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Superluminal Signal Velocity and Causality

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A superluminal signal velocity (i.e. faster than light) is said to violate causality. However, superluminal signal velocities have been measured in tunneling experiments recently. The classical dipole interaction approach by Sommerfeld and Brillouin results in a complex refractive index with a finite real part. For the tunneling process with its purely imaginary refractive index this model obtaines a zero-time traversing of tunneling barriers in agreement with wave meechanics. The information of a signal is proportional to the product of its frequency band width and its time duration. The reasons that superluminal signal velocities do not violate causality are: (i) physical signals are frequency band limited and (ii) signals have a finite time duration.

KEY WORDS: Tunneling, superluminal velocity, signals, causality.

1. INTRODUCTION

Franco Selleri investigated the problem of causality from several approaches in Ref. (1). He questioned, whether this fundamental principle can be violated? In 1992 Achim Enders and the author have demonstrated that photonic tunneling proceeds at superluminal signal velocities.(2) The result was in agreement with quantum mechanical investigations(3–6) Later superluminal amplitude modulated (AM) and frequency modulated (FM) microwave and single photon experiments were carried out using different photonic barriers.(7–9) Mozart's 40th symphony was FM tunneled at a speed of $4.7 \cdot c$.(9) Recently infrared digital signals were tunneling with superluminal velocities along photonic barriers built in a fiber.(10, 11)

Many physicists are terribly anxious about superluminal signal velocities. They are teaching that the special theory of relativity (STR) prohibits such a phenomenon. Otherwise a time machine would be possible and

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man could manipulate the past. These physicists do not accept that quantum mechanics allows instantaneous tunneling and superluminal velocities. In addition they do not know the properties of a physical signal see e.g. Refs. (12–17). A physical signal has a limited frequency band width $\Delta\nu$ and a finite duration Δt. Signals are represented by wave packets with

$$\Delta\nu \cdot \Delta t > 1. \tag{1}$$

Definitions of the frequency bandwidth, of the time duration, and of the bandwidth–time interval product are introduced in Refs. (18, 19) for example.

The optical tunneling process is often called evanescent mode propagation. It represents the mathematical analogy of quantum mechanical tunneling, see Refs. (20, 21) for instance. Several quantum mechanical studies on tunneling came to the conclusion, that tunneling a barrier proceeds in zero time. A short time is spent at the barrier front boundary.(3–6,22) This result is in agreement with the photonic tunneling experiments: zero-time is spent inside a barrier.(9)

The dispersion relations of the wavenumber k can be described by a refractive index n in the case of the electromagnetic field and in the case of a wave particle as:

$$k = k_0 \cdot n, \tag{2}$$

$$k = k_0 \cdot (\mu\epsilon)^{1/2} \tag{3}$$

$$k = k_0 \cdot (2m(W - U)/\hbar^2)^{1/2}, \tag{4}$$

where k_0 is the wave number in free space, μ and ϵ are the permeability and the permittivity, respectively, m is the rest mass of the wave packet, W and U are the wave packet's energy and the potential, respectively.

Brillouin defined in his book on *Wave propagation and group velocity* a signal velocity.(23) The basis for his calculations is a finite real part of the complex refractive index n, which is zero in the case of tunneling. Brillouin investigated the signal velocity of an example with 7 sinusoidal oscillations as sketched in Fig. 1. The signal is assumed to be frequency band unlimited. According to quantum mechanics every frequency component has an energy minimum of $h \cdot \nu$, i.e. a photon, where h is the Planck constant and ν the frequency. The assumption of an unlimited frequency band would result in an infinite energy of a signal since a frequency component does exist and can be detected only if there is at least one photon with the energy $h \cdot \nu$.(25)

