Correlation of Experimental Data and Three-Dimensional Finite Element Modeling of a Spinning Magnet Array

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***Abstract*—A magnet array was spun above an aluminum disk, and the drag torque was measured for various speeds and gap sizes. Drag torques calculated using a three-dimensional finite element program were consistent with measured values. The finite element model was also used to determine the effects of the polarity and position of magnets in the source array. The peak torque was shown to occur when magnets are located at a radius equal to 70% of the disk radius. A magnet array with alternate magnets reversed was shown to produce more than twice the drag of an array comprised of parallel magnets. An approximation for fields under the magnet centers was obtained using a two-dimensional analytical solution.**

INTRODUCTION

Drag forces due to eddy currents induced by the relative motion of a conductor and a magnetic field occur in many practical devices: motors, brakes, magnetic bearings, and magnetically levitated vehicles. Recently, finite element codes have included solvers for 3-D eddy current geometries and have the potential to be very useful in the design and analysis of these devices. In this paper, numerical results from three-dimensional modeling of a magnet array spinning above a conductor are compared to experimental results in order to assess the capabilities of these codes.

EXPERIMENTAL PROCEDURE

A set of experiments was performed in which four Nd-Fe-B permanent magnets mounted in a non-conducting cylindrical drum were rotated over a cylindrical aluminum conductor by means of a variable speed motor, as shown in Fig. 1. The cylinder of aluminum was attached to, and electrically isolated from, a torque transducer. A thin barrier was placed between the rotating magnets and the aluminum so that aerodynamic drag did not influence the measurements. Physical dimensions of the apparatus are given in Table I.

FINITE ELEMENT SOLUTION

The finite element program ELEKTRA 3D by Vector Fields Ltd. was used to model the spinning magnets. The solution uses the magnetic vector potential formulation and Lorentz gauge to solve Maxwell's equations [1]. Fields resulting from the motion of a conductor relative to a fixed dc coil are described mathematically by (1).

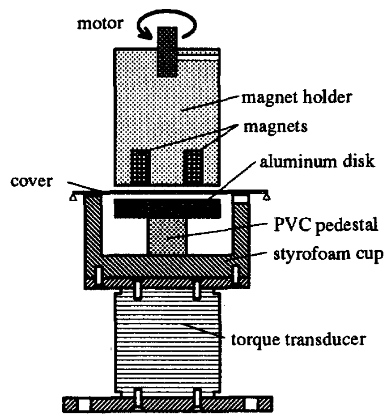

Fig. 1. Experimental Apparatus

TABLE I
PHYSICAL DIMENSIONS OF APPARATUS

Disk radius	19.1	mm
Disk thickness	6.4	mm
Conductivity	$2.5\cdot10^7$	mhos/m
Gap width	1-8	mm
Radius of magnet centers	9.5	mm
Magnet length	12.7	mm
Magnet diameter	6.4	mm

$$\nabla\times\left[\frac{1}{\mu}\nabla\times\mathbf{A}\right]-\sigma\cdot\left[\mathbf{v}\times(\nabla\times\mathbf{A})\right]-\nabla\frac{1}{\mu}\nabla\cdot\mathbf{A}=0 \quad (1)$$
$$\nabla\cdot\sigma\nabla V-\sigma\nabla\cdot\left[\mathbf{v}\times(\nabla\times\mathbf{A})\right]=0$$

A magnetic vector potential μ permeability
V electric scalar potential σ conductivity
v velocity

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The finite element model consisted of approximately 18,000 linear elements and 18,000 nodes. Materials were assumed to be linear. Each permanent magnet was modeled as a thin-walled solenoid. Fig. 2 shows flux density B predicted by the solenoid model and actual flux density measured for a permanent magnet. The four-magnet array was modeled in two orientations--parallel magnetic moments and alternate poles reversed. Fig. 3 shows flux density calculated on a circle intersecting the centers of the solenoids for parallel and alternating magnet configurations.

ANALYTICAL SOLUTION

A two-dimensional analytical model was developed based on a solution used by Connor and Tichy to predict forces in an eddy current journal bearing [2]. Eddy currents in the model, shown in Fig. 4, are driven by motion of a conductive layer at constant velocity parallel to a sinusoidally distributed dc current sheet.

