Force exerted on a stationary charge by a moving electric current or by a moving magnet

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Retarded fields are used for deriving the force exerted on a stationary charge by a slowly moving current-carrying conductor or a slowly moving magnet. The derivation makes use of the fact that a neutral current-carrying conductor, when it is moving, creates an external electric field. Although the presence of this field is generally regarded as a strictly relativistic effect, it is shown that this field is deducible from the basic laws of classical electrodynamics, without any recourse to relativity. In terms of classical electrodynamics, a moving neutral current-carrying conductor creates this electric field because of unequal retardation of the electric potentials associated with the positive and negative charges of the conductor. This field must be taken into account for the correct computation of the force exerted by moving current-carrying conductor or a moving magnet on a stationary charge.

I. INTRODUCTION

Although most textbooks on electricity and magnetism provide some discussion of retarded electric and magnetic fields and potentials, examples on practical applications of such fields or potentials are usually limited to electromagnetic radiation. Actually, however, the use of retarded fields and potentials is indispensable for the solution of many problems of classical electromagnetic theory. One such problem is the computation of the force exerted by a moving current-carrying conductor on a stationary electric charge. It is important to note that if, without analyzing the relevant effects of retardation, one makes a plausible assumption that the external electric field of a moving *neutral* current-carrying loop is zero, then one finds that the force exerted by the loop on a stationary charge is only *half as large* as the force experienced by the same charge moving relative to the same stationary loop.[1,2] This is a disturbing result, since it conflicts with Galilean relativity and, if correct, may indicate that classical electromagnetic theory is less reliable than we assume.

The purpose of the present paper is to provide, on the basis of classical electrodynamics, a rigorous and reasonably complete analysis of the force exerted by a slowly moving electric current or by a moving magnet on a stationary electric charge, to compare this force with the force experienced by the same charge when it is moving relative to the same stationary current, and to demonstrate the crucial role of retarded electric fields and potentials for correctly determining the force on the stationary charge.

II. BASIC THEORY

It is generally accepted that all magnetic fields are created by electric currents. Therefore, in discussing the force effects of magnetic fields, we only need to examine the corresponding force effects of the currents that produce the magnetic fields under consideration. The general equation for the *electric field* produced in a vacuum by an electric current and by the charges that form the current is[3–5]

$$\mathbf{E}=\frac{1}{4\pi\epsilon_0}\int_{\text{All space}}\left\{\frac{[\rho]}{r^2}+\frac{1}{rc}\frac{\partial[\rho]}{\partial t}\right\}\mathbf{r}_u dv' -\frac{1}{4\pi\epsilon_0 c^2}\int_{\text{All space}}\frac{1}{r}\left[\frac{\partial\mathbf{J}}{\partial t}\right]dv'. \tag{1}$$

The square brackets in this equation are the retardation symbol indicating that the quantities between the brackets are to be evaluated for the time $t'=t-r/c$, where t is the time for which $\mathbf{E}$ is evaluated, ρ is the electric charge density, $\mathbf{J}$ is the current density, r is the distance between the field point x, y, z (point for which $\mathbf{E}$ is evaluated) and the source point x', y', z' (volume element dv'), and c is the velocity of light; the integrals are extended over all space.

It is important to note that in terms of retarded potentials, the first integral in Eq. (1) originates from the retarded electric scalar potential φ_{ret}, while the second integral originates from the retarded magnetic vector potential $\mathbf{A}_{\text{ret}}$, according to[4]

$$\frac{1}{4\pi\epsilon_0}\int\left\{\frac{[\rho]}{r^2}+\frac{1}{rc}\frac{\partial[\rho]}{\partial t}\right\}\mathbf{r}_u dv'=-\nabla\varphi_{\text{ret}}, \tag{2}$$

and

$$-\frac{1}{4\pi\epsilon_0 c^2}\int\frac{1}{r}\left[\frac{\partial\mathbf{J}}{\partial t}\right]dv'=-\frac{\partial\mathbf{A}_{\text{ret}}}{\partial t}. \tag{3}$$

(The integrals are over all space, but, for simplicity, we are omitting the subscript "All space" in these and in all following integrals.)