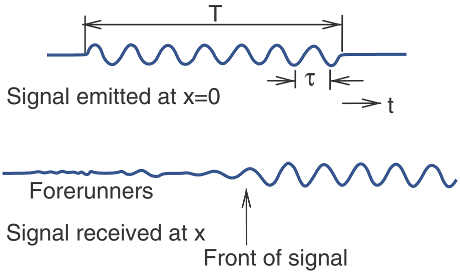

Fig. 1. Brillouin's sketch of a signal on top of the figure.[23] Seven frequency band unlimited sinusoidal oscillations represent the signal. It traverses a dispersive medium, where the high frequency components experience a refraction index of 1 due to the lack of interaction with the dipole oscillator of the model. The high frequency components of the signal shown in the figure are traversing the medium at c, the velocity of light in vacuum. Other frequency components are slowed down due to their interaction, which is expressed by a refractive index $n > 1$ in this example. The assumed unlimited frequency band signal would have an infinite energy according to quantum mechanics. The signal is reshaped and its front is arbitrarily defined for 6 of the 7 original oscillations.

Due to the dispersion relation of the interaction of an electromagnetic wave with a Lorentz-Lorenz dipole oscillator the high frequency components of a wave packet are propagating at the velocity of light, whereas the lower frequencies near the dipole oscillation frequency are propagating slower as illustrated in Fig. 1. The high frequency components are called forerunners, they do not interact with the dipole. The signal is significantly reshaped in consequence of the dispersion relation of the oscillator.

In this case the imaginary part of the refractive index represents the dissipation of the interaction process, which is opposite to the imaginary refractive index in the case of tunneling. The tunneling process is free of losses, the transmission is reduced by reflection only. Brillouin has not investigated the behavior of the wave propagation in the case of a purely imaginary refractive index.

A physical signal is frequency band limited as mentioned above. A modern and well defined series of signals is presented in Fig. 2. The digital information is represented by the half-width of the pulses (half-maximum of the power). In order to receive the information (i.e. the number of

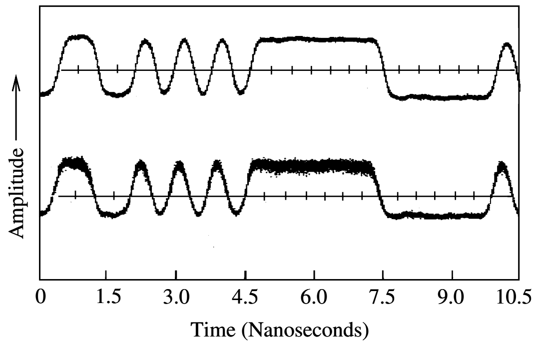

Fig. 2. Digital infrared signals travelled 9000 km through a loop of ordinary fiber and several erbium-doped fiber amplifiers. The half-width in units of 0.2 ns corresponds to the number of bits. From left to right: 1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 1, 1, 1, 1, ... Finally, after 9000 km distance there is emerging noise (bottom). The infrared carrier frequency of the signal is 2×10^{14} Hz (wavelength 1.5 μm). The frequency–band-width of the signal is about 2×10^{10} Hz corresponding to a relative frequency–band-width of 10^{-4}.(34)

digits) the complete envelope of the signal has to be measured. Thus the signal velocity is represented by the velocity of the envelope. The model used for defining the signal velocity is based on a refractive index $n(\omega)$ with a finite real part. The tunneling process, however, has a purely imaginary refractive index, which results in zero barrier traversing time and thus in an infinite velocity inside barriers.

On the other hand an emerging of waves as shown in Fig. 1 can not be identified as the arriving or delay time of a signal. An observer could also interprete the forerunners as the arrival of a signal. The forerunners are the high frequency components of the signal, consequently they are part of the signal.

2. VELOCITIES AND DELAY TIMES

In the following definitions of velocities and delay times are reminded. They are presented in many text books see Refs. (18, 19, 40), for instance. We shall recognice that they result in infinite values in the case of the quantum mechanical tunneling process with a purely imaginary refractive index n.

$$\text{Phase velocity } v_\varphi = \omega/k = c/n(\omega), \tag{5}$$

$$\text{Group veloctiy } v_g = d\omega/dk = c/(n(\omega) + \omega dn(\omega)/d\omega), \tag{6}$$

$$\text{Signal velocity}(in\ vacuum)\ v_{\text{s}} \equiv v_g, \tag{7}$$

where ω and k are the angular frequency and the wave number, respectively. The complex refractive index n is given by the relation:

$$n = n' - i\kappa, \tag{8}$$

where n' and κ are the real and the imaginary parts of the refractive index, respectively. κ is in charge of the wave attenuation. In the case of tunneling the attenuation is caused by reflection and not by dissipation. The energy velocity has to equal the signal velocity, because a signal is detected by its energy.