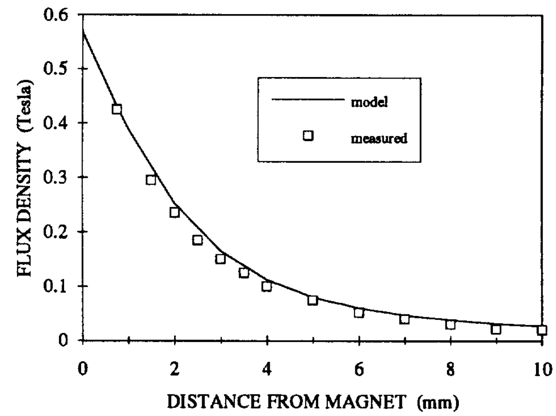


Fig. 2. Axial flux density along magnet centerline.

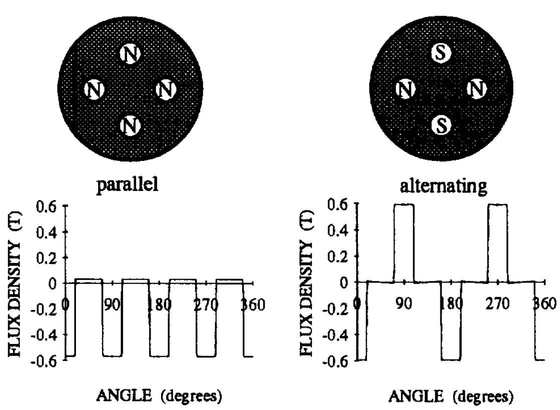


Fig. 3. Axial flux density for parallel and alternating magnet configurations.

Excitation by a stepped source field (such as the one produced by the permanent magnet array) may be approximated by superposing solutions obtained at each spatial harmonic in the spectrum of the source.

The analysis involves finding a solution to (2) in each of the four regions. Solutions for fields in the conductive region are given by (3). Expressions for the constants A_3 and B_3, which have been omitted for brevity, are determined by interface and boundary conditions.

$$\begin{aligned} \nabla^2 H_x &= -\mu\sigma v \frac{\partial H_y}{\partial y} \\ \nabla^2 H_y &= \mu\sigma v \frac{\partial H_y}{\partial x} \end{aligned} \tag{2}$$

$$\begin{aligned} H_x(x,y) &= \mathrm{Re}\left[j(\lambda/\gamma)\left(A_3 e^{\lambda y} - B_3 e^{-\lambda y}\right) e^{j\gamma x}\right] \\ H_y(x,y) &= \mathrm{Re}\left[\left(A_3 e^{\lambda y} + B_3 e^{-\lambda y}\right) e^{j\gamma x}\right] \\ J_z(x,y) &= \mathrm{Re}\left[j\gamma\left\{1-(\lambda/\gamma)^2\right\}\left(A_3 e^{\lambda y} + B_3 e^{-\lambda y}\right) e^{j\gamma x}\right] \end{aligned} \tag{3}$$

$\gamma = n/r$ $\quad$ r radius of magnet centers

$\lambda = \gamma\sqrt{1 + j(\mu\sigma v)/\gamma}$ $\quad$ n # north poles (2 or 4)

Analytical models of this sort have been used successfully to predict eddy current fields in cylindrical geometries with magnetic poles aligned in the radial direction, e.g., eddy current bearings [2] and induction motors [3]. The model is less appropriate when magnetic poles are aligned parallel to the axis of rotation, since "unwrapping" the disk to a linear configuration involves warping of the azimuthal coordinate. In this case, the dependence of source current, disk geometry, and disk velocity on the radial coordinate is ignored. Thus, the domain of validity for the 2-D model is restricted to points on a cylinder passing through the magnet centers. Furthermore, the model is limited to configurations where eddy currents are restricted neither by lack of space near the center of the disk, nor interrupted by the outer edge of the disk.

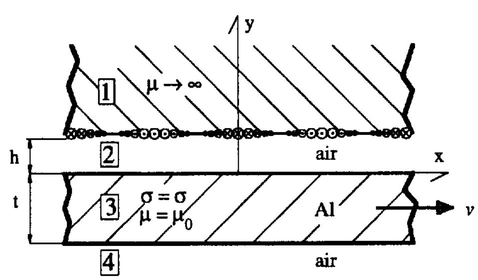


Fig. 4. Analytical model.

RESULTS

Torque results for an aluminum disk at room temperature are given in Fig. 5. Agreement between experimental and finite element results is within an acceptable range of 10% for gap sizes of 2 mm and larger. Increased errors were observed for smaller gaps. The high flux gradient present at the disk surface in a small gap configuration was believed to be the major source of error. A refined mesh in the gap and conductive disk, however, failed to reduce the error. For all gap sizes tested, the model consistently predicted lower torque values than those measured.

