Electromagnetically induced transparency

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I. SCOPE

Imagine that you are taking a quick walk and you are in fact traveling faster than light. This is what has been achieved in a recent experiment by Lene Hau and her co-workers.[1] In that experiment, the group velocity of a pulsed laser was effectively reduced to about 1 mile per hour (0.45 m/s) in a cold, laser-dressed sodium atom cloud. In an earlier experiment, the same group had successfully slowed the group velocity of light to 38 miles per hour (17 m/s) in a similar system.[2]

Propagation of light in a medium is a well-studied subject even though there have been some very tough questions, such as the group velocity exceeding the speed of light in vacuum.[3] Five years ago, it was demonstrated for the first time that the speed of light can be reduced significantly[4] in a cold atom cloud that is in a state called electromagnetically induced transparency.[5,6] What has made the experimental work of Hau *et al.* so unique is that the orders of magnitude in the reduction of speed of light, which cannot commonly be accomplished by increasing the index of refraction, is achieved instead by the extremely rapid variation of the index with the frequency, and the elimination of light absorption at resonance frequency—a quantum phenomenon resulting from the coupling and interaction between lasers and electrons at different atomic levels. Here we would like to highlight some basic understanding of this exciting phenomenon.

From the Maxwell equations for a propagating electromagnetic wave with angular frequency ω and complex wave vector κ in a nonconducting medium, we have[7]

$$\kappa^2=\mu\epsilon\omega^2, \tag{1}$$

where μ is the magnetic permeability and ϵ is the electric permittivity of the medium. If we assume that ω is real and

$$\kappa=k+i\frac{\alpha}{2}, \tag{2}$$

where k is the real propagating wave vector and α is the absorption coefficient, we have

$$n=\frac{ck}{\omega}=\mathrm{Re}\sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}}, \tag{3}$$

$$\alpha=2\omega\,\mathrm{Im}\sqrt{\mu\epsilon}, \tag{4}$$

with $c=1/\sqrt{\mu_0\epsilon_0}$ being the speed of light in vacuum and μ_0 and ϵ_0 the vacuum permeability and permittivity, respectively. In most cases, we have $\mu\simeq\mu_0$, a condition that is assumed here. From the above relation, we find the phase velocity of the wave,

$$v_p=\frac{\omega}{k}=\frac{c}{n}, \tag{5}$$

which characterizes how fast the wave changes its phase. So the field propagating along the z direction is proportional to $e^{ikz-\alpha z/2-i\omega t}$.

Now let us consider that the electric field is a nondecaying $(\alpha=0)$ wave packet with a range of frequencies $\omega=\omega(k)$:

$$E(z,t)=\int dk\,A(k)e^{ikz-i\omega t}, \tag{6}$$

where $A(k)$ is a narrow function peaked at $k=k_0$. We can then expand $\omega(k)$ as

$$\omega(k)=\omega(k_0)+(k-k_0)\left.\frac{d\omega}{dk}\right|_{k=k_0}+O[(k-k_0)^2], \tag{7}$$

if it is well behaved, that is, a smooth function around the given wave vector k_0. If only the zeroth- and first-order terms are kept in the expansion, we have

$$\begin{aligned}E(z,t)&\simeq e^{i[k_0v_g-\omega(k_0)]t}\int dk\,A(k)e^{ikz-ikv_gt}\\&=e^{i[k_0v_g-\omega(k_0)]t}E(z-v_gt,0),\end{aligned} \tag{8}$$

where

$$v_g=\left.\frac{d\omega}{dk}\right|_{k=k_0} \tag{9}$$

is termed the group velocity of the wave at $k=k_0$ because the packet acts if it is traveling in space with such a velocity without changing its shape, with an overall phase change. Using Eq. (3), we obtain

$$v_g=\frac{v_p}{1+(\omega/n)(dn/d\omega)}. \tag{10}$$

Note that we have assumed that n is a function of ω through k. We can consider the group velocity to be the velocity of the wave packet, that is, the velocity of the energy and information contained in the packet, if the linear term in the above Taylor expansion is the dominant term. However, the meaning of the group velocity can change if the wave packet becomes incoherent. Care must be taken when the angular frequency of the wave is near a resonance or $dn/d\omega<0$.[3]

Quantum mechanically, the resonance occurs when the frequency of light matches the energy difference between two allowed quantum levels in the system and is typically accompanied by strong absorption under normal circumstances. This is why normal matter that can be well approximated by a two-level model can never slow the light very much. For laser-dressed atom clouds, the coupling and interaction between a three-level atom and two lasers can drastically alter the behavior of the system, including effectively eliminating the absorption at the resonance frequency and therefore creating electromagnetically induced transparency, as shown in the problems given here.