Let us call the field represented by Eq. (2) the (pseudo) *electrostatic* field $\mathbf{E}_s$ and the field represented by Eq. (3) the *electrokinetic* field $\mathbf{E}_k$.[6] We then have

$$\mathbf{E}_s=\frac{1}{4\pi\epsilon_0}\int\left\{\frac{[\rho]}{r^2}+\frac{1}{rc}\frac{\partial[\rho]}{\partial t}\right\}\mathbf{r}_u\, dv', \tag{4}$$

and

$$\mathbf{E}_k=-\frac{1}{4\pi\epsilon_0 c^2}\int\frac{1}{r}\left[\frac{\partial\mathbf{J}}{\partial t}\right]dv'. \tag{5}$$

Consider now an initially stationary current $\mathbf{J}(x',y',z')$. Let this current move as a whole with a velocity v relative to a stationary observer. The current is then a function of $(x'-v_x t)$, $(y'-v_y t)$, and $(z'-v_z t)$, or

$$\mathbf{J}=\mathbf{J}(x'-v_x t,\ y'-v_y t,\ z'-v_z t). \tag{6}$$

The time derivative of the current is

$$\frac{\partial \mathbf{J}}{\partial t}=-\frac{\partial \mathbf{J}}{\partial x'}v_x-\frac{\partial \mathbf{J}}{\partial y'}v_y-\frac{\partial \mathbf{J}}{\partial z'}v_z=-(\mathbf{v}\cdot\nabla')\mathbf{J}. \tag{7}$$

The electrokinetic field caused by the moving current is therefore, by Eqs. (5) and (7),

$$\mathbf{E}_k=\frac{\mu_0}{4\pi}\int\frac{[(\mathbf{v}\cdot\nabla')\mathbf{J}]}{r}\,dv'. \tag{8}$$

The integral in Eq. (8) can be transformed by using the vector identity

$$\nabla'(\mathbf{v}\cdot\mathbf{J})=(\mathbf{v}\cdot\nabla')\mathbf{J}+\mathbf{v}\times(\nabla'\times\mathbf{J})+(\mathbf{J}\cdot\nabla')\mathbf{v}+\mathbf{J}\times(\nabla'\times\mathbf{v}). \tag{9}$$

Taking into account that $\mathbf{v}$ is a constant vector, we then obtain

$$\mathbf{E}_k=\frac{\mu_0}{4\pi}\int\frac{[\nabla'(\mathbf{v}\cdot\mathbf{J})]}{r}\,dv'-\frac{\mu_0}{4\pi}\int\frac{[\mathbf{v}\times(\nabla'\times\mathbf{J})]}{r}\,dv'. \tag{10}$$

If we compare Eq. (10) with the equation representing the magnetic flux density field produced by a time-variable current,[3–5]

$$\mathbf{B}=\frac{\mu_0}{4\pi}\int\frac{[\nabla'\times\mathbf{J}]}{r}\,dv', \tag{11}$$

we find that Eq. (10) can be written as

$$\mathbf{E}_k=\frac{\mu_0}{4\pi}\int\frac{[\nabla'(\mathbf{v}\cdot\mathbf{J})]}{r}\,dv'-\mathbf{v}\times\mathbf{B}, \tag{12}$$

where $\mathbf{B}$ is the magnetic flux density field created by the moving current distribution $\mathbf{J}$.

Consider now a small square-shaped loop carrying a current I and slowly moving past a distant stationary point charge q. Let the loop be electrically neutral at all its points. The moving loop exerts a force on the stationary charge. How does this force compare with the force experienced by the same charge when the loop is at rest and the charge is moving?

The force experienced by the charge when it moves past the stationary loop is simply

$$\mathbf{F}=q\mathbf{v}\times\mathbf{B}, \tag{13}$$

where $\mathbf{B}$ is the magnetic field produced by the loop. The force experienced by the stationary charge when the loop moves past it is $\mathbf{F}=q\mathbf{E}_s+q\mathbf{E}_k$. However, since the loop is everywhere neutral, E_s [given by Eq. (4)] appears on first examination to be zero, so that the entire force appears to be caused by $\mathbf{E}_k$. Thus, according to Eq. (12), the force experienced by the stationary charge appears to be

$$\mathbf{F}=q\mathbf{E}_k=q\frac{\mu_0}{4\pi}\int\frac{[\nabla'(\mathbf{v}\cdot\mathbf{J})]}{r}\,dv'-q\mathbf{v}\times\mathbf{B}, \tag{14}$$

which differs from the force experienced by the moving charge by the value of the integral in Eq. (14). Observe that, according to Eq. (14), the two forces are different regardless of how small the velocity of the charge is. This is a curious result, since it is in an obvious violation of one of the most sacrosanct principles of classical physics: The principle of Galilean relativity, according to which, for $v \ll c$, the magnitude of the two forces should be exactly the same. Clearly, either our calculation is wrong, or Maxwellian electrodynamics is very deficient.