Comparisons of eddy current and drag force densities on the surface of the disk calculated using the 2-D analytical and 3-D finite element models are shown in Figs. 6 and 7. The finite element fields were calculated on a circle directly under the magnet centers. Analytical fields were determined by superposition of the first four spatial harmonics in the field for the parallel magnet configuration. The 2-D analysis gives an adequate estimate of "centerline" fields, i.e., those lying on a cylinder intersecting the magnet centers. It cannot, however, determine fields at points off that cylinder, predict global forces, or model edge effects.

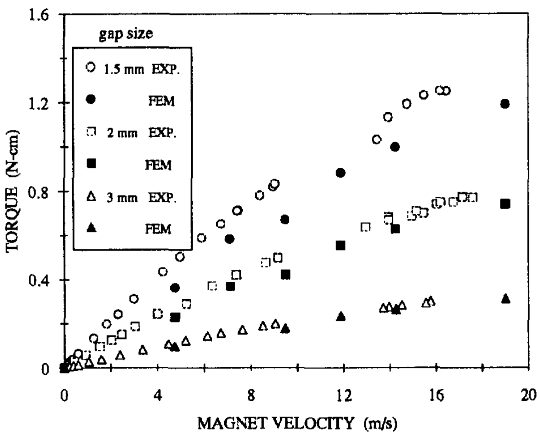


Fig. 5. Torque vs. average magnet velocity

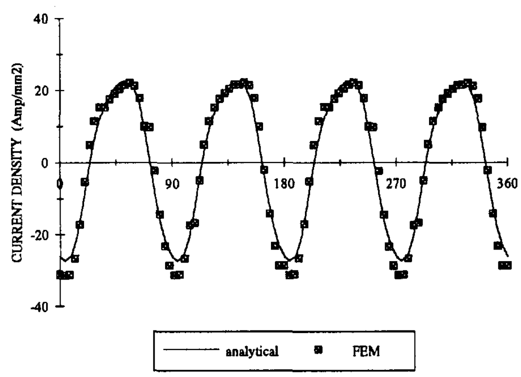


Fig. 6. Analytical and numerical eddy current density on surface of disk

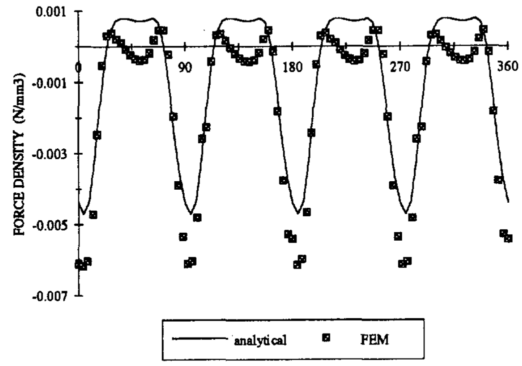
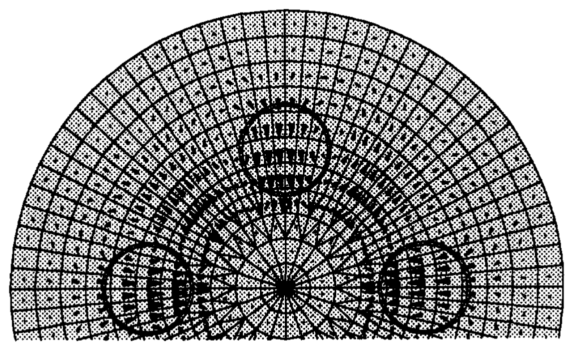


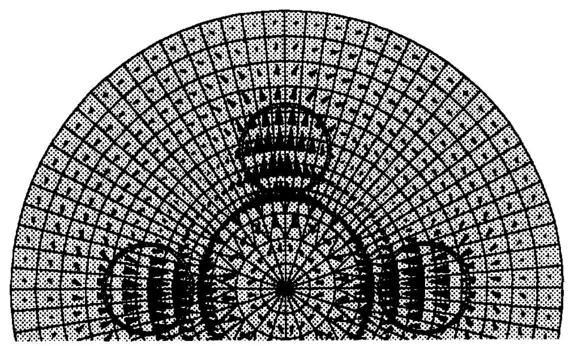
Fig. 7. Analytical and numerical drag force density on surface of disk

Effect of Magnet Polarity

The effect of magnet polarity was investigated using the finite element model. Eddy current flows induced by parallel and alternating magnet configurations with a 2-mm gap at a speed of 14,300 rpm (14.25 m/s magnet velocity) are shown in Fig. 8. The effect of alternating polarity on drag torque for a range of speeds is shown in Fig. 9. Alternating polarity produces an increase in torque by a factor of 2.4 at all speeds.



