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Dear Kirk,

You asked if I could explain how I arrived at figures 4 and 5 of my article "Why an Antenna Radiates." I hope the following information will be useful.

The Current Distribution

My first task was was to compute the current distribution on the antenna. In the 1992 era I didn't have access to NEC software, so I worked this problem myself. I divided the dipole lengthwise into 101 equal segments of length L_S, numbered from -50 to 50. I expressed the current i on segment k as a quadratic function of the of the current $I(k)$ at the center of the segment and the distance $x - x_k$ from the center of the segment:

$$i = I(k) + \mu_1(k)(x - x_k) + \tfrac{1}{2}\mu_2(k)(x - x_k)^2 .$$

I derived expressions for the coefficients $\mu_1(k)$ and $\mu_2(k)$ in terms of the currents at the centers of segments $k-1$, k, and $k+1$. This was done by writing equations stating that the current and its first and second spatial derivatives were continuous across the segment boundaries.

I then developed an expression for the tangential component $E(k)$ of the electric field on the surface of the antenna at the center of segment k. This field arises from the currents and charges on all 101 segments. In the resuting equations for all except the center segment, I set $E(k) = \rho I(k)$, where ρ is the ohmic resistance per unit length of the antenna conductor. I assumed that the power source had emf V_S and internal impedance Z_S and was distributed uniformly along the center segment. I set the field at the center of this segment equal to $-V_S / L_S + (Z_S / L_S) I(0) + \rho I(0)$. There resulted a set of 51 linear equations in the 51 center currents $I(0)$ to $I(50)$. I used a desktop computer to solve these equations for the center currents by Gauss reduction. Values of i between the segment centers were found, when needed, by cubic spline interpolation.

During this and subsequent work I made extensive use of two formulas I had developed, one for the total vector electric field $\vec{E}_T$ and the other for the vector magnetic field $\vec{B}$, which were valid at all points on and off the antenna. The formulas are

$$\vec{E}_T = -\frac{1}{4\pi\varepsilon_0 r_0^2}\iint\left(\frac{1+ju}{u^2}\sigma\vec{r}_1 + \frac{j}{cu}\vec{i}\right)e^{-j\omega r/c}\,ds$$

and

$$\vec{B} = \frac{\mu_0}{4\pi}\iint\left(\frac{1}{r^3}+\frac{j\omega}{r^2c}\right)\vec{r}\times\vec{i}\,e^{-j\omega r/c}ds\,.$$

In these formulas, $c = 2.99793\times10^8$, the speed of light

$\varepsilon_0 = 10^7/4\pi c^2$, the permittivity of space

$\mu_0 = 4\pi\times10^{-7}$, the permeability of space

$j = \sqrt{-1}$

ω is the angular frequency of the power source

$\lambda = 2\pi c/\omega$

$r_0 = \lambda/2\pi$

ds is the element of surface area on the antenna

$\vec{r}_1$ is the unit vector directed from the point of observation toward the element ds

$\vec{r}$ is the vector from the point of observation to the element ds

r is the length of the vector $\vec{r}$

$u = r/r_0$

σ is the surface charge density on the antenna (charge per unit area)

$\vec{i}$ is the vector surface current density on the antenna (current per unit path width)

In the foregoing equations, some of the variables represent time-varying physical quantities that have phase as well as amplitude, such as the field $\vec{E}$ and the surface charge density σ. These variables are complex numbers whose magnitude is the peak amplitude of the quantity and whose angle is the phase. In my work the phase of the power-source emf was used as the phase reference.

Like a good engineer, I used rationalized mks units throughout.

Some time I'd like to know how NEC4 carries out this analysis.

The Charge Distribution

The complex surface current density was now available throughout the antenna by interpolation between segment centers. Since it was always pointed in the x direction I could treat it as a scalar complex quanty, i. I computed the surface charge density σ from the equation of continuity,

$$\sigma = -\frac{1}{j\omega}\frac{di}{dx}.$$

The Coulomb Field

One normally thinks of a coulomb field as an electrostatic field emanating from a distribution of non-varying charges. If the distribution consists of surface charges, the static coulomb field $\vec{E}_C$ is given by the inverse square relation

$$\vec{E}_C = -\frac{1}{4\pi\varepsilon_0}\iint \frac{\sigma'}{r^2}\vec{r}_1\, ds$$

in which σ' is the distribution of non-varying surface charge and the rest of the variables are as defined previously. My antenna has charge σ that varies sinusoidally with time, but the above equation is still perfectly valid if I replace σ' by the oscillating antenna charge σ. I have now defined the coulomb field of the antenna. This field is sinusoidal instead of electrostatic, but I can still call it the coulomb field, right?

You might prefer to see the the antenna's coulomb field defined as

$$\vec{E}_C = -\frac{1}{4\pi\varepsilon_0}\iint \frac{\sigma}{r^2}\vec{r}_1 e^{-j\omega r/c}\, ds \qquad \text{(wrong)}$$

in which I've introduced the phase retardation factor $e^{-j\omega r/c}$ to include a propagation delay proportional to distance. But the coulomb field is a non-propagating field, and the definition I used doesn't include a retardation factor. A sudden change in σ results in a simultaneous change in the entire coulomb field. This seems contrary to Mr. Einstein's teachings, but some well-informed authors have explained why there's no contradiction.

