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THE RADIATION RESISTANCE OF BEAM ANTENNAS*

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Summary—*In this paper a new method proposed by Brillouin for the calculation of radiation resistance is applied to several types of beam antennas. New formulas are deduced and some interesting results are obtained showing the distribution of the radiated power among the different wires of beam antennas and giving the numerical value of the radiation resistance in various cases (synphase antenna, antiphase antenna, Marconi three-stage antenna). The radiation resistance in the presence of a perfect conducting plane is also considered. A table of values of the components of radiation resistance is added to the paper for practical use.*

THERE are two methods of computing the power radiated by an antenna. In the first we calculate the Poynting-vector for each point of space and integrate the normal energy flow through any surface enclosing the antenna. This method might be called the Poynting-vector method, and has been used in the well known works of G. W. Pierce,[1] B. van der Pol,[2] S. Ballantine,[3] M. A. Bontsch-Bruewitsch,[4] and S. Levin and C. Young.[5] It may be noticed that we cannot by this method obtain the contributions to the radiated power of different parts of the antenna, which it is sometimes desirable to know when dealing with some practical cases.

The other method is based upon the study of the emfs induced in the antenna by the currents in the wires of which the antenna is constructed. Let AB (Fig. 1) be a wire in which flows a current of frequency f and assume that the distributions of the current and of the charges in the wire are known. We may then calculate the electric force in each point M of the space. It depends upon the combined action at this point of all elements of the wire. In particular we may take the point M_0 to lie in the surface of the wire itself and calculate the emf due to the electromagnetic field of the wire. If we assume the action at a distance of the current to be instantaneous, this emf would

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[1] G. W. Pierce, *Proc.* Amer. Acad. **52,** 192, 1916.
[2] Balth. van der Pol, Jr., *Proc.* Phys. Soc. of London, **13,** **217,** 1917.
[3] Stuart Ballantine, Proc. I. R. E., **12,** 823, 1924; **15,** 245, 1927.
[4] M. A. Bontsch-Bruewitsch, *Annalen der Physik,* **81,** 425, 1926.
[5] S. A. Levin and C. J. Young, Proc. I. R. E., **14,** 675, 1926.

be a purely reactive one and we may speak of it as arising from the capacity or the inductance of the wire. If the wire is of a length comparable with that of the electromagnetic wave we must take into account the propagation velocity of the field. Then at a moment t there will act in the point M the current and the charge which have existed at the element dx_I at the earlier moment $\left(t-\frac{r_1}{c}\right)$, r_1 being the distance of the point from dx_1, and c the light-velocity. Similarly for the element dx_2 we must use the values of current and charge existing at the earlier moment $\left(t-\frac{r_2}{c}\right)$. Under these conditions the calculated emf (or, strictly speaking, the corresponding potential drop) will have a watt component which may be called the radiation emf.

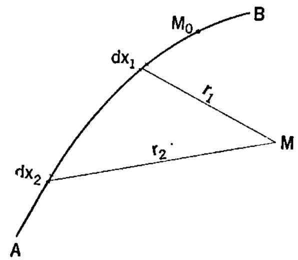

Fig. 1

The product of this emf and the current in an element of the wire gives the radiation from the element. We may find by integration the expenditure of power for radiation in the whole antenna or in its different parts.

We shall call this method the induced emf method. It was proposed by Brillouin[6] and applied by Kliatzkin[7] in the analysis of the radiation of a vertical earthed wire. It is based upon the electromagnetic field equations in the form employing the retarded potentials of Lorentz.[8]

This paper deals with the radiation resistance of antennas, composed of parallel half-wave vibrators.

[6] *Radioélectricité,* April, 1922.
[7] *Telegrafia i telefonia bez provodov* (TITBP) **1(40),** 33, 1927.
[8] Lorentz, "The Theory of Electrons," 2 ed., Chap. 1, paragraphs 13 and 14.

1. Outline of the Method. Radiation EMF. We have first to solve the following problem: a single wire with a known distribution of current being specified, a formula is to be found for the component of electric force parallel to the wire at any point of space.