There is nothing wrong with Maxwellian electrodynamics, of course. We have obtained Eq. (14) by assuming that the only electric field produced by a neutral current-carrying loop is the $\mathbf{E}_k$ field. It is a plausible assumption, and yet, as we shall presently see, it is very incorrect.

III. RETARDED POTENTIAL AND THE APPARENT ELECTRIC CHARGE OF NEUTRAL CURRENT-CARRYING CONDUCTORS

The retarded potential of a point charge moving with velocity v relative to a stationary observer located at a distance r from the charge is[7]

$$\varphi_{\text{ret}}=\frac{q}{4\pi\epsilon_0 r\{1-(v^2/c^2)\sin^2\theta\}^{1/2}}, \tag{15}$$

where θ is the angle between the direction of v and the direction of r. For $v \ll c$, the potential can be written as

$$\varphi_{\text{ret}}=(q/4\pi\epsilon_0 r)\{1+(v^2/2c^2)\sin^2\theta\}. \tag{16}$$

To the stationary observer this potential appears to be caused by *two* charges: by the original charge q, whose contribution to the potential is

$$\varphi_1=q/4\pi\epsilon_0 r, \tag{17}$$

and by an additional charge $q'=q(v^2/2c^2)\sin^2\theta$, whose contribution to the potential is

$$\varphi_2=(qv^2/8\pi\epsilon_0 c^2 r)\sin^2\theta. \tag{18}$$

Consider now a short segment of length L of a neutral current-carrying wire initially at rest on the x axis of rectangular coordinates. Let the midpoint of the segment be at the origin. Let the line density of the positive and the negative charges in the wire be $\lambda=q/L$ and $\lambda^-=-q/L$, respectively. Let the positive charges be at rest, and let the current in the wire be due to the negative charges moving with velocity $u \ll c$ in the negative x direction. The current in the wire is then

$$I=\lambda u=qu/L, \tag{19}$$

and is in the positive x direction.

An observer located at a point of the z axis at a distance $r \gg L$ from the wire measures the electric potential produced by the wire. Let us see what this potential is. Since the wire is short, and since the point of observation is far from the wire, all positive charges of the wire may be considered to constitute a single positive point charge q, and all negative charges of the wire may be considered to constitute a single negative point charge $-q$. The positive (stationary) charges of the wire produce at the location of the observer an electric potential

$$\varphi^+=q/4\pi\epsilon_0 r. \tag{20}$$

By Eq. (16), taking into account that $\sin\theta=1$, the negative (moving) charges of the wire produce a rather different potential

$$\varphi^-_{\text{ret}}=-(q/4\pi\epsilon_0 r)(1+u^2/2c^2). \tag{21}$$

The two potentials do not cancel each other. The resulting potential is

$$\varphi=\varphi^{+}+\varphi_{\rm ret}^{-}=-(q/4\pi\epsilon_0 r)(u^2/2c^2). \quad (22)$$

The observer attributes this potential to a charge

$$q_{\rm apparent}=-qu^2/2c^2=-I^2L^2/2qc^2, \quad (23)$$

which the neutral wire appears to have acquired as a result of the current in it.[8] It may be useful to note that, although this charge is important for our derivations, it is of no practical significance, except perhaps in superconductors, since the velocity of charge carriers in common conductors is many orders of magnitude smaller than the velocity of light.[9]

