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# Do Magnetometers Measure B or H?

F. J. Lowes

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*Summary*

It is shown by simple arguments that real magnetometers, whether scaled in units of **B** or of **H**, all have calibrations which depend to some finite extent on the permeability of the ambient medium in which the field is measured.

## 1. Introduction

Geomagnetic measurements are usually made in media which have a relative permeability very close to unity, and in the cgs emu system of units we also had $\mu_0 = 1$. It did not really matter therefore whether our magnetometers measured **B** or **H**, or whether we expressed the results in Gauss or Oersted, as the numerical values were very nearly equal. In fact it is no longer clear whether the gamma was introduced as $10^{-5}$ Gauss or as $10^{-5}$ Oersted!

However in SI $\mu_0$ is very different from unity, so at the very least we must decide whether to express our results in terms of **B** or of **H**. Also the introduction of the mks system, with the subsequent emphasis on electromagnetism rather than magnetostatics, had led to controversy as to which of the fields was more ' fundamental ', and which was actually measured, and these arguments have now spread into geomagnetism. Further the accuracy of magnetic field measurements is improving, so the effect of the ambient medium may soon not be negligible. It is therefore necessary to attempt to answer the question posed in the title of this paper.

## 2. Units

For the purpose of this paper **B** and **H** are simply two different vector fields with properties which are useful in describing the behaviour and effects of electric circuits and magnetized materials—if the reader wishes to think of them as different macroscopic averages of the spatially rapidly varying atomic field, or through the cavity definitions, he is free to do so; which, if either, field is more ' fundamental ' is irrelevant.

The relative units and dimensions of **B** and **H** are to a large extent arbitrary, but in SI they are such that $\mathbf{B} = \mu_0 \mathbf{H}$ in ' free space '. Inside a magnetic material, **B** and $\mu_0 \mathbf{H}$ differ by a third vector field which specifies the intensity of magnetization, and in (rationalized) SI we have

$$\mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M} = \mu_0 \mathbf{H} + \mathbf{J}, \tag{1}$$

where $\mu_0 \mathbf{M}$ and **J** are simply two different ways of expressing the same quantity. In ' linear ' media we find that **M** and **J** are proportional to **H**:

$$\mathbf{J} = \mu_0 \mathbf{M} = \mu_0 \chi \mathbf{H} \tag{2}$$

where $\chi$ is the susceptibility. We can therefore write

$$\mathbf{B} = \mu_r \mu_0 \mathbf{H} \tag{3}$$

where

$$\mu_r = 1+\chi \tag{4}$$

is the relative permeability.

In (non-rationalized) cgs emu we had essentially

$$\mathbf{B} = \mathbf{H}+4\pi\mathbf{M}, \tag{1a}$$

$$\mathbf{M} = \chi\mathbf{H}, \tag{2a}$$

$$\mathbf{B} = \mu_r \mathbf{H}, \tag{3a}$$

$$\mu_r = 1+4\pi\chi. \tag{4a}$$

$\mu_r$ is non-dimensional, and for a given material has the same value in all systems of units; however because of the normalization, although $\chi$ is also non-dimensional its value is $4\pi$ times larger in SI than in emu.

## 3. The ideal magnetometer

We might try to measure **B** with a magnetometer using the induction law $E = -dB/dt$. Alternatively we might try to measure **H** with a feedback magnetometer using Ampere's law $\oint \mathbf{H}.\mathrm{ds} = I$. However, as we shall see, the materials from which a magnetometer is constructed distort the ambient field. In addition many magnetometers themselves produce considerable magnetic fields when they are operating. What we want is an instrument whose output meter registers the value of the ambient field which was present *before* the magnetometer was inserted. Let this ambient field, for convenience assumed uniform, be $\mathbf{B}_0 = \mu_e \mu_0 \mathbf{H}_0$, where $\mu_e$ is the relative permeability of the (*e*xternal) ambient medium.

If the output meter is calibrated in units of **B**, and if the calibration is correct, *regardless of* $\mu_e$*, the permeability of the medium in which* $\mathbf{B}_0$ *is measured*, then we have an ideal magnetometer measuring **B**; similarly for **H**.

In the next section it is shown that a wide variety of magnetometers are *not* ideal in this sense, and it is most unlikely that such an ideal magnetometer, measuring either **B** or **H**, does in fact exist. In practice a very good approximation to such an ideal instrument can be made, but with a real instrument this is never more than an approximation, no matter how good.

## 4. Real magnetometers

Let us consider various magnetometers made using one or more of the following: a flat coil, a magnet, a Mumetal rod, and a bottle of water, together with a few accessories.

(a) The induced emf in the coil gives the rate of change of the flux of **B** through the coil. For a single turn of infinitesimally thin wire the area of the coil is easily calculated. For a multiple turn coil using wire of finite cross-section the geometrical area turns $(An)_0$ can, at least in principle, still be calculated.