The following terms are used to describe the delay of the various parts of a signal envelope crossing a media with the refractive index n Ref. (18), for instance:

$$\text{Phase time delay } t_\varphi(\omega) = \varphi(\omega)/\omega, \tag{9}$$

$$\text{Group time delay } t_g(\omega) = d\varphi(\omega)/d\omega, \tag{10}$$

$$\text{Front time delay } t_{fr}(\omega) = \lim_{\omega \to \infty} \varphi(\omega)/\omega, \tag{11}$$

where φ is the phase of the wave. The strikingly different results of the mathematical and the physical treatment of the front time delay and the signal arriving time becomes evident. A physical signal is frequency band limited and thus the front time delay, the arrival of a signal according Eq. (11) is not defined. A signal starts gradually within a time span given by its frequency bandwidth. But more crucial, the definitions Eqs. (5)–(11) for velocities and for delay times are depending on the real part of the refractive index and on φ, which are zero and constant respectively inside a tunneling barrier.

3. TUNNELING

Tunneling is the wave mechanical analogy of evanescent modes.[20,21] Evanescent modes are found in undersized waveguides, in the forbidden frequency bands of periodic dielectric hetero-structures, and with double

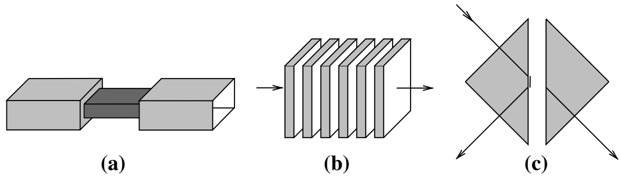

Fig. 3. Sketch of three prominent photonic barriers. (a) illustrates an undersized wave guide (the central part of the wave guide has a cross-section being smaller than half the wavelength in both directions perpendicular to propagation); (b) a photonic lattice (periodic dielectric hetero structure); and (c) the frustrated total internal reflection (FTR) of a double prism, where total reflection takes place at the boundary from a denser to a rarer dielectric medium.

prisms in the case of frustrated total reflection.(7,9) Prominent examples of photonic tunneling barriers are sketched in Fig. 3. Dielectric lattices are analogous to electronic lattices of semiconductors with forbidden energy gaps.

Each of the three barriers introduced in Fig. 3 has a different dispersion relation. A simple one describes the frustrated total internal reflection (FTIR) of a double prism. In this case the tunneled electric field E_t and the imaginary wave number κ are given by the relations:(21)

$$E_t = E_0 e^{(i\omega t - \kappa x)} \tag{12}$$

$$\kappa = \left[\frac{\omega^2}{c^2}\left(\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta - 1\right)\right]^{1/2}, \tag{13}$$

where θ is the angle of the incident beam, E_0 the amplitude of the electric field at the barrier entrance, n_1 and n_2 are the refractive indexes, and $(n_1/n_2)\sin\theta > 1$ holds. The wave equation yields for the electric field $E(z)$

$$E(t,x) = E_0\, e^{i(\omega t - kx)} \Rightarrow E(t,x) = E_0\, e^{i\omega t - \kappa x}, \tag{14}$$

where ω is the angular frequency, t the time, x the distance, k the wave number, and κ the *imaginary* wave number of the evanescent mode.