Let us now assume that the wire moves with velocity $v \ll c$ along the x axis. When the wire moves, the velocity of the positive charges in the wire, as seen by the stationary observer, becomes v, and that of the negative charges becomes $(v-u)$. Therefore, by Eq. (16), the potentials produced by the positive and the negative charges of the wire at the point of observation are now, respectively,

$$\varphi_{\rm ret}^{+}=\frac{q}{4\pi\epsilon_0 r}(1+v^2/2c^2), \quad (24)$$

and

$$\varphi_{\rm ret}^{-}=-\frac{q}{4\pi\epsilon_0 r}\{1+(v-u)^2/2c^2\}. \quad (25)$$

The total potential produced by the charges of the moving wire is therefore

$$\varphi_{\rm ret}=\varphi_{\rm ret}^{+}+\varphi_{\rm ret}^{-}=\frac{q}{4\pi\epsilon_0 r}\left\{1+\frac{v^2}{2c^2}-1-\frac{(v-u)^2}{2c^2}\right\}$$
$$=\frac{q}{4\pi\epsilon_0 r}\frac{2uv-u^2}{2c^2}. \quad (26)$$

The stationary observer notices that, in addition to the charge given by Eq. (23), the wire appears to have acquired in consequence of its motion a charge which, by Eqs. (23), (26), and (19), is

$$q_{\rm apparent}^{+}=quv/c^2=ILv/c^2. \quad (27)$$

A similar calculation shows that, if the current is in the direction opposite to the direction in which the wire moves, the wire appears to acquire a charge

$$q_{\rm apparent}^{-}=-quv/c^2=-ILv/c^2. \quad (28)$$

Let us now assume that the wire under consideration is the top side of a square-shaped current-carrying loop of length and width L whose plane is in the xy plane of the coordinates. To the observer at $r \gg L$ the stationary loop constitutes a magnetic dipole of moment[10]

$$\mathbf{m}=-IL^2\mathbf{k}. \quad (29)$$

But when the loop is moving along the x-axis, the top side of the loop appears to acquire a charge given by Eq. (27), while the bottom side appears to acquire a charge given by Eq. (28). For the stationary observer the moving loop constitutes therefore not only a magnetic dipole but also an electric dipole of moment[11–13]

$$\mathbf{p}=\frac{quvL}{c^2}\mathbf{j}=\frac{IL^2v}{c^2}\mathbf{j}=\frac{mv}{c^2}\mathbf{j}=\frac{\mathbf{v}\times\mathbf{m}}{c^2}. \quad (30)$$

IV. FORCE EXERTED BY A MOVING CURRENT-CARRYING LOOP ON A STATIONARY CHARGE

It is now clear that Eq. (14) cannot possibly give the correct result for the force exerted by the moving current-carrying loop on the stationary charge: contrary to our initial assumption, the neutral loop does create a nonvanishing electric potential and therefore a nonvanishing field $\mathbf{E}_s$, which must be included in Eq. (14) to give the correct force on the charge.[14,15] To determine the correct expression for the force, we can proceed as follows.

First, since $v \ll c$, we can neglect retardation in Eq. (14) and can transform the integral in Eq. (14) by using the vector identity

$$\frac{\nabla'(\mathbf{v}\cdot\mathbf{J})}{r'}=\nabla\frac{\mathbf{v}\cdot\mathbf{J}}{r}+\nabla'\frac{\mathbf{v}\cdot\mathbf{J}}{r}, \quad (31)$$

where the unprimed ∇ operates upon field-point coordinates only. We obtain

$$\mathbf{E}_k=\frac{\mu_0}{4\pi}\int\nabla\frac{\mathbf{v}\cdot\mathbf{J}}{r}dv'+\frac{\mu_0}{4\pi}\int\nabla'\frac{\mathbf{v}\cdot\mathbf{J}}{r}dv'-\mathbf{v}\times\mathbf{B}. \quad (32)$$

Next, using the vector identity

$$\oint U d\mathbf{S}'=\int\nabla' U\, dv', \quad (33)$$

we can transform the second volume integral in Eq. (32) into a surface integral. But, because there are no currents at infinity, the surface integral vanishes, and so does the volume integral. Thus the electrokinetic field is

$$\mathbf{E}_k=\frac{\mu_0}{4\pi}\int\nabla\frac{\mathbf{v}\cdot\mathbf{J}}{r}dv'-\mathbf{v}\times\mathbf{B}. \quad (34)$$