parallel; magnets at radius 9.5 mm



alternating; magnets at radius 9.5 mm

Fig. 8. Eddy currents for parallel and alternating magnet arrays

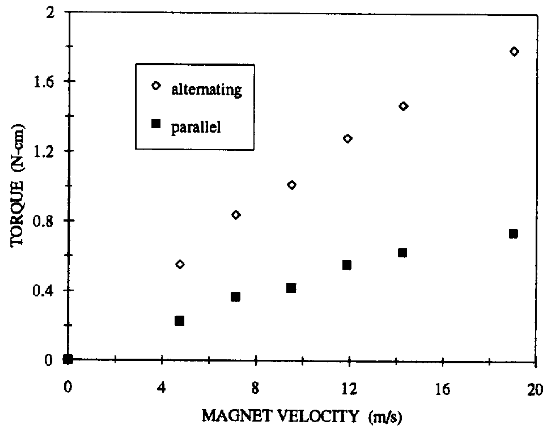
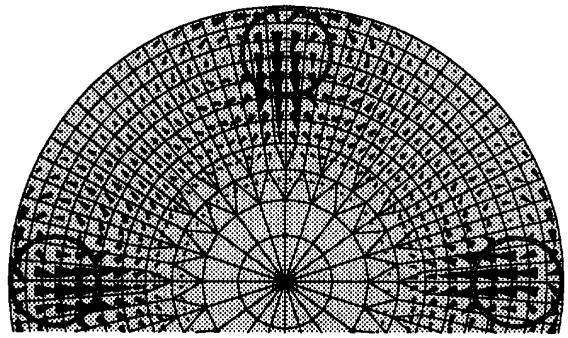


Fig. 9. Calculated effect of alternating polarity on torque.



parallel; magnets at radius 15.9 mm

Fig 11. Eddy currents influenced by disk edge.

Effect of Magnet Center Radius

Fig. 10 shows the calculated effect of magnet center radius on drag torque for a rotating speed of 14,300 rpm. Peak drag occurs when the ratio of magnet center radius to disk radius is in the range of 0.65 to 0.75. At larger ratios, torque decreases as the disk edge begins to influence eddy current flow. The effect is illustrated for the case of parallel magnets at 14,300 rpm in Fig. 11. The magnet center radius for peak torque with alternating magnets is slightly less than for parallel magnets.

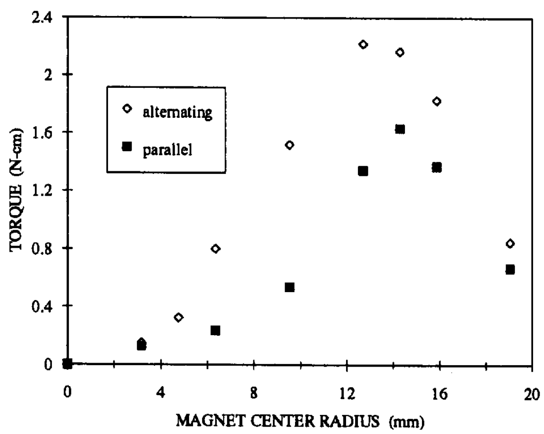


Fig. 10. Calculated effect of magnet center radius on torque.

CONCLUSIONS

The finite element solution of the three-dimensional code ELEKTRA has accurately predicted the torque created by an array of four permanent magnets with parallel magnetic moments that rotate at low velocity over a conductor. The geometry of the experiment, like that of many electromechanical devices, does not readily admit two-dimensional analysis. By combining the results of several 2-D harmonic solutions, one may obtain a reasonable estimate for current and force densities at points on a cylinder intersecting the magnet centerlines, but such an analysis fails to provide any information regarding the influence of the radial coordinate. Edge effects cannot be determined without a full 3-D solution.

The finite element results indicate that the drag torque would be considerably increased if the magnet array had alternating polarity. The maximum drag torque for a particular rotating speed occurs when magnet centers are located at approximately 70% of the disk radius.

REFERENCES

[1] OPERA-3D User Guide, Vector Fields Ltd., Oxford, England, 1992.

[2] K. A. Connor and J. A. Tichy, "Analysis of an eddy current journal bearing," *ASME Journal of Tribology*, vol. 110, pp. 320-6, April 1988.

[3] H. H. Woodsen and J. R. Melcher, *Electromechanical Dynamics*, Part II. New York: Wiley, 1968, pp. 355-70.