

the argumentation aiming to bring out the essential ambiguity involved in a reference to physical attributes of objects when dealing with phenomena where no sharp distinction can be made between the behavior of the objects themselves and their interaction with the measuring instruments.”²

Both Bohr and Heisenberg repeatedly cautioned that ingrained ways of thinking and speaking may be inappropriate in a quantum context. In spite of this, the C.I. has been widely misunderstood and misrepresented, leading to unjustified claims that it is vacuous, circular, and subjective.

The reasoning underlying the C.I. is perhaps most easily understood by considering its application in a context where physics as such plays no role. Thus, in the case of the unexpected hanging, the prisoner can avoid paradox if he recognizes a distinction between the assumption that the decree may be taken as a true premise, and the lesser assumption that it will correctly describe the total situation. Referring to the paradox of the liar, the medieval philosopher John Buridan said:

“Those propositions are indeed in conflict with regard to being true, but they are not in conflict with regard to the case being as they signify.”³

This characterization applies equally well to the statements of the decree in the paradox of the unexpected hanging, and to the claim about the atom in the paradox of Schrödinger’s cat.

Ballentine apparently does not like the C.I., and hints at his own preference, namely the statistical interpretation (S.I.). It must be recognized, however, that this approach is distinct from the three interpretations discussed in my paper. In particular, it conflicts with the C.I., because it assumes the quantum mechanical state to refer to an ensemble of systems. The artificiality of this assumption probably accounts for the fact that the S.I. is not widely accepted, though it does find adherents among those who for some reason cannot accept the C.I.⁴

Finally, I turn to Ballentine’s contention that the point of the cat paradox was not as I stated, but rather “to test whether or not the theoretical predictions of quantum mechanics (including an interpretation of the meaning of a state function) are in accord with experience.” This does convey the deep questions which Schrödinger had regarding his own theory by 1935, when he wrote the three-part article which introduced his cat.⁵ However, I have two reservations. First, I repeat that the cat paradox itself was meant only as a heuristic device. If Schrödinger’s intentions are not made clear by his presentation of the paradox (at the end of part one), then they are by his subsequent allusion to it (at the end of part two) in conjunction with an explanation of why the measurement process which it exemplifies need not be considered paradoxical. Second, I argue that Ballentine’s desire to derive an interpretation of the meaning of the state function from quantum mechanics cannot be met.⁶ Certainly the predictions of quantum mechanics can help to differentiate it from other theories (such as one with local hidden variables or one with nonlinear correction terms), but they cannot differentiate among interpretations of the meaning of the state function which are consistent with those predictions. The continuing debate on the interpretation of quantum mechanics serves to remind us of this fact.

¹W. Heisenberg, in *Niels Bohr and the Development of Physics*, edited by W. Pauli (McGraw-Hill, New York, 1955), p. 27.

²N. Bohr, in *Quantum Theory and Measurement*, edited by J. A. Wheeler and W. H. Zurek (Princeton University, Princeton, NJ, 1983), p. 42.

³J. Buridan, *Sophisms on Meaning and Truth*, translated by T. K. Scott (Appleton-Century-Crofts, New York, 1966), p. 183.

⁴T. Bastin, in *Quantum Theory and Beyond*, edited by T. Bastin (Cambridge University, Cambridge, 1971), p. 5.

⁵E. Schrödinger, *Die Naturwissenschaften* **23**, 807–812, 823–828, 844–849 (1935); translation in *Quantum Theory and Measurement* (see Ref. 2), p. 152.

⁶F. Suppe, in *The Structure of Scientific Theories*, edited by F. Suppe (University of Illinois, Urbana, IL, 1977), pp. 6–61.

Field angular momentum in Feynman’s disk paradox

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The angular momentum in static electromagnetic fields has always been a fascinating subject.^{1–10} Feynman’s disk paradox¹ is a striking example of this. In their recent article on Feynman’s disk paradox, Bahder and Sak² calculated the angular momentum in two ways, one by using Faraday’s law, the other by using Lorentz force law. Although it was asserted that the angular momentum is stored in the fields, the direct approach to calculate this angular momentum from the fields has not been shown. We present this calculation so that students may understand the reason angular momentum is said to be stored in the fields.

A brief summarization of the paradox is in order. The same configuration as in Ref. 1 will be adopted. A current carrying small solenoid of radius a is located at the center of an insulator disk of radius $R \gg a$ (Fig. 1). Embedded at the edge of the disk there is a ring of charged particles. We

will take the continuum limit so that the number of these charged particles approaches infinity, while the total charge remains Q . The coil carries a steady current I to begin with and the disk is initially at rest. The initial angular momentum thus seems to be zero. Now the current in the coil is decreased to zero. Faraday’s law dictates that there be an induced electric field around the disk in the counterclockwise direction looking from top (in the negative z direction). The charged particles will thus rotate accordingly. Now the system has a nonzero angular momentum. It appears that angular momentum conservation for a closed system is violated. The resolution to this is that initially there is angular momentum in the static electromagnetic fields.^{1–10}

Let us calculate this field angular momentum. The magnetic field due to the solenoid may be taken as that of a