But the wire, insulation, and coil former will in general have a permeability different from that of the surrounding medium. (Even if these constructional materials were perfectly non-magnetic (which they never are), on the Earth's surface the surrounding medium, air or water, is not.) The lines of force of the previously uniform $\mathbf{B}_0$ are therefore now distorted, the nature and amount of the distortion depending on the geometry of the coil and on the permeability contrast. The flux linkage of $\mathbf{B}_0$ with the coil therefore is *not* $B_0(An)_0$, but an 'effective' area-turns can be defined such that the flux linkage is $B_0(An)_{\text{eff}}$. We can write

$$(An)_{\text{eff}} = (An)_0[1+f(\mu_c/\mu_e)] \tag{5}$$

where $f$ is a small correction function whose nature depends on the coil geometry; $\mu_c$ is the relative permeability of the coil material (assumed uniform for simplicity) and $\mu_e$ that of the ambient medium.

We see that the magnetometer reading depends not only on $dB_0/dt$ but also to a finite, if small, extent on the permeability of the ambient medium.

(b) We can increase the sensitivity of our induction magnetometer by inserting the Mumetal rod. Now almost all the flux linked with the coil is carried by the highly permeable rod, so that the effect of the coil materials is probably negligible. But what determines the flux through the rod? For generality we will now call this rod the 'core'.

For an ellipsoidal core having relative permeability $\mu_i$ (for *i*nternal), there will be a uniform field inside it given by (see e.g. Lowes 1974)

$$\mathbf{B}_i = \frac{\mu_i}{N\mu_i+(1-N)\,\mu_e}\,\mathbf{B}_0 = \frac{\mu_i\,\mu_0}{N(\mu_i/\mu_e)+(1-N)}\,\mathbf{H}_0. \tag{6}$$

$N$ is the 'demagnetizing factor', a geometrical constant which is approximately zero for a long thin core, $\frac{1}{3}$ for a sphere, and approximately unity for a short fat core. (If the core is not ellipsoidal $\mathbf{B}_i$ will not be uniform, but (6) can be applied to other shapes by using an appropriate average $B_i$, and determining an effective $N$ empirically.) At the limit $N = 0$ we would have the flux proportional to $H_0$ regardless of $\mu_e$, but for finite length cores $N$ is finite, and because $\mu_i$ is large the flux will depend significantly on $\mu_e$, the permeability of the ambient medium. $\mathbf{B}_i$ *is* almost independent of $\mu_e$ near the other limit $N \to 1$, but then we have not gained by using the Mumetal!

(c) Our induction magnetometer measures only change of flux, but we can use it to measure a constant $\mathbf{B}_0$ by varying $\mu_i$; this is done by wrapping an exciting coil round the core, and passing through it an alternating current large enough to take the core into saturation. In practice two such cores are used, with antiphase excitation so that there is no net flux linkage between the exciting coils and the pick up coil; we now have a 'fluxgate' sensor. The instantanteous emf in the pick-up coil is proportional to the rate of change of an expression like (6), where now $\mu_i$ is the (cyclically varying) *differential* permeability $dB/dH$ of the core material (Burger 1972). The output of the magnetometer is essentially some average of the instantaneous emf, and so will be an even more complicated function of the permeability of the external medium.

(d) Now take our original coil, pass a fixed current $I_0$ through it, and measure the torque on it when it is placed in $\mathbf{B}_0$. The torque is determined by the magnetic dipole moment produced by $I_0$ (see e.g. Lowes 1974), but this is affected to some extent by the materials of the coil, which distort the field produced by $I_0$. (For example, the wire on one side of the coil will be magnetized by the field of the current in the opposite side.) The maximum torque is

$$T = (An)_0[1+h(\mu_c/\mu_e)]\,I_0\,B_0, \tag{7}$$

where $h$ is a small correction function whose nature depends on the coil geometry and whose value on the permeability contrast between the coil and the surrounding medium.

(e) Let us measure the ambient field by suspending our magnet and measuring the torque on it. There has been a long controversy as to what is measured, but Lowes (1974) has shown that for a well-stabilized magnet of ellipsoidal shape the torque is in fact

$$\mathbf{T} = \frac{1}{N\mu_i + (1-N)\,\mu_e}\,\mathbf{m}_0 \times \mathbf{B}_0 = \frac{1}{N(\mu_i/\mu_e) + (1-N)}\,\mathbf{j}_0 \times \mathbf{H}_0 \tag{8}$$

where $\mathbf{j}_0 = \mu_0\,\mathbf{m}_0$ is a constant; $\mu_i$ is now the 'recoil' permeability of the magnet (the differential permeability at the working point), typically of order of magnitude 10.

For a short fat magnet, $N \simeq 1$, the torque is proportional to $B_0$ and very nearly independent of the medium. For a long thin magnet, $N \simeq 0$, the torque is proportional to $H_0$ and very nearly independent of the medium; this is a much better approximation than in equation (6). However, in neither case can the limit actually be obtained.