This may be derived from the expression given by Lorentz:

$$E=-\frac{1}{c}\frac{\partial A}{\partial t}-\text{grad}\,\phi\,; \tag{1}$$

where

$$\phi=\frac{1}{4\pi}\int\frac{1}{r}[\rho]dv \tag{2}$$

is a scalar potential at the given point due to the fixed charges distributed in the space and

$$A=\frac{1}{4\pi c}\int\frac{1}{r}[\rho v]dv \tag{3}$$

is a vector potential due to the charges, moving with the velocity v.

Applying this to the case of a very thin straight wire and passing from the Lorentz units to the absolute system of units we obtain

$$\phi=\frac{1}{\epsilon}\int_0^l\frac{|\sigma|_{t-r/c}}{r}dx \tag{4}$$

$$A=\mu c\int_0^l\frac{|i|_{t-r/c}}{r}dx \tag{5}$$

where σ is the charge per unit length, i is the current in the element dx of the wire, l is the length of the wire, and r is the distance of the given point from dx. The values of charge and current must be taken at the time $\left(t-\dfrac{r}{c}\right)$.

Let the origin of coordinates lie at the one end of the wire and let the OX axis lie along the wire (Fig. 2). The component of the electric force parallel to the wire will then be at the point M (∂, ξ) as follows:

$$E=-\frac{1}{c}\frac{\partial A}{\partial t}-\frac{d\phi}{d\xi}=-\int_0^l\frac{\mu}{r}\frac{\partial i_{(t-r/c)}}{\partial t}dx-\frac{\partial}{\partial\xi}\int_0^l\frac{\sigma_{(t-r/c)}}{\epsilon r}dx \tag{6}$$

If the angular frequency is ω then $i=I_x\sin\omega t$ and

$$\frac{\partial i(t-r/c)}{\partial t}=\omega I_x\cos\omega\left(t-\frac{r}{c}\right) \tag{7}$$

$$\sigma_{(t-r/c)}=-\int\frac{\partial i(t-r/c)}{\partial x}dt=\frac{1}{\omega}\frac{\partial I_x}{\partial x}\cos\omega\left(t-\frac{r}{c}\right) \tag{8}$$

from the equation $-\dfrac{\partial\sigma}{\partial t}=\dfrac{\partial i}{\partial x}$ giving the relationship between the

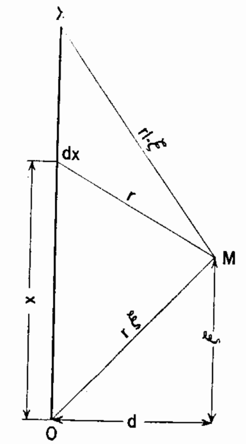

Fig. 2

current and the charge along the wire.

The instantaneous value of electric force will be as follows

$$e_d=-\mu c\left[m^2\int_0^l\frac{\cos(\omega t-mr)}{mr}I_x dx+\frac{\partial}{\partial\xi}\int_0^l\frac{\cos(\omega t-mr)}{mr}\frac{\partial I_x}{\partial x}dx\right] \tag{9}$$

where $m=\dfrac{\omega}{c}=\dfrac{2\pi}{\lambda}$ and $r=\sqrt{d^2+(x-\xi)^2}$.

Assume that the current is distributed sinusoidally along the wire and is zero at the origin of coordinates. Thus

$$I_x = I_0 \sin mx \tag{10}$$

where I_0 is the amplitude of the current at the loop.

The expression for e_d will then be

$$e_d = -\mu c\left[m^2 \int_0^l \frac{\cos(\omega t - mr)}{mr} I_0 \sin mx dx + m\frac{\partial}{\partial \xi} \int_0^l \frac{\cos(\omega t - mr)}{mr} I_0 \cos mx\, dx \right] \tag{11}$$

After integration we obtain:

$$e_d = \mu c I_0\left[\frac{\cos(\omega t - mr_{l-\xi})}{r_{l-\xi}} \cos ml - \frac{\cos(\omega t - mr_\xi)}{r_\xi}\right] \tag{12}$$

where $r_{l-\xi} = \sqrt{d^2 + (l-\xi)^2}$ and $r_\xi = \sqrt{d^2 + \xi^2}$.