II. PROBLEMS

A. Coherent population trapping

The key to keeping the group velocity at the vicinity of a resonance frequency meaningful lies in the properties of the laser-dressed atomic cloud. Without such an effect, absorption would be too strong to have any transmitted light.

Consider that each atom in the medium has three levels. The presence of a coupling (dressing) laser ($\omega_c \simeq \omega_2 - \omega_1$) and a probe laser ($\omega \simeq \omega_2 - \omega_0$) causes a mixing of the three levels, $|0\rangle$, $|1\rangle$, and $|2\rangle$.

The Hamiltonian of such a system is

$$H = H_0 + H_1. \tag{11}$$

Here the unperturbed Hamiltonian H_0 is given by

$$\langle l|H_0|l'\rangle = \hbar\omega_l \delta_{ll'}, \tag{12}$$

with l, $l' = 0,1,2$. The perturbation H_1 is restricted to be

$$\langle l|H_1|l'\rangle = \langle l'|H_1|l\rangle^* = \hbar\Omega_{ll'} e^{-i\omega_{ll'}t}, \tag{13}$$

with $\omega_{ll'} = \omega_l - \omega_{l'}$ and $\Omega_{ll} = \Omega_{01} = \Omega_{10} = 0$. Note that $\omega_2 > \omega_1 > \omega_0 = 0$ and $\omega_{l0} = \omega_l$. This is a so-called ''Λ'' system with the highest level coupled to two lower levels.

For the Hamiltonian given, find the time-dependent wave function

$$|\psi(t)\rangle = \sum_{l=0}^{2} c_l(t)|l\rangle, \tag{14}$$

if $|\psi(0)\rangle = c_0(0)|0\rangle + c_1(0)|1\rangle$ with $|c_0(0)|^2 + |c_1(0)|^2 = 1$. Discuss the condition for $c_2(t) \equiv 0$ and its implication.

B. Electromagnetically induced transparency

If we define a density matrix

$$\rho(t) = |\psi(t)\rangle\langle\psi(t)|, \tag{15}$$

whose diagonal elements are the probabilities of occupying specific states and off-diagonal elements represent the transition rates between two given states, we have

$$i\hbar\frac{\partial\rho}{\partial t} = [H,\rho], \tag{16}$$

from the Schrödinger equation. The interactions between atoms in the cloud can cause a finite linewidth and decay of each level, which can be accounted for by a relaxation matrix:

$$\langle l|\Gamma|l'\rangle = 2\gamma_l \delta_{ll'}, \tag{17}$$

and change Eq. (16) into

$$i\hbar\frac{\partial\rho}{\partial t} = [H,\rho] - \frac{i\hbar}{2}(\Gamma\rho + \rho\Gamma). \tag{18}$$

Assuming that only the dominant decaying factor is nonzero, that is, $\gamma_2 = \gamma$ and $\gamma_{0,1} = 0$, and that the atom is in the ground state at $t = 0$, show that

$$\epsilon(\omega) = \epsilon_0\left[1 + \frac{\omega_d(\omega - \omega_2)}{\omega_R^2/4 - (\omega - \omega_2)^2 - i\gamma(\omega - \omega_2)}\right], \tag{19}$$

where $\omega_d = n_a|p_{20}|^2/\hbar\epsilon_0$ with $|p_{20}|$ being the coupling dipole strength between $|2\rangle$ and $|0\rangle$ and $\omega_R = 2|\Omega_{21}|$ the Rabi angular frequency between $|2\rangle$ and $|1\rangle$.