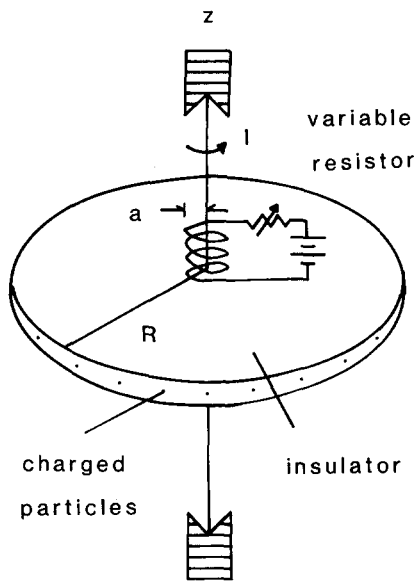


Fig. 1. Disk with coil, battery, and charged particles is mounted on frictionless bearings. The variable resistor allows the change of current.

dipole \mathbf{u} at the center of the disk²

$$\mathbf{B}(\mathbf{r}) = \frac{3(\mathbf{u}\cdot\mathbf{r})\mathbf{r} - r^2\mathbf{u}}{r^5} = -\nabla\left(\frac{\mathbf{u}\cdot\mathbf{r}}{r^3}\right) = \nabla\left(\nabla\cdot\frac{\mathbf{u}}{r}\right). \quad (1)$$

The ring of charged particles at $r = R$, $\theta = \pi/2$ produces a static electric field $\mathbf{E} = -\nabla V$, where V is, in spherical coordinate system, the solution to Laplace's equation¹¹

$$V(r, \theta) = Q \sum_{l=0}^{\infty} \frac{r_{<}^l}{r_{>}^{l+1}} P_l(0) P_l(\cos \theta), \quad (2)$$

where $r_{<}$ ($r_{>}$) is the smaller (greater) of r and R . The field angular momentum is well known to be

$$\mathbf{l} = \frac{1}{4\pi c} \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) = \frac{1}{4\pi c} [\mathbf{E}(\mathbf{r}\cdot\mathbf{B}) - \mathbf{B}(\mathbf{r}\cdot\mathbf{E})]. \quad (3)$$

The symmetry of the problem suggests that the angular momentum has only z component if it is calculated for spherical shells. The radial field angular momentum distribution function is given by

$$l_z(r) = \int_{-1}^1 d(\cos \theta) \int_0^{2\pi} d\phi \mathbf{l} \cdot \hat{\mathbf{z}}. \quad (4)$$

Using Eqs. (1), (2), and (3) this can be calculated to be

$$l_z(r) = \frac{2}{5} \frac{uQ}{c} \frac{r_{<}^2}{r_{>}^3} - \frac{2}{3} \frac{uQ}{c} \frac{d}{dr} \left(\frac{1}{r_{>}} \right) - \frac{1}{15} \frac{uQ}{c} \frac{d}{dr} \left(\frac{r_{<}^2}{r_{>}^3} \right). \quad (5)$$

One may then calculate the angular momentum $L_{<}$ within the sphere of radius R by doing the r integration of $l_z(r)$ from 0 to R :

$$L_{<} = \int_0^R dr \left[\frac{2}{5} \frac{uQ}{c} \frac{r}{R^3} - \frac{1}{15} \frac{uQ}{c} \frac{d}{dr} \left(\frac{r^2}{R^3} \right) \right] = 2uQ/15cR. \quad (6)$$

Outside of the sphere the field angular momentum $L_{>}$ may also be found by integrating from R to ∞ :

$$L_{>} = \int_R^\infty dr \left[\frac{2}{5} \frac{uQ}{c} \frac{R^2}{r^4} - \frac{2}{3} \frac{uQ}{c} \frac{d}{dr} \left(\frac{1}{r} \right) - \frac{1}{15} \frac{uQ}{c} \frac{d}{dr} \left(\frac{R^2}{r^3} \right) \right] = 13uQ/15cR. \quad (7)$$

The total angular momentum is thus found to be uQ/cR as in Ref. 2. It may be surprising to find that there is a finite fraction of the total field angular momentum within a sphere of finite volume.

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³R. H. Romer, *Am. J. Phys.* **53**, 15 (1985).

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⁶E. Corinaldesi, *Am. J. Phys.* **48**, 83 (1980).

⁷G. G. Lombardi, *Am. J. Phys.* **51**, 213 (1983).

⁸F. L. Boos, *Am. J. Phys.* **52**, 756 (1984).

⁹I. Adawi, *Am. J. Phys.* **44**, 762 (1976).

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¹¹J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1969), p. 64.

Stable oscillating orbits of a charged particle moving parallel to a current

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It is well known that two long parallel currents attract one another. By the same token a charged positive particle moving parallel to a long wire carrying a current is attracted to the current. If the charged particle also has angular momentum about the wire then there will be an effective repulsive centrifugal potential in the radial coordinate and

hence the possibility of a stable radius and oscillations about this radius. Although the calculation of the charged particle motion is a textbook-type exercise which uses standard techniques,¹ it seems to be relatively novel as example (compare, e.g., the examples in Ref. 2).

In cylindrical coordinates (r, θ, z) we can write the accel-