \tag{1}$$

Following Ref. 1, I assume that the potentials are caused solely by a point particle of charge q moving at a constant velocity $\mathbf{v}=v\mathbf{i}_1$, where $\mathbf{i}_1$ is a unit vector. I will derive φ for this particle. The derivation for $\mathbf{A}$ is almost identical.

In the Lorentz gauge, the scalar potential in SI units is given by

$$\nabla^2\varphi-\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}=-\frac{q\,\delta(\mathbf{x}-\mathbf{v}t)}{\epsilon_0}. \tag{2}$$

Let the three-dimensional spatial Fourier transform of φ be $\tilde{\varphi}(\mathbf{k},t)\equiv\int d\mathbf{x}\varphi(\mathbf{x},t)\exp(-i\mathbf{k}\cdot\mathbf{x})$. The spatial Fourier transform of Eq. (2) gives[2]

$$\frac{1}{c^2}\frac{d^2\tilde{\varphi}}{dt^2}+k^2\tilde{\varphi}=\frac{q}{\epsilon_0}e^{-i\mathbf{k}\cdot\mathbf{v}t}. \tag{3}$$

The complete solution of a linear, inhomogeneous, second-order ordinary differential equation includes the homogeneous and particular (or inhomogeneous) parts.[3] The homogeneous solution of Eq. (3) corresponds to potentials caused by other particles, which vanishes by the assumption that the potentials are due solely to the charge q. The particular solution is[4]

$$\tilde{\varphi}(\mathbf{k},t)=\frac{q}{\epsilon_0}\frac{e^{-i\mathbf{k}\cdot\mathbf{v}t}}{k^2-(\mathbf{v}\cdot\mathbf{k}/c)^2}. \tag{4}$$

We take the inverse Fourier transform of Eq. (4), change variables to $k_1=\gamma k_1'$ (where $\gamma=[1-(v^2/c^2)]^{-1/2}$), and apply Eq. (1), and find

$$\begin{aligned}\varphi(\mathbf{x},t)&=\int\frac{d\mathbf{k}}{(2\pi)^3}\tilde{\varphi}(\mathbf{k},t)e^{i\mathbf{k}\cdot\mathbf{x}}\\&=\frac{q}{\epsilon_0}\int\frac{dk_1}{2\pi}\int\frac{dk_2}{2\pi}\int\frac{dk_3}{2\pi}\frac{e^{ik_1(x_1-vt)+k_2x_2+k_3x_3}}{k_1^2(1-v^2/c^2)+k_2^2+k_3^2}\\&=\frac{q}{\epsilon_0}\int\frac{\gamma dk_1'}{2\pi}\int\frac{dk_2}{2\pi}\int\frac{dk_3}{2\pi}\frac{e^{ik_1'\gamma(x_1-vt)+k_2x_2+k_3x_3}}{k_1'^2+k_2^2+k_3^2}\\&=\frac{\gamma q}{4\pi\epsilon_0[\gamma^2(x_1-vt)^2+x_2^2+x_3^2]^{1/2}},\end{aligned} \tag{5}$$

which is the SI version of the desired result, Eqs. (18) and (19) in Ref. 1.

Which method, the one in Ref. 1 or the one presented here, the reader finds easier is a matter of personal taste. In any case, it does not hurt to have an alternative derivation.

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[1]Valery P. Dmitriyev, ‘‘The easiest way to the Heaviside ellipsoid,’’ Am. J. Phys. **70**, 717–718 (2002).
[2]One can easily show, using integration by parts and assuming that φ vanishes at $|\mathbf{x}|=\infty$ (which can be checked later for consistency), that after taking the spatial Fourier transform of φ, $\partial\varphi/\partial x_\alpha$ is replaced by $ik_\alpha\tilde{\varphi}$.
[3]See, for example, Mary L. Boas, *Mathematical Methods in the Physical Sciences*, 2nd ed. (Wiley, New York, 1983), p. 362.
[4]One can obtain this unique solution by guessing it has the form $a\exp(-i\mathbf{k}\cdot\mathbf{v}t)$. If we substitute this form into Eq. (3), we obtain the algebraic equation $-a(\mathbf{k}\cdot\mathbf{v})^2/c^2+ak^2=q/\epsilon_0$, from which a can be determined.

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