C. The slowest light

In the recent experiment, Hau and co-workers have successfully reduced the group velocity of light in a cold, laser-dressed sodium atom cloud to 1 mile per hour (0.45 m/s).[1] Each sodium atom can be approximated well by a three-level system. Assume that the permittivity of such a laser-dressed atom cloud is given by Eq. (19) and the frequency of the probe laser ($\omega/2\pi$) is near the resonance frequency ($\omega_2/2\pi \simeq 5.1\times10^{14}$ Hz for sodium atom). Estimate the number density of the atom cloud needed in order to have $v_g \simeq 0.45$ m/s. Assume that the Rabi angular frequency is about $\omega_R = 3.5\times10^7$ rad/s and the coupling dipole strength is about $|p_{20}| \simeq 2.5\times10^{-29}$ C m.

III. SOLUTIONS

A. Coherent population trapping

From the time-dependent Schrödinger equation

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = H|\psi(t)\rangle, \tag{20}$$

we have

$$i\dot{c}_0(t) = \omega_0 c_0(t) + \Omega_{20} e^{i\omega_2 t} c_2(t), \tag{21}$$

$$i\dot{c}_1(t) = \omega_1 c_1(t) + \Omega_{21} e^{i\omega_{21} t} c_2(t), \tag{22}$$

$$i\dot{c}_2(t) = \omega_2 c_2(t) + \Omega_{02} e^{-i\omega_2 t} c_0(t) + \Omega_{12} e^{-i\omega_{21} t} c_1(t). \tag{23}$$

If we redefine the coefficients by

$$c_l(t) = e^{-i\omega_l t} b_l(t), \tag{24}$$

the equation set is simplified to

$$i\dot{b}_0(t) = \Omega_{20} b_2(t), \tag{25}$$

$$i\dot{b}_1(t) = \Omega_{21} b_2(t), \tag{26}$$

$$i\dot{b}_2(t) = \Omega_{02} b_0(t) + \Omega_{12} b_1(t). \tag{27}$$

Multiplying Eq. (25) with Ω_{02} and Eq. (26) with Ω_{12} and adding them together, and substituting the resulting equation into Eq. (27) after taking one more time derivative, we obtain

$$\ddot{b}_2(t) = -(|\Omega_{20}|^2 + |\Omega_{21}|^2) b_2(t). \tag{28}$$

We have used $\Omega_{02} = \Omega_{20}^*$ and $\Omega_{12} = \Omega_{21}^*$. So we have

$$b_2(t) = Ae^{i\Omega t} + Be^{-i\Omega t}, \tag{29}$$

with $\Omega = \sqrt{|\Omega_{20}|^2 + |\Omega_{21}|^2}$. Taking the initial condition $b_2(0) = c_2(0) = 0$, we arrive at

$$b_2(t) = C\sin\Omega t, \tag{30}$$

with C being a constant. Substituting this result back into Eqs. (25) and (26), we have

$$b_0(t) = [c_0(0) - \alpha]\cos\Omega t + \alpha, \tag{31}$$

$$b_1(t) = [c_1(0) - \beta]\cos\Omega t + \beta, \tag{32}$$

where α and β are constants constrained by

$$\Omega_{02}\alpha+\Omega_{12}\beta=0. \tag{33}$$

We have used the initial conditions $b_0(0)=c_0(0)$ and $b_1(0)=c_1(0)$. The coefficient C is given by

$$C=\frac{i}{\Omega}[\Omega_{02}c_0(0)+\Omega_{12}c_1(0)]. \tag{34}$$

If $c_0(0)$ and $c_1(0)$ are such that $C\equiv 0$, we have $c_2(t)\equiv 0$ all the time. A typical case is $|\Omega_{02}|=|\Omega_{12}|$ and $|c_0(0)|=|c_1(0)|=1/\sqrt{2}$, with the total phase difference between the two terms being π. So the state $|2\rangle$ will stay empty and the atoms are trapped in the lower states. The effect of such a coherent population trapping is that the absorption or emission of light is completely eliminated.