In the three introduced barriers the modes are characterized by an exponential attenuation of transmission due to reflection by photonic barriers and by a lack of a phase shift inside the barrier. The latter means a zero-time barrier traversal according to the phase time approach

$$t_g = d\varphi/d\omega, \tag{15}$$

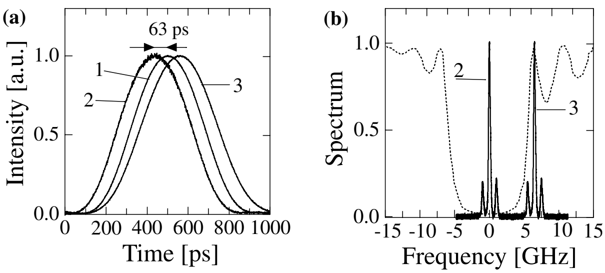

Fig. 4. Measured propagation time of three digital signals.[10] (a) Pulse trace 1 was recorded in vacuum. Pulse 2 traversed a photonic lattice in the center of the frequency band gap (see part (b) of the figure), and pulse 3 was recorded for the pulse travelling through the fiber outside the forbidden band gap. The tunneling barrier was a photonic lattice of a periodic dielectric hetero-structure fiber.

where t_g is the phase-time, which equals the group delay time. In fact, this zero-time was measured and the observed short tunneling time τ arises at the barrier front.[9] A digital signal used in communication systems is displayed in Fig. 4. Longhi *et al.*[10,11] performed superluminal tunneling of infrared pulses over distances up to 50 mm at a infrared signal wavelength of 1.5 μm. Results are presented in Fig. 4. The frequency band width is of the order of 10^{-4}. The measured velocity was $2 \cdot c$ and the transmission intensity of the barrier was 1.5%. The narrow band width of the signal is displayed in Fig. 4(b). The narrow frequency bandwidth avoids a pulse reshaping.

The apparently classical evanescent modes represent the world of mesoscopic quantum mechanics. The potential barriers in the case of microwave frequencies are of the order of a meter. Amazingly, that is on a logarithmic scale in the middle of the microcosmos and the dimensions of the cosmos.

The evanescent or tunneling modes display some non-classical properties as:

1. Evanescent modes are represented by nonlocal fields as shown by transmission and partial reflection experiments. Tunneling and reflection times are equal and independent of barrier length.[3,9,27]
2. Evanescent modes have a negative energy, they cannot be measured inside a barrier.[24,26,28,39]
3. Evanescent modes can be described by virtual photons.[29]

4. Evanescent modes are not Lorentz invariant as $(v_g/c)^2 \rightarrow \infty$ holds, where $v_g = x/\tau$ is the phase time group velocity, x represents the barrier length, and τ the tunneling time arising from the barrier entrance boundary.

Evanescent modes are not fully describable by the Maxwell equations and quantum mechanics have to be taken into consideration. Non-local fields, virtual photons, and zero-time spreading are properties of quantum mechanics.

4. SUPERLUMINAL SIGNALS DO NOT VIOLATE CAUSALITY

According to many text books and review articles, a superluminal signal velocity violates Einstein causality, implying that cause and effect can be interchanged and time machines known from science fiction can be constructed.[17,38] On the other hand it can be shown for frequency band *unlimited groups* that the front travels always at a velocity $\leq c$, and only the peak of the signal has travelled with a superluminal velocity. Such calculations were carried out by several authors, for example Refs. (32, 33, 42). In this case the tunneled pulse is reshaped and its front has propagated at luminal velocity. This behavior is different from frequency band limited signals composed of evanescent frequency components only. An example is presented in Fig. 4. In this case the pulse (i.e. the signal) has gradually formed a front tail. A pulse reshaping did not happen and the envelope and thus the signal travelled at a superluminal velocity.

The not well defined front, more precise the beginning of a physical signal propagates always slower than c due to its decrease of intensity with distance. For instance a dipole radiation intensity decreases with the second power of distance, and in the case of tunneling even exponentially.

Recently Winful calculated superluminal transport of pulses.[33] He believes that a superluminal signal velocity would violate the principle of causality and provided a calculation to resolve the *mystery of apparent superluminality*. He claimed that the incoming pulse is not related to the superluminal outcoming pulse contrary to the experimental data (see e.g. Fig. 4 and he did not consider the non-local property of the tunneling process in consequence of the imaginary refractive index. He denied both the negative energy of photons and that tunneling fields are not detectable inside a barrier.

