ONE CENTURY LATER: REMARKS ON THE BARNETT EXPERIMENT

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Abstract

The Barnett experiment (rotation of either a solenoid or short-circuited cylindrical condenser about a common axis) was performed at the beginning of 20^{th} century with a goal to select either the Hertz', or Maxwell's, or Lorentz electrodynamics theory. The results obtained by Barnett led him to the conclusion that both Hertz and Maxwell electrodynamics should be rejected, while the Lorentz' one is solely correct. The present paper analyzes this experiment from a modern point of view, and it shows that its result can be explained within the framework of Maxwell electrodynamics and relativity theory. At the same time, it seems that the explanation cannot be accepted as physically satisfactory.

1. Introduction

The experiment by Barnett [1, 2] was realized at the beginning of 20^{th} century for resolution of the problem of unipolar induction. At that time is was an excited problem closely related with the question: do the lines of magnetic induction move as if rigidly attached to the magnet, or do the lines remain fixed in space under rotation of magnet? It was expected that experimental solution of this problem (formulated in archaic language of the beginning of 20^{th} century) would allow to choose either the Hertz', or Maxwell's, or Lorentz' electrodynamics theory. The first (Hertz) theory was associated with a fully entrained ether; the Lorentz theory was associated with non-entrained ether (Lorentz ether theory), while the Maxwell electrodynamics was based on the special relativity theory, rejecting any ether models. It seems that Barnett invented the most interesting experimental scheme for resolution of the above mentioned problem, which allowed to get the most convincing results in comparison with other authors. His conclusion was that the lines of magnetic induction remain to be fixed in space under rotation of a magnet that is in agreement only with the Lorentz electrodynamics and Lorentz ether theory. Now it is difficult to say why this experiment did not essentially influence further development of space-time physics and electrodynamics theory. It is rather a question to a history of physics. Next section represents a description of the Barnett experiment, and section 3 gives its modern explanation.

2. Description of the experiment by Barnett

Simplified scheme of the experiment is shown in Figs. 1 and 2.

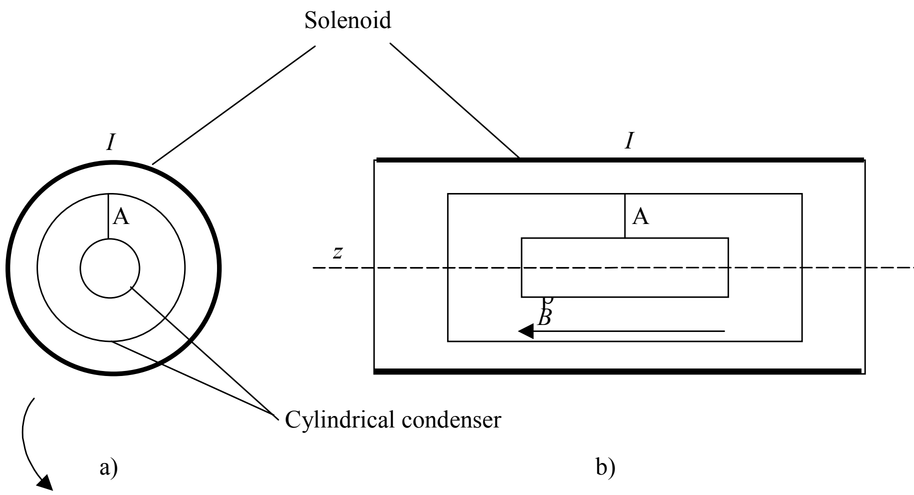

Fig. 1. Scheme of the Barnett experiment: a – top view; b – side view.