Since ∇ in Eq. (34) does not operate on source-point coordinates, we can factor it out from under the integral sign, obtaining

$$\mathbf{E}_k=\frac{\mu_0}{4\pi}\nabla\int\frac{\mathbf{v}\cdot\mathbf{J}}{r}dv'-\mathbf{v}\times\mathbf{B}. \quad (35)$$

Since the current in the loop is filamentary, we can replace $\mathbf{J}\,dv'$ by $Id\mathbf{l}'$ and can factor I out from under the integral sign. This gives

$$\mathbf{E}_k=\frac{\mu_0 I}{4\pi}\nabla\oint\frac{\mathbf{v}\cdot d\mathbf{l}'}{r}-\mathbf{v}\times\mathbf{B}, \quad (36)$$

where $d\mathbf{l}'$ is a length element vector in the direction of I. Since $\mathbf{v}$ is constant, it too can be factored out, and the circulation integral in Eq. (36) can then be transformed into a surface integral by means of the vector identity,

$$\oint U d\mathbf{l}'=-\int\nabla' U\times d\mathbf{S}', \quad (37)$$

which gives [observe that $\nabla'(1/r)=+(1/r^2)\mathbf{r}_u$],

$$\mathbf{E}_k=-\frac{\mu_0 I}{4\pi}\nabla\left(\mathbf{v}\cdot\int\frac{\mathbf{r}_u}{r^2}\times d\mathbf{S}'\right)-\mathbf{v}\times\mathbf{B}. \quad (38)$$

Since the linear dimensions of the loop are much smaller than r, we can replace the integration over the surface of the loop by the vector area of the loop, $\mathbf{S}'=-L^2\mathbf{k}$, so that

$$\mathbf{E}_k = -\frac{\mu_0 I}{4\pi} \nabla \left(\mathbf{v} \cdot \frac{\mathbf{r}_u}{r^2} \times \mathbf{S}' \right) - \mathbf{v} \times \mathbf{B}. \quad (39)$$

Transposing $\mathbf{v}$ and $\mathbf{r}_u$, we finally obtain

$$\mathbf{E}_k = \nabla \left(\frac{\mu_0 I}{4\pi} \frac{\mathbf{r}_u}{r^2} \cdot \mathbf{v} \times \mathbf{S}' \right) - \mathbf{v} \times \mathbf{B}. \quad (40)$$

Since our current-carrying loop is small and is far from the point of observation, it constitutes a magnetic dipole. But, as we know from Eq. (30), when such a dipole moves, it generates an electric dipole field. The field of an electric dipole of moment $\mathbf{p}$ is given by[16]

$$\mathbf{E}_{\text{dipole}} = -\nabla \left(\frac{\mathbf{p} \cdot \mathbf{r}_u}{4\pi\epsilon_0 r^2} \right). \quad (41)$$

Substituting $\mathbf{p}$ from Eq. (30) into Eq. (41), and using Eq. (29), we obtain (observe that $1/c^2 = \epsilon_0 \mu_0$)

$$\mathbf{E}_{\text{dipole}} = -\nabla \left(\frac{\mu_0 I}{4\pi} \frac{\mathbf{r}_u}{r^2} \cdot \mathbf{v} \times \mathbf{S}' \right). \quad (42)$$

But this dipole field is the field $\mathbf{E}_s$, which we have failed to include in Eq. (14). Since for the system under consideration Eq. (14) is the same as Eq. (40), we obtain, adding Eqs. (42) and (40), for the force exerted by the moving loop on the stationary point charge,

$$\mathbf{F} = q\mathbf{E} = q(\mathbf{E}_s + \mathbf{E}_k) = q(\mathbf{E}_{\text{dipole}} + \mathbf{E}_k) = -q\mathbf{v} \times \mathbf{B}, \quad (43)$$

which, by Eq. (13), is the same (except, of course, for the sign) as the force experienced by the charge when it is moving and the loop is stationary.[17]

V. SUMMARY

In Sec. II, where we used the very plausible (although very wrong) assumption that a neutral current-carrying loop produces no external electric field, we found that the force exerted on a stationary electric charge by the moving loop appears to be different from the force experienced by the same charge when the loop is stationary but the charge is moving. This result is in conflict with Galilean relativity and, if it were correct, would indicate that classical electrodynamics may have a major flaw.