The causal correlation of the components of an outcoming signal with its incoming one is most obvious in the case of FM signal. FM is given by the relation:

$$V = V_0 \cos[\omega_c(1 + a_m \cos \omega_{mod} t)], \tag{16}$$

where ω_c and ω_{mod} are the carrier angular frequency and the modulation angular frequency, respectively. a_m presents the modulation amplitude.

Figure 5 displays an FM signal as described by Eq. (16). The time duration between the zeros of the oscillations represents the information. The frequency components of the information of the tunneled signal travelled faster than light. A computer simulation of the time advance of the demodulated signal of a tunneled FM carrier is presented in Fig. 6. (Experimentally this was shown by tunneling an FM Symphony by Mozart, which travelled at a speed of $4.7 \cdot c$ without any significant distortion.[9])

The frequency distribution is at the input the same as at the output. The output frequencies of the signal are connected by causal propagation to the input frequency components.

Does the measured superluminal signal velocities violate the principle of causality? The line of arguments showing how to manipulate the past in the case of superluminal signal velocities is illustrated in Fig. 7. There are displayed two frames of reference. In the first one at the time $t = 0$ lottery numbers are presented as points on the time coordinate without duration. At $t = -0.5$ s the counters are closed. Mary (A) sends the lottery numbers to her friend Susan (B) with a signal velocity of $4 \cdot c$. Susan, moving in the second inertial system at a relative speed of $0.75 \cdot c$, sends the numbers back at a speed of $2 \cdot c$, to arrive in the first system at $t = -1$ s, thus

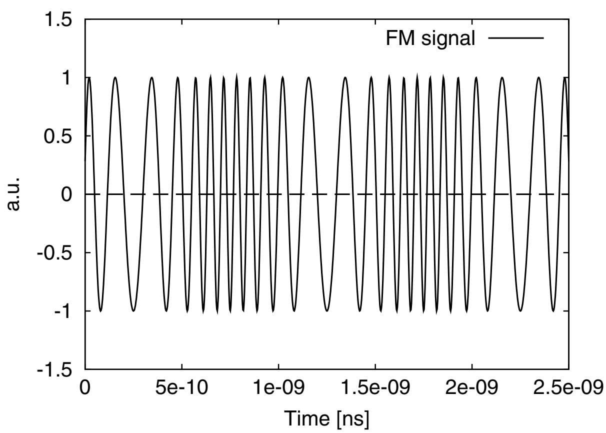

Fig. 5. A numerical example of an FM signal described by Eq. 16, where $\nu_c = 10$ GHz and $\nu_{mod} = 1$ GHz.[43]

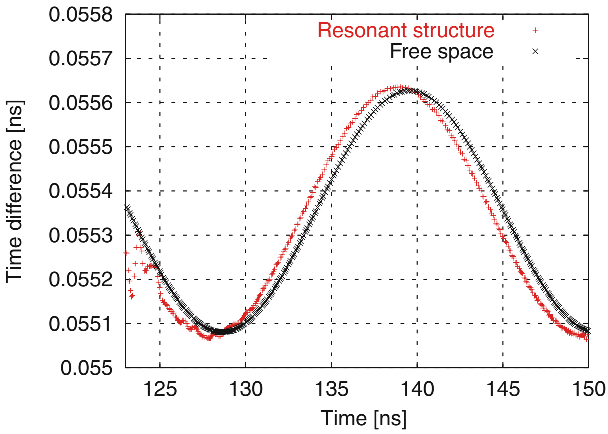

Fig. 6. Calculated time difference of the FM modulation (ν_c = 10 GHz, ν_{mod} = 46 MHz) of an airborne signal (x) and of a tunneled signal (+) front. The tunneled signal, i.e. the modulation is about 0.7 ns faster than the airborn signal. The barrier length was 279.4 mm.(43)