The Dynamic Electric Field

I now had expressions and computer programs for the total electric field $\vec{E}_T$, the coulomb field E_C, and the magnetic field $\vec{B}$ at all points, both on and off the antenna. I then defined another electric field, $\vec{E}_D$, which I needed for describing the flow of power from the antenna. In my article I called this field the dynamic electric field. It is given by the formula

$$\vec{E}_D = \vec{E}_T - \vec{E}_C$$

and it is what remains of the electric field after the coulomb field is subtracted off. The dynamic electric field is the field that is produced by electron acceleration in the antenna.

It was illuminating to separate $\vec{E}_D$ into two components, one in phase with $\vec{B}$ and the other 90 degrees out of phase: $\vec{E}_D = \vec{E}_{RAD} + \vec{E}_{IND}$. I called $\vec{E}_{RAD}$ the radiation field because it carries real (unidirectional) power from the surface of the antenna out into the surrounding universe. I called $\vec{E}_{IND}$ the induction field; it carries energy in alternating directions with a net flow of zero. The induction field plays a role in the transfer of energy back and forth between the stored electric and magnetic fields that alternately surround the antenna.

Power Flow

The Poynting vector, which is equal to the complex vector cross product $\vec{E}_T \times \vec{B}$, gives the instantaneous power density (the power per unit area) flowing past any point. The power flows in the direction of the Poynting vector with a power density equal to the instantaneous magnitude of the Poynting vector. The Poynting vector lies in any plane that contains the axis of the antenna, as does the electric field vector. The magnetic field vector is perpendicular to that plane. The direction of power flow is perpendicular to the electric field vector and to the magnetic field vector.

I evaluated and plotted the power flow, averaged over one rf cycle, in various spatial regions. I found that in the far field (more than a few wavelengths from the antenna, where the radiation field predominates) the plot was as expected—the power flow was generally away from the antenna and its average value was in accord with the well-known directional radiation pattern of a dipole antenna. So far, so good.

In the near field I got an entirely different story. The power flow plots were difficult to understand. All the flow lines ended on the power source instead of on the body of the antenna as one would like. It was as if the antenna itself was playing a minor part in the activity. This was a most unsatisfying result. I then decided to plot the power flow attributable to each of the three electric fields—coulomb, induction, and radiation—individually. The whole picture suddenly came into sharp focus after I found that each of the electric fields by itself plays a simple and well-defined part in the game. Only when the three fields are jumbled together does the game become disorganized.

You asked about figures 4 and 5 of the article. Figure 4, the map of the coulomb field, is a computer-generated plot based on numerical evaluation of the above formula for the field. The plot is exactly as generated by the computer, without any fudging on my part to support a point of view (as some may have imagined). You mention difficulty in accepting the fact that the field lines don't meet the antenna at right angles. It's true that a civilized *electrostatic* field is always perpendicular to any conducting surfaces. If it weren't, it would drag free electrons along the conducting surfaces until until the field did come in at right angles. The difference here is that the charges which produce my coulomb field are *oscillating sinusoidally in strength,* and the phase of the oscillation varies along the antenna, as you implied in your current distribution at the top of page 2 of your letter. In any event, my coulomb field has to be off perpendicularity in order to drag each free electron to and fro and thereby expend on it the same amount of energy that it's radiating. The field lines also have to lean away from perpendicularity enough to propel the free electrons through any ohmic resistance that is present in the conductor.

More on the Poynting Vector

Now at last to figure 5 showing the the flow of power in the coulomb field. You broke the Poynting vector into more than one part. I did too, but in a different way. If $\vec{P}$ is the total complex Poynting vector, we have

$$\begin{aligned} \vec{P} &= \vec{E}_T \times \vec{B} \\ &= (\vec{E}_C + \vec{E}_I + \vec{E}_R) \times \vec{B} \\ &= (\vec{E}_C \times \vec{B}) + (\vec{E}_I \times \vec{B}) + (\vec{E}_R \times \vec{B}) \\ &= \vec{P}_C + \vec{P}_I + \vec{P}_R \end{aligned}$$

where the three $\vec{P}$ s are the Poynting vectors associated with the three electric fields. For the part of $\vec{P}$ associated with the coulomb field we have

$$\vec{P}_C = \vec{E}_C \times \vec{B} \,.$$

It was the time average of this vector that I evaluated and plotted in figure 5.

Validity Tests

I applied several cross checks to test the validity of my rather unorthodox conclusions.

One was to compute the input impedance of the antenna from the center current and the source voltage. I did this for a wide range of input frequencies and compared the result with that published by Schelkunoff. The agreement was good.

Another check was to re-do the entire analysis, this time assuming that the antenna contained a dense distribution of free electrons and positive ions. I used the fields of the vibrating electrons and the ions rather than the continuums of charge and current that I assumed for my article. The results of the two methods agreed precisely.

Still another check was to test the power balance in various ways. For example, I used the computed Poynting vector $\vec{E}_R \times \vec{B}$ of the radiation field to find the power radiated from the entire surface of the antenna, including the center segment. This power agreed with the power flowing into the antenna from the power source. A variation of this test was to compare the power radiated from a given 10-cm section of the antenna's surface with the power "sprayed" onto that same section of the antenna by the coulomb field. Obviously these two amounts of power must be equal, for otherwise that section would quickly become either extremely hot or extremely cold!

Thank you, Kirk, for your interest in my article. I hope this explanation will serve to clarify the methods I used in preparing it.

Sincerely,

Ken Macleish