(f) If we take our bottle of water and measure the Larmor precession frequency of the individual protons in it we have a proton precession magnetometer. The field 'seen' by the precessing protons is not of course the original $B_0$. Quite apart from the constant shielding effect of the electrons of the water molecule in which the proton sits (the 'diamagnetic correction'), there is also the 'bulk paramagnetic correction' due to the non-zero susceptibility of the 'water' (in practice a dilute paramagnetic solution). Precision laboratory measurements in air do in fact already need to apply corrections for the shape and permeability of the 'water' (Dickinson 1951); when allowance is made for the permeability of the external medium the correction is similar to equation (6).

## 5. Feedback magnetometers

Any of the devices (c)–(f) discussed above can be used as the null-detector in a feedback magnetometer; the output reading is then the position of a magnet or the current through a coil. With large enough feedback the net field at the detector is effectively zero, and whether the detector 'sees' **B** or **H** is irrelevant. If $\mathbf{B}_F = \mu_e\,\mu_0\,\mathbf{H}_F$ is the feedback field (in the absence of the detector), we have $\mathbf{B}_F = -\mathbf{B}_0$ and $\mathbf{H}_F = -\mathbf{H}_0$ at the position of the detector.

If the feedback field is produced by our magnet we have (Lowes 1974)

$$B_F = \mu_e\,\mu_0\,H_F = \frac{\mu_e\,\mu_0}{N\mu_i + (1-N)\,\mu_e}\,m_0\,M \tag{9}$$

where $M$ is a purely geometric factor (which is varied to adjust $B_F$). (Note the difference from equation (8); a long thin magnet ($N \simeq 0$) produces a **B** field, but experiences a torque in a given external **H** field, almost independent of the medium.)

If the feedback field is produced by passing a current $I_F$ through our coil, then the coil materials will (slightly) distort the field produced by an amount again proportional to the permeability constant. We can put

$$B_F = \mu_e\,\mu_0\,H_F = \mu_e\,\mu_0\,k[1 + g(\mu_c/\mu_e)]\,I_F \tag{10}$$

where $k$ is a purely geometric factor, and $g$ is a small correction function which now depends not only on the coil geometry but also on the observation point.* (Again there is a contrast between the situations of equations (10) and (7); a coil of small cross-section carrying a fixed current produces a field **H**, but experiences a torque proportional to **B**, almost independent of the medium.)

* As the function $h$ of (7) was defined for uniform $B_0$, $g = h$ only at large distances.

We see that with real feedback systems there is again a finite, if small, dependence of output reading on the permeability of the medium.

## 6. Discussion

In order to obtain the comparatively simple results of the last two sections, a large number of simplifying assumptions have been made, which for brevity were not listed. (For an example of a more rigorous treatment see e.g. Section 3 of Lowes 1974.) In real situations not all these assumptions will be valid, and the magnetometer output readings will depend in even more complicated ways on the various parameters.

Also, most magnetometer heads are ' potted ' in, or enclosed by, a container which excludes the ambient medium; there will then be an additional factor of the form of (6) involving the permeability of the potting material.

We see that real magnetometers are not ' ideal ' in that they do not in fact measure exactly **B** or **H**; they have calibrations which depend to some extent on the permeability of the medium in which they are measuring.

In geomagnetic practice however, to present measurement accuracy we can make good enough approximations to the ideal. This is because the permeabilities of the usual ' non-magnetic ' materials of construction differ from those of air or water by only a few parts in $10^5$.

Also in practice the distinction between **B** and $\mu_0$ **H** does not matter, as for air $\mu_r = 1+4\cdot10^{-7}$ and for water $\mu_r = 1-9\cdot10^{-6}$. So even in water, to an accuracy of 1 in $10^5$, about 0·5nT, we have $\mathbf{B} = \mu_0 \mathbf{H}$. (If we wished to measure more accurately than this in water, we would in any case have to remember that the Earth's field will have been distorted by the presence of the water.)

However, quite apart from these approximations, we see that from the instrumentation aspect there is no reason for preferring to express our results in terms of either **B** or **H**. For simplicity of conversion from emu values to SI values the International Association of Geomagnetism and Aeronomy has in fact recommended that magnetic fields be expressed as **B**, for which 1 tesla (abbreviation T) equals $10^4$ Gauss, so that by defining $1\gamma = 10^{-5}$ G we have $1\gamma = 1$nT. Similarly it has recommended that the intensity of magnetization be expressed as **M**, for which 1 ampere/metre (abbreviation $\mathrm{Am}^{-1}$) equals $10^{-3}$ emu. (Although dimensionally the emu unit of magnetization is the oersted, it differs from it by $4\pi$; see equation (1a).)

*School of Physics,*
*The University,*
*Newcastle upon Tyne NE1 7RU, U.K.*

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