There are a solenoid with a current I and a cylindrical condenser placed inside of solenoid, where a magnetic induction $\vec{B}$ is parallel to the axis z. For elongated solenoid the value of $\vec{B}$ can be taken constant in inner space region being far from the boundaries of solenoid. The condenser is short-circuited by a conducting wire A. Both the solenoid and condenser are mounted coaxially, and they can rotate about the common axis z. During rotation an electric connection between facings of the condenser is broken, and after annuling of current and stop of rotation a charge of condenser is measured. It is known that under rotation of condenser it certainly acquires some charge Q due to the Lorentz force in the wire A, moving in magnetic field $\vec{B}$. Barnett did not perform the experiment with rotation condenser (Fig. 2, a), because similar experiments were already realized by Faraday and many others. In this case the charge Q can be easy calculated. The goal of Bernett experiment was to measure a charge of condenser after rotation of the solenoid (Fig. 2, b) and to compare it with the calculated value of Q. The inner armature of the air condenser was a brass tube 14.9 cm long and 3.97 cm in external diameter. The outer armature was a brass tube 28.0 cm long and 6.67 cm in internal diameter. The length of solenoid exceeded the length of outer armature. During rotation of solenoid about the axis z at the frequency 20 revolutions per second a connection between the armatures was broken by a special key system, after that the field was annuled. Then a sensitive electrometer was used to measure a charge of the condenser. The results of the described above and other modified Barnett experiments showed that the condenser acquired a charge not more than 1.4 % of Q within a measuring precision. It allowed to conclude him that the relativity principle is violated, and only the Lorentz electrodynamics is able to explain the obtained results.

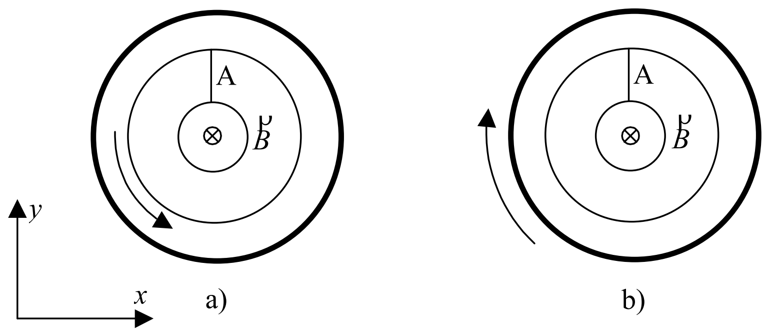

Fig. 2: a – cylindrical condenser rotates around the axis z at some constant angular frequency ω;
b – solenoid rotates around the same axis at the angular frequency $-\omega$.

3. Modern explanation of Barnett experiment

In further calculations we take for simplicity $\Delta r << r$, where Δr is the length of the wire A in laboratory frame, and r is the modulus of the radius-vector of any point upon the wire A. We will carry out all calculations to the order of approximation $\omega r/c$ (in Barnett experiment it was about 10^{-8}); and we will use a system of units with c=1 (c is the speed of light in vacuum). Under these approximations let us consider, first of all, the experiment in Fig. 2, a. Under rotation of the condenser in laboratory frame, the wire A has a linear velocity $\vec{u} \approx \vec{\omega} \times \vec{r}$ (ω is the rotational frequency), which is orthogonal to the vector $\vec{B}$ at any moment. Hence, the conduction electrons in A are subjected to the Lorentz force

$$F = euB \qquad (1)$$

(e is the charge of electron) in radial direction. This force charges the condenser to the potential difference of facings

$$\Delta V = \Delta r u B \ln\frac{R_0}{r_0} = \omega r \Delta r B \ln\frac{R_0}{r_0}.$$

The measured charge is

$$Q = C\Delta V = C\omega r \Delta r B \ln \frac{R_0}{r_0}. \qquad (2)$$

Now let us calculate a charge of condenser for the experiment in fig. 2, b. Under rotation of the solenoid an electric $\vec{E}$ and magnetic $\vec{B}$ fields, in general, change inside it. Since the condenser rests in the laboratory frame, that the magnetic field is not essential. In order to determine the electric field $\vec{E}$ inside of solenoid in the laboratory frame, we notice that moving conductor with non-zero current acquires a non-zero charge density ρ. A physical reason is a different scale contraction for the systems of negatively and positively charged particles in conductor, because under $I\neq 0$ they have different circular speeds. (Formally, ρ can be found under transformation of four-vector of current density). At the same time, due to a symmetry of the Barnett's experimental scheme to rotations and translations with respect to the axis z, the charge should be distributed homogeneously upon the surface of solenoid. In such a case in the regions being far from boundaries of solenoid, the electric field inside of it is equal to zero for any value of the charge density ρ, according to well-known result of electrostatics. Hence, we conclude that inside of solenoid the electric field is equal to zero. Therefore, there is no electric forces, acting on the electrons in wire A, and the condenser does not acquire any charge. It explains the result of Barnett experiment. Different results of the experiments in Figs. 2,a and 2,b simply mean that a conception of "relative motion", in general, is not applicable to accelerated motion.