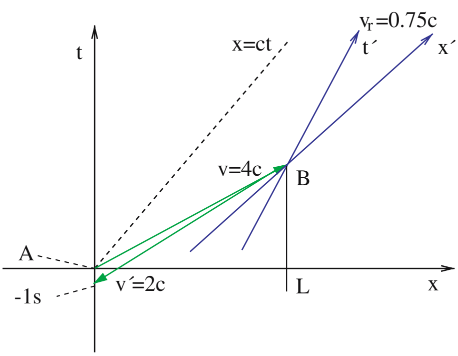

Fig. 7. Coordinates of two inertial observers **A** (0, 0) and **B** with $O(x, t)$ and $O'(x', t')$ moving with a relative velocity of $0.75 \cdot c$. The distance L between **A** and **B** is 2000000 km. **A** makes use of a signal velocity $v_s = 4 \cdot c$ and **B** makes use of $v_s' = 2 \cdot c$ (in the sketch is $v \equiv v_s$). The numbers in the example are chosen arbitrarily. The signal returns -1 s in the past in **A**.

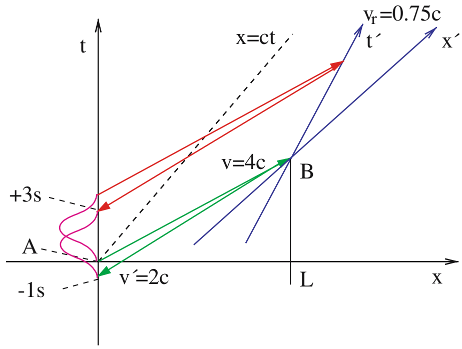

Fig. 8. In contrast to Fig. 7 the pulse-like signal has now a finite duration of 4 s. This data is used for a clear demonstration of the effect. In all superluminal experiments, the signal length is long compared with the measured negative time shift. In this sketch the signal envelope ends in the future with 3 s (in the sketch is $v \equiv v_s$).

in time to deliver the correct lottery numbers before the counters close at $t = -0.5$ s.

The time shift of a point on the time axis of reference system A into the past is given by the relation,[38,44]

$$t_A = -\frac{L}{c} \cdot \frac{(v_r - c^2/v_s - c^2/v_s' + c^2 v_r/v_s v_s')}{(c - c v_r/v_s')}, \tag{17}$$

where L is the transmission length of the signal, v_r is the velocity between the two inertial systems A and B. The condition for the change of chronological order is $t_A, < 0$, the time shift between the systems A and B. This interpretation assumes, however, a signal to be a point in the time dimension neglecting its temporal width. Several tunneling experiments have revealed superluminal signal velocity in tunneling photonic barriers.[9] Nevertheless, the principle of causality has not been violated as will be explained in the following.

In the example with the lottery data, the signal was assumed to be a point in space-time. However, a physical signal has a finite duration like the pulses sketched along the time axis in Fig. 8.

The general relationship for the bandwidth-time interval product of a signal, i.e. a packet of oscillations is given by Eq. (1).

A zero time duration of a signal would require an infinite frequency bandwidth. Taking into consideration the dispersion of the transmission of tunneling barriers, the frequency band of a signal has to be narrow in order to avoid nonsuperluminal frequency components and thus a pulse reshaping.

Assuming a signal duration of 4 s the complete information is obtained with superluminal signal velocity at 3 s in positive time as illustrated in Fig. 8. The compulsory finite duration of all signals is the reason that a superluminal velocity does not violate the principle of causality. A shorter signal with the same information content would have an equivalently broader frequency bandwidth (Eq. (1)). That means an increase of v_s or v_s' can not violate the principle of causality.

For instance, the dispersion relation of FTIR (Eq. 13) elucidate this universal behavior: Assuming a wavelength $\lambda_0 = c/\nu$, a tunneling time $\tau = T = 1/\nu$, and a tunneling gap between the prisms $d = j \cdot \lambda_0$ (j = 1, 2, 3, ...) the superluminal signal velocity is $v_s = j \cdot c$, (remember the tunneling time is independent of barrier length). However, with increasing v_s the bandwidth $\Delta\nu$ (that is the tolerated imaginary wave number width $\Delta\kappa$) of the signal decreases $\propto$ 1/d in order to guarantee the same amplitude distribution of all frequency components of the signal. In spite of an increasing superluminal signal velocity $v_s \to \infty$ the general causality can not be violated because the signal time duration increases analogously $\Delta t \to \infty$ (Eq. (1)).