However, as we now know from Sec. III, a moving current-carrying conductor does produce a nonvanishing external electric potential and field, even when the conductor is completely neutral in all its parts, that is, even when the net electric charge in any volume element of the conductor is zero. When the electric field of the moving current is included into the force equations, the force on a stationary charge in the field of a slowly moving current, and the force on a slowly moving charge in the field of a stationary current are found to be the same.

The creation of an external electric field by a moving neutral current-carrying conductor is generally recognized only as a relativistic effect. But as we have seen, it is also a classical effect. Classically, it is a consequence of unequal retardation of the electric fields (potentials) of charges moving with different velocities relative to a stationary observer. Just as the equations for the retarded potentials and fields, this effect is a direct consequence of Maxwell's equations.

Our derivations did not explicitly involve moving magnets. However, since a magnet is essentially a conglomeration of magnetic dipoles formed by microscopic currents, and since our calculations do apply to such dipoles, there is no doubt that our analysis of forces is applicable not only to moving currents but also to moving magnets. In either case a proper use of retarded fields is crucial for obtaining correct classical force expressions.

[1]Umberto Bartocci and Marco Mamone Capria, "Some remarks on classical electromagnetism and the principle of relativity." Am. J. Phys. **59**, 1030–1032 (1991); see also W. Rinder, "Relativity and electromagnetism: The force on a magnetic monopole," Am. J. Phys. **57**, 993–994 (1989).

[2]Umberto Bartocci and Marco Mamone Capria, "Symmetries and asymmetries in classical and relativistic electrodynamics," Found. Phys. **21**, 787–801 (1991).

[3]Oleg D. Jefimenko, *Electricity and Magnetism* (Electret Scientific, Star City, 1989), 2nd ed., p. 516.

[4]David J. Griffiths and Mark A. Heald, "Time-dependent generalization of the Biot-Savart and Coulomb laws," Am. J. Phys. **59**, 111–117 (1991).

[5]Tran-Cong Ton, "On the time-dependent, generalized Coulomb and Biot-Savart laws," Am. J. Phys. **59**, 520–528 (1991).

[6]A discussion of properties and applications of the electrokinetic field can be found in Oleg D. Jefimenko, *Causality, Electromagnetic Induction, and Gravitation* (Electret Scientific, Star City, 1992), pp. 28–66.

[7]See, for example, David J. Griffiths, *Introduction to Electrodynamics* (Prentice-Hall, Englewood Cliffs, 1981), 2nd ed., p. 421.

[8]The total potential of the stationary wire is zero and the apparent charge is absent, if the current in the wire is caused by equal positive and negative charges moving with equal velocities in opposite directions. However, even in this case, the total potential of a *moving* wire does not vanish, as can be seen from considerations presented below. In particular, Eq. (30) remains unchanged regardless of the velocities of the positive and negative charges forming the current. See also Ref. 6, pp. 166–168.

[9]For a related experimental study see W. F. Edwards, C. S. Kenyon, and D. K. Lemon, "Continuing investigation into possible electric fields arising from steady conduction currents," Phys. Rev. D **14**, 922–938 (1976).

[10]See, for example, Ref. 7, p. 237.

[11]The vertical sides of the loop make no contribution to the electric dipole. This is because $v \perp u$ along the vertical sides.

[12]Our derivation is for the current moving at right angles to line joining the current with the observer. This is the geometry used in Refs. 1 and 2.

[13]In the literature, the appearance of additional electric charges in moving neutral current-carrying conductors is explained as a strictly relativistic effect (see, for example, Ref. 7, pp. 489–491). However, the derivations just presented, based on classical retarded scalar potential, show clearly that this effect is caused by the finite speed of the electric field propagation, and thus is explainable without any recourse to the relativity theory.

[14]For moving charges, the word *neutral* does not have quite the same connotation as it has for stationary charges. Whereas in the case of stationary charges *neutral* usually means both "zero total charge" and "producing no electric field," in the case of moving charges it only means "zero total charge." The electric field of a moving charge is concentrated near the equatorial plane normal to the direction of motion of the charge. The concentration increases with the speed of the charge [this effect was first reported and explained by Oliver Heaviside in "The Electro-magnetic effects of a moving charge," The Electrician **22**, 147–148 (1888); see also Oliver Heaviside, *Electrical Papers* (MacMillan, New York, 1894), Vol. II, pp. 495–496, 511]. Thus, when charges of equal magnitude and opposite polarity are moving with different velocities, their fields have a different structure and cannot cancel each other.