Further, it is interesting to analyze these experiments from a viewpoint of observer in accelerated frame, attaching to either the condenser (Fig. 2, a – case 1: $Q\neq 0$), or to solenoid (Fig. 2, b – case 2: $Q=0$).

Under rotation about the axis z at the constant angular frequency ω, a relationship between space coordinates of the laboratory (resting) frame (x', y', z') and space coordinates in rotating frame (x, y, z) has the form

$$\begin{aligned} x' &= x\cos(\omega t) - y\sin(\omega t), \\ y' &= x\sin(\omega t) + y\cos(\omega t), \\ z' &= z. \end{aligned} \qquad (3)$$

One can show that non-zero metric coefficients in such rotating frame are:

$$g_{00} = 1-\omega^2 r^2,\ \ g_{11} = g_{22} = g_{33} = -1,\ \ g_{01} = g_{10} = \omega y,\ \ g_{02} = g_{20} = -\omega x. \qquad (4)$$

In order to determine the electric and magnetic fields in rotating frame, it is necessary to implement a corresponding transformation for the tensor of electromagnetic field

$$F_{ik} = \frac{\partial x'^l}{\partial x^i}\frac{\partial x'^m}{\partial x^k} F'_{lm}, \qquad (5)$$

taking into account its form in the laboratory frame under resting solenoid:

$$F'_{lm} = \begin{Bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -B & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & 0 & 0 \end{Bmatrix}. \tag{6}$$

Substituting eq. (6) into eq. (5) with account of eq. (3), we get the non-zero components of this tensor in the rotating frame in the adopted accuracy of calculations:

$$F'_{01} = -F'_{10} = -Bx\omega,\ \ F'_{02} = -F'_{20} = -By\omega,\ \ F'_{12} = -F'_{21} = -B. \tag{7}$$

Non-zero components of the electric and magnetic fields are found according to known equations (see, e.g., [3]):

$$E_1 = -F_{01} g^{11} \sqrt{\frac{\gamma_{11}}{g_{00}}},\ \ E_2 = \frac{F_{02}}{\sqrt{g_{00}}},\ \ B_3 = \left(g^{11}F_{12} + g^{01}F_{02}\right)\sqrt{\gamma_{11}}\ , \tag{8}$$

where **γ** is the metric tensor of three-dimensional space, related with the metric tensor of four-dimensional space-time **g** by the relationship

$$\gamma_{\alpha\beta} = -g_{\alpha\beta} + \frac{g_{0\alpha} g_{0\beta}}{g_{00}}\ , \qquad \alpha, \beta = 1\ldots3. \tag{9}$$

Now let us consider the experiment in Fig. 1, a (case 1, $Q \neq 0$) from a viewpoint of observer being in rotation together with condenser. (We conditionally designate this case 1R). For this observer the wire A is immovable, and the magnetic field is not essential. Hence, the condenser can only be charged due to action of an electric field to conduction electrons of the wire A. Using the relationships (8), (7), (9) and (4), one gets

$$E_x = -Bx\omega,\ \ E_y = -By\omega$$

in the taken order of approximation. It means that the vector of electric field lies in radial direction, and its value within the wire is

$E = uB$.

Hence, we get the same expression (1) for the force, charging the condenser:

$F = eE = euB$.