B. Electromagnetically induced transparency

Consider that the traveling (probing) laser is described by a time-dependent electric field $E(t)=E_0e^{-i\omega t}$ with ω very close to ω_2. The perturbation from such a field is

$$\langle 2|H_1|0\rangle=-\langle 2|p|0\rangle E_0e^{-i\omega t}=\hbar\Omega_{20}e^{-i\omega t}, \tag{35}$$

where p is the dipole moment induced by the field. Now if we examine the density matrix elements between two states, $\rho_{ll'}=\langle l|\rho|l'\rangle$, we have

$$i\frac{\partial\rho_{20}}{\partial t}=(\omega_2-i\gamma)\rho_{20}+\Omega_{21}e^{-i\omega_{21}t}\rho_{10}+\Omega_{20}e^{-i\omega t}(\rho_{00}-\rho_{22}), \tag{36}$$

$$i\frac{\partial\rho_{10}}{\partial t}=\omega_1\rho_{10}+\Omega_{12}e^{-i\omega_{12}t}\rho_{20}-\Omega_{20}e^{-i\omega_2 t}\rho_{12}. \tag{37}$$

We have used

$$\sum_{l=0}^{2}|l\rangle\langle l|=1 \tag{38}$$

in deriving the above equations. We can then replace ρ_{00}, ρ_{22}, and ρ_{12} by their values at $t=0$, that is, $\rho_{00}=1$, $\rho_{22}=0$, and $\rho_{12}=0$, and change a variable with $\zeta_{10}=\rho_{10}e^{-i\omega_{21}t}$, because we are only looking for the linear solution. Then we have

$$i\frac{\partial\rho_{20}}{\partial t}=(\omega_2-i\gamma)\rho_{20}+\Omega_{21}\zeta_{10}+\Omega_{20}e^{-i\omega t}, \tag{39}$$

$$i\frac{\partial\zeta_{10}}{\partial t}=\omega_2\zeta_{10}+\Omega_{12}\rho_{20}. \tag{40}$$

This equation set resembles a harmonic oscillator under damping and driving forces. The steady solutions are therefore given by

$$\rho_{20}(t)=Ae^{-i\omega t}, \tag{41}$$

$$\zeta_{10}(t)=Be^{-i\omega t}. \tag{42}$$

Substituting the above solutions into the equations, we obtain

$$A=\frac{\Omega_{20}(\omega-\omega_2)}{(\omega-\omega_2+i\gamma)(\omega-\omega_2)-|\Omega_{21}|^2}. \tag{43}$$

Because ρ_{20} represents the dipole transition rate between $|2\rangle$ and $0\rangle$, the polarization of the system is given by $P=n_a\rho_{20}p_{02}=(\epsilon-\epsilon_0)E(t)$ with $p_{02}=\langle 0|p|2\rangle=p_{20}^*$. Then we reach Eq. (19).

C. The slowest light

We know that the group velocity is given by

$$v_g=\frac{d\omega}{dk}=\frac{c}{n+\omega(dn/d\omega)}. \tag{44}$$

For all known materials, $n\sim O(1)$. So if $v_g\ll c$, we must have

$$v_g\simeq\frac{c}{\omega(dn/d\omega)}. \tag{45}$$

For $v_g=0.45$ m/s, as observed in the experiment by Hau's group,[1] one must have

$$\omega(dn/d\omega)\simeq 6.7\times 10^8. \tag{46}$$

From the given permittivity, we have

$$n+i\frac{c\alpha}{2\omega}\simeq 1+\frac{1}{2}\frac{\omega_d(\omega-\omega_2)}{\omega_R^2/4-(\omega-\omega_2)^2-i\gamma(\omega-\omega_2)}. \tag{47}$$

Considering that ω is very close to ω_2, we have

$$n+i\frac{c\alpha}{2\omega}\simeq 1+\frac{2\omega_d(\omega-\omega_2)}{\omega_R^2}\times\left[1+\frac{4(\omega-\omega_2)^2}{\omega_R^2}+\frac{i4\gamma(\omega-\omega_2)}{\omega_R^2}+\cdots\right], \tag{48}$$

which gives

$$\omega(dn/d\omega)\simeq\frac{2\omega}{\hbar\epsilon_0}\frac{n_a|p_{20}|^2}{\omega_R^2}. \tag{49}$$

We have used $\omega_d=n_a|p_{20}|^2/\hbar\epsilon_0$. With the numerical values of the quantities given, we then obtain $n_a\simeq 2\times 10^{20}\,\mathrm{m}^{-3}$, a density quite difficult to achieve experimentally.

Note that the absorption coefficient α is zero at the resonance frequency. This is the essence of the electromagnetically induced transparency, a condition that must be met in order to have a significant light transmission at the resonance frequency. Otherwise, the drastically slowed group velocity of light observed by Hau's group would not have been possible.

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