[15]The fact that the moving neutral ($\rho = 0$) loop produces $\mathbf{E}_s \neq 0$ appears to be in conflict with Eq. (4). However, one should not forget that Eq. (4) contains the *retarded* charge density $[\rho]$ rather than the ordinary ρ. Since the retardation for charges moving with different velocities is different, one may not lump all moving charges into a single $[\rho]$, but should perform separate integrations for charges moving with different velocities. (In this connection, it is instructive to note that when

charges having the same present position are moving with different velocities, their retarded positions are different). A similar situation with charge densities is encountered in the special relativity theory. There, too, charge distributions moving with different velocities must be considered separately, since such distributions have different relativistic charge densities, even if the densities are the same when measured in reference frames in which each particular distribution is at rest. It is the difference of the relativistic charge densities of the positive and negative charges in the moving current-carrying loop that is at the core of the relativistic explanation of the electric dipole moment of the loop (see, for example, Ref. 7, pp. 489–491).

[16]See, for example, Ref. 3, p. 130.

[17]Actually, there is an additional electric field, albeit insignificant, due to the charge given by Eq. (23). Since this field does not depend on the velocity of the current-carrying conductor, it should be included in Eq. (13) as well as in Eq. (14), and therefore it does not affect the equality of forces experienced by the moving or by the stationary charge. Note also that we are only considering the nonrelativistic case of $v \ll c$; for relativistic velocities the force on a moving charge is, of course, not the same as the force on a stationary charge. There is also a possible contribution to the electric field from the accelerated charges in the corners of the loop (where the velocity of the charges changes direction). This contribution is proportional to the acceleration of the charges and to the volume occupied by each corner. Since the acceleration is finite (an infinite acceleration is physically impossible), and since the volume of the corners can be assumed as small as one pleases, the contribution of the corners to the electric field of the loop is negligible.

Student programming in the introductory physics course: M.U.P.P.E.T.

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Since 1983, the Maryland University Project in Physics and Educational Technology (M.U.P.P.E.T.) has been investigating the implication of including student programming in an introductory physics course for physics majors. Many significant changes can result. One can rearrange some content to be more physically appropriate, include more realistic problems, and introduce some contemporary topics. One can begin training the student in professional research-related skills at an earlier stage than is traditional. An interesting point to note is that the inclusion of carefully considered computer content requires an increased emphasis on qualitative and analytic thinking.

I. INTRODUCTION

Since 1983, in an effort we refer to as the Maryland Project in Physics and Educational Technology[1] (M.U.P.P.E.T.), the authors and their colleagues at the University of Maryland have been studying what the impact is of introducing beginning students to programming at the start of the traditional calculus-based introductory physics course.

The computer is more than simply a powerful calculator that allows students to multiply and add more quickly. The computer adds orders of magnitudes to the individual's computational abilities. As we know well in physics, when scales change by orders of magnitude, we have to look carefully for qualitatively new phenomena.

The question we address in this paper is the following:

What is the implication of the computer for teaching physics majors at the introductory level?

To answer this we must consider the answer to two related questions:

What is it we want our students to learn?

In what way is the current introductory physics course inadequate?

We assume that the broad general goal of the introductory physics course is to begin to prepare students to be professional physicists as well as to introduce them to the basic physics content. We find that having the computer as part of the course affects significantly both the skills we can begin to train and the specific content.

A. Problems with the traditional physics curriculum

The physics curriculum as presently taught was developed over thirty years ago.[2] Although the change was a significant advance at the time, the curriculum has failed to develop since then.

This might not be a problem if physics were a static field. It is not. In the past 30 years we have seen an explosion of new understanding and power in a variety of subfields of physics ranging in scale from the substructure of the proton to the clustering of galaxies. There have even been major breakthroughs in fields long thought to be understood. Current developments in Newtonian mechanics are evolving into a theory of nonlinear systems and chaotic behavior that may produce profound changes in the way we think about physics.[3]