Now it is interesting to find an origin of this electric field. A single reason for creation of the electric field inside of solenoid is a non-zero charge density ρ on its surface. However, due to the mentioned above symmetry of Barnett experiment (which is kept in the rotating frame), the charge density ρ should be constant over the surface of solenoid[1]. However, we already

[1] This conclusion can be derived from direct calculations under transformation of the four-vector of current density $j^i = \left(\frac{\rho}{\sqrt{-g}}, \frac{\rho u^\alpha}{\sqrt{-g}} \right)$ from the laboratory to rotating frame, applying the transformation law $j_i = \frac{\partial x'^k}{\partial x^i} j'_k$ for transformation (3). (Here g=det**g**). Omitting simple calculations, we present a final result

mentioned above that for such a constant ρ, the electric field inside of elongated solenoid should be equal to zero for any value of ρ. Thus, we have to admit that in case 1R we are failed to indicate an origin of the electric field, which is responsible for the charge of condenser.

Now let us go to the experiment in Fig. 2, b (rotating solenoid, case 2), and explain the obtained result (Q=0) for observer attached to the solenoid (case 2R). In his reference frame the condenser rotates with the angular velocity ω, and both the electric and magnetic fields can act to the wire A. Using eqs. (4), (7), (8) we get the non-zero components of these fields:

$$E_x = -\omega x B,\ \ E_y = -\omega y B,\ \ B_z = B$$

in the adopted order of approximation. In such a case the vector $\vec{E}$ lies in radial direction, and the forces $e\vec{E}$ and $e(\vec{u}\times\vec{B})$ act in the opposite directions. The modulus of the vector of electric field is uB. Hence, we see that the total Lorentz force to each electron of the wire A

$$\vec{F} = e\vec{E} + e(\vec{u}\times\vec{B}) = 0. \qquad (10)$$

It means that in case 2R the condenser remains uncharged. At the same time, for observer attached to the solenoid, a charge surface density of the solenoid is again constant over its surface, and an origin of the electric field $\vec{E}$ inside of solenoid, which compensates an action of the magnetic field to the wire A, remains unclear.

Further, the result of Barnett experiment seems useful for an analysis of the principle of local Lorentz invariance (LLI), if we imagine that in this experiment we are able to create a radially symmetrical with respect to the axis z gravitation field. Let under rotation of the condenser (case 1), we choose a gravitational potential to be equal to centripetal acceleration of an observer on the outer surface of condenser (case 1RG). And under rotation of the solenoid, the gravitational potential is equal to centripetal acceleration of an observer on the surface of solenoid (case 2RG). If the radii of solenoid and outer armature of condenser are close to each other, we can assume the same gravitation field in both cases 1RG and 2RG. Since the gravitation field changes a metrics of space-time, it can change the electric and magnetic fields due to re-distribution of currents and charges in space. However, it is not the case for Barnett experiment: the electric field inside of solenoid should be equal to zero under any gravitation field with axial symmetry due to keeping of homogeneous distribution of the charge upon the surface of solenoid. However, the latter physical requirement comes into certain contradiction with the requirement of local Lorentz invariance (LLI). Indeed, the LLI requires that the cases 2 and 1RG should be equivalent. Hence, in case 1RG the condenser remains uncharged. Further, comparing the cases 1R and 1RG we conclude that for observer attached to rotating condenser, the gravitation field eliminates the electric field $\vec{E}$, existing in case 1R. The same conclusion is derived under comparison of the cases 2R and 2RG. Indeed, according to LLI the case 2RG is equivalent to the case 1, and $Q\neq0$. It means that the gravitation field eliminates the electric field in eq. (10). The LLI also signifies that from a viewpoint of observer in the laboratory frame, the gravitation field reverses the results of Barnett experiment: under rotation of condenser it remains uncharged (case 1G), while under rotation of solenoid condenser is charged (case 2G). It is due to the appearance of electric field inside of solenoid in both these cases: it compensates an action of the magnetic force to the wire A under rotation of condenser (case 1G), and it charges the condenser under rotation of solenoid (case 2G). However, according to electrostatics laws, such the electric field cannot appear inside of solenoid under any homogeneous distribution of ρ over its surface.

