NEW PROBLEMS

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''New Problems'' solicits interesting and novel worked problems for use in undergraduate physics courses beyond the introductory level. We seek problems that convey the excitement and interest of current developments in physics and that are useful for teaching courses such as Classical Mechanics, Electricity and Magnetism, Statistical Mechanics and Thermodynamics, ''Modern'' Physics, and Quantum Mechanics. We challenge physicists everywhere to create problems that show how contemporary research in their various branches of physics uses the central unifying ideas of physics to advance physical understanding. We want these problems to become an important source of ideas and information for students of physics and their teachers. All submissions are peer-reviewed prior to publication. Send manuscripts directly to Christopher R. Gould, *Editor*.

Magnetars

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(Received 6 July 1999; accepted 1 October 1999)

I. PROBLEM

The x-ray pulsar SGR1806-20 has recently been observed[1] to have a period T of 7.5 s and a relatively large ''spindown'' rate $\dot{T}=8\times10^{-11}$.

Calculate the maximum magnetic field at the surface of this pulsar, assuming it to be a standard neutron star of mass $1.4M_{\odot}=2.8\times10^{30}$ kg and radius $R=10$ km, that the mass density is uniform, that the spindown is due to electromagnetic radiation, and that the angular velocity vector is perpendicular to the magnetic dipole moment of the pulsar.

Compare the surface magnetic field strength to the so-called quantum electrodynamic critical field strength $m^2c^3/e\hbar=4.4\times10^{13}$ gauss, at which electron–positron pair creation processes become highly probable.

II. SOLUTION

In Gaussian units, the rate of magnetic dipole radiation is

$$\frac{dU}{dt}=\frac{2}{3}\frac{\ddot{\mathbf{p}}^2}{c^3}=\frac{2}{3}\frac{p^2\omega^4}{c^3}, \tag{1}$$

where $\omega=2\pi/T$ is the angular velocity, taken to be perpendicular to the magnetic dipole moment $\mathbf{p}$.

The energy of rotation is $U=I\omega^2/2$, so

$$\frac{dU}{dt}=I\omega\dot{\omega}=\frac{2}{5}MR^2\omega\dot{\omega}, \tag{2}$$

assuming the pulsar is a sphere of uniform mass density. Combining (1) and (2), we have

$$p^2=\frac{3}{5}\frac{MR^2\dot{\omega}c^3}{\omega^3}. \tag{3}$$

Substituting $\omega=2\pi/T$, and $\dot{\omega}=2\pi\dot{T}/T^2$, we find

$$p^2=\frac{3}{20\pi^2}MR^2T\dot{T}c^3. \tag{4}$$

The magnetic field $\mathbf{B}$ due to dipole $\mathbf{p}$ is

$$\mathbf{B}=\frac{3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}-\mathbf{p}}{r^3}, \tag{5}$$

so the peak field at radius R is

$$\mathbf{B}=\frac{2\mathbf{p}}{R^3}. \tag{6}$$

Inserting this in (4), the peak surface magnetic field is related by

$$\begin{aligned} B^2&=\frac{3}{5\pi^2}\frac{MT\dot{T}c^3}{R^4}\\ &=\frac{3}{5\pi^2}\frac{(2.8\times10^{33})(7.5)(8\times10^{-11})(3\times10^{10})^3}{(10^6)^4}\\ &=2.8\times10^{30}\ \text{gauss}^2. \end{aligned} \tag{7}$$

Thus, $B=1.7\times10^{15}$ gauss $=38B_{\text{crit}}$, where $B_{\text{crit}}=4.4\times10^{13}$ gauss.

When $B>B_{\text{crit}}$, electrons and photons of kinetic energies greater than 1 MeV rapidly lose this energy via electron–position pair creation.

Kouveliotou *et al.* report[1] that $B=8\times10^{14}$ gauss without discussing details of their calculation.

[1] C. Kouveliotou *et al.*, ''An x-ray pulsar with a superstrong magnetic field in the soft-ray repeater SGR1806-20,'' Nature (London) **393**, 235–237 (1998).