5. SUMMING UP

The tunneling process shows amazing properties to which we are not used to from classical physics. The tunneling time is short and arises from the barrier front. It equals approximately the reciprocal frequency of the carrier frequency or of the wave packet energy divided by the Planck constant h. Inside a barrier the wave packet does not spent any time. This property results in superluminal signal and energy velocities, as a signal is detected by its energy, i.e. by photons. The latter becomes obvious in a single photon experiment, where the detector measured the superluminal energy velocity of a single photon.[8] Evanescent fields are not fully describable by the Maxwell equations. They carry a *negative* energy, which makes it impossible to detect them,[24,26,28,37] and they are non-local.[27] Incidentally, the properties of an evanescent mode are in agreement with wave mechanical tunneling.

In the review on *The quantum mechanical tunnelling time problem—revisited* by Collins *et al.*,[4] the following statement has been made: *the phase-time-result originally obtained by Wigner and by Hartman is the best expression to use for a wide parameter range of barriers, energies and wave-*

packets. The experimental results of photonic tunneling have confirmed this statement.[9]

As mentioned above the energy of signals is always finite resulting in a limited frequency spectrum. This is a consequence of Planck's quantization of radiation with an energy minimum of $\hbar\omega$.[25] An electric field cannot be measured directly. All detectors need at least one energy quantum $\hbar\omega$ in order to respond. This is a fundamental deficiency of classical physics, which assumes any small amount of field and charge is measurable.

A physical signal has not a well defined envelope front. The latter would need infinite high frequency components.[18,41] In addition signals are not presented by an analytical function, otherwise the complete information would be contained in the forward tail of the signal, see for instance Ref. (36).

Another consequence of the frequency band limitation of signals is, if they have only evanescent mode components, they can violate Einstein causality, which claims that signal and energy velocities have to be $\leq$c.

In spite of so much arguing about violation of Einstein causality,[12–16] all the properties introduced above are useful for novel fast devices, for both photonics and electronics.[45] As mentioned above according to Collins *et al.*[4] the disputes on zero tunneling time (the time spent inside a barrier) are redundant after reading the papers by Wigner and Hartman.

The discussion about superluminal tunneling reminds me to the historical causality problem of the multiplex signal transmission technique

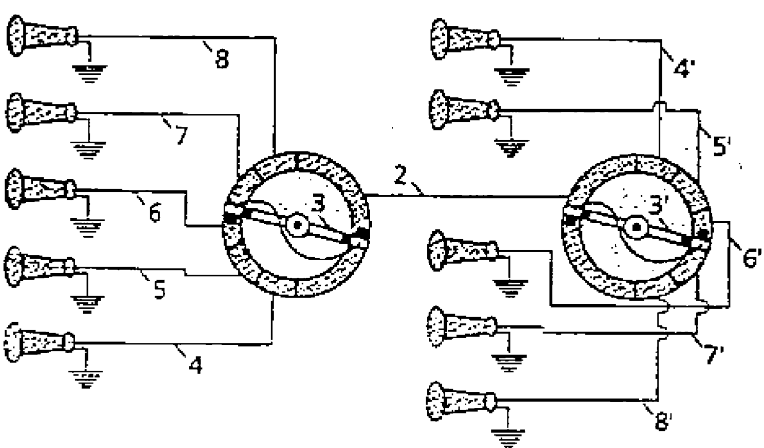

Fig. 9. Historical picture of a multiplex transmission system Ref. (35). Every phone pair has only a fraction of the total rotation time of the two connecting switching wheels.

displayed in Fig. 9.[35] In this widely used technique a signal's finite time duration and frequency band limitation violate causality according to Fourier transform. There should exist a negative time component. However, no one had a ringing-up before the other phone was switched on. This is an established example of the fundamental role of finite frequency bands and finite time duration of physical signals without violating the principle of causality in spite of the Fourier transform.[41]

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