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within the adopted accuracy of calculations: the charge density at any fixed point of solenoid is equal to $\rho = j\omega R$, where $\vec{j}$ is the current density of the solenoid in the laboratory frame, and R is the radius of the solenoid. We see that ρ is indeed constant over the surface of solenoid.

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Conclusion

Thus, for a laboratory frame the Maxwell's classical electrodynamics derives the correct results under calculation of a charge of condenser for the Barnett experiment ($Q \neq 0$ in case 1 (Fig. 2, a) and $Q=0$ in case 2 (Fig. 2, b)). At the same time, a consideration of this experiment for the observers being attached to either the rotating condenser (case 1R), or rotating solenoid (case 2R) reveals a difficulty in its physical explanation. Namely, the appeared electric field inside of solenoid in both mentioned cases cannot be produced by elongated solenoid with constant charge density ρ over the surface of solenoid: for any ρ an electric field in the inner volume of solenoid should be equal to zero according to known result of electrostatics. Nevertheless, this field exists, and it disappears under applying of appropriate gravitation fields in the rotating frames, corresponding to the cases 1RG and 2RG. According to the principle of LLI, application of such gravitation field reverses the results of Barnett experiment: rotating condenser remains uncharged, while under rotation of solenoid the condenser acquires a charge. However, the electric field to be responsible for such inversion of the results of Barnett experiment cannot appear inside of solenoid under any homogeneous distribution of charge over its surface. In a visible future it will be impossible to realize the Barnett experiment with an axially symmetrical gravitation field, in order to resolve experimentally a revealed contradiction between the requirements of LLI and electrostatic laws. Nevertheless, it seems interesting for further analysis of the LLI principle.

All the results obtained and short comments are summarized in Table 1.

References

1. Barnett, S.J., Phys. Rev. **35**, 323 (1912).
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3. Logunov, A.A., “Lectures on the Theory of Relativity” (Moscow State University, Moscow 1984) (in Russian).

Table 1

Summary to the Barnett experiment

Conditions	Charge	Comment
Case 1 (Faraday's and others experiments)	$Q\neq0$	Condenser is charged by the force $e(\vec{u}\times\vec{B})$
Case 2 (Barnett experiment)	$Q=0$	Condenser is uncharged, because $E=0$ inside of solenoid under homogeneous distribution of the charge over the surface of solenoid
Case 1R	$Q\neq0$	Condenser is charged by the force $e\vec{E}$. At the same time, according to electrostatics laws, the field $\vec{E}$ should be equal to zero inside of solenoid
Case 2R	$Q=0$	Condenser is uncharged, because the resultant Lorentz force $e\vec{E}+e(\vec{u}\times\vec{B})=0$ inside of solenoid. Here $\vec{E},\vec{B}\neq0$. At the same time, the field $\vec{E}$ should be equal to zero inside of solenoid under homogeneous distribution of charge over the surface of solenoid
Case 1RG (equivalent to case 2 according to LLI)	$Q=0$	Condenser is uncharged, because gravitation field eliminates the electric field, existing in case 1R. At the same time, due to symmetry of the experiment with respect to z axis, the gravitation field with the same symmetry cannot influence an electric field inside of solenoid.
Case 2RG (equivalent to case 1 according to LLI)	$Q\neq0$	Condenser is charged by the force $e(\vec{u}\times\vec{B})$, because gravitation field eliminates the electric field $\vec{E}$, existing in case 2R. At the same time, due to symmetry of the experiment with respect to z axis, the gravitation field with the same symmetry cannot influence an electric field inside of solenoid.
Case 1G	$Q=0$	Condenser is uncharged, because the resultant Lorentz force $e\vec{E}+e(\vec{u}\times\vec{B})=0$ inside of solenoid. Here $\vec{E},\vec{B}\neq0$. At the same time, according to electrostatics laws the field $\vec{E}$ should be equal to zero inside of solenoid
Case 2G	$Q\neq0$	Condenser is charged by the force $e\vec{E}$. At the same time, according to electrostatics laws the field $\vec{E}$ should be equal to zero inside of solenoid