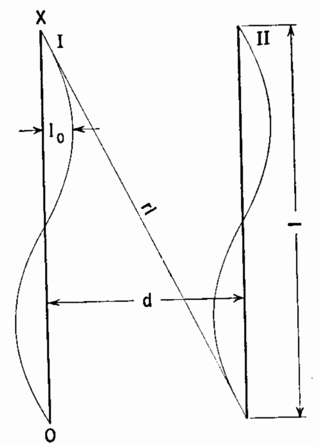

Fig. 3

In the particular case in which the point M lies on the wire itself, $d=0$ and we shall have:

$$e_0 = \mu c I_0\left[\frac{\cos(\omega t - ml + m\xi)}{l-\xi} \cos ml - \frac{\cos(\omega t - m\xi)}{\xi}\right]. \tag{13}$$

2. Case of Two Parallel Wires. We shall now study the problem of the radiation of power from a system formed by two parallel wires. For this purpose we may assume some conditions which simplify the solution. We shall consider (Fig. 3) equal wires whose lengths are multiples of the half-wavelength. The wires are not displaced in height, that is, their ends must lie on the straight line perpendicular to the direction of the wires. The distribution, the phase and the values of current we assume to be identical in both wires.

Under these conditions the tangential component of E at any point along either wire comprises two parts, the first produced in each one by its own current and a second produced by the current in the other wire: i.e., $e = e_0 + e_d$. This electric force produces an emf in the wire, and the power needed to suppress it will be the radiation power. For the element dx of the wire this power will be

$$dP_\Sigma = -EI_x \cos\phi dx \tag{14}$$

where E and I_x are the effective values of the electric force and current and ϕ is the phase angle.

The total power for one wire having the length l will be

$$P_\Sigma = -\int_0^l EI_x \cos\phi dx = -\int_0^l E_0 I_x \cos\phi_0 dx - \int_0^l E_d I_x \cos\phi_d dx = P_0 + P_d. \tag{15}$$

We shall first calculate P_d. Let the full expression of E and I_x be written. To obtain the power in watts we must take the current in amperes and E_d, E_0 in volts. Then

$$I_x = I_0 \sin mx \tag{16}$$

where I_0 is the effective value of the current at the loop in amperes.

E_d is also the rms value and from the formula (12) we obtain two components of it

$$E' = 30 I_0 \frac{\cos ml}{r_{l-x}} \quad \text{and} \quad E'' = 30 I_0 \frac{1}{r_x} \tag{17}$$

Each of these components has a different phase angle ϕ_1 and ϕ_2. Let us find them.

From the expression (12) we have

$$e_d' = E_d' \cos(\omega t - mr_{l-x}) = E_d' \sin\left(\omega t - mr_{l-x} + \frac{\pi}{2}\right). \tag{18}$$

The current in the wire is $i_x = I_x \sin \omega t$

Therefore $\phi_1 = mr_{l-x} - \frac{\pi}{2}$ and

$$\cos \phi_1 = \cos\left(mr_{l-x} - \frac{\pi}{2}\right) = \sin mr_{l-x} \qquad (19)$$

Similarly

$$\cos \phi_2 = \sin mr_x .$$

Thus we obtain for P_d

$$P_d = -30 I_0^2 \int_0^l \left(\frac{\sin mr_{l-x}}{r_{l-x}} \cos ml - \frac{\sin mr_x}{r_x}\right) \sin mx\, dx \qquad (20)$$

By integrating we obtain the following expression for the radiated power *when the length of the wire is a multiple of the half-wavelength.*

$$P_d = 30 I_0^2 [2 Ci\, md - Ci\, m(\sqrt{d^2+l^2}+l) - Ci\, m(\sqrt{d^2+l^2}-l)] = 30 I_0^2 M_d \quad (21)$$

Here $Ci(x)$ denotes the integral cosine, d is the distance between the wires, l is the length of the wire.

P_d is merely one of the components of radiation power, depending upon the current in the other wire. The second component P_0 we may obtain as limit of P_d when the distance between the wires approaches O.

$$P_0 = \lim |P_d| = 30 I_0^2 (E + \log 2ml - Ci 2ml) = 30 I_0^2 M_0 \quad (22)$$

where $E = 0.577 \cdots$ is the Eulers constant.

The whole radiation power in one wire will be

$$P = P_0 + P_d \qquad (23)$$

and the radiation power of the system of two wires

$$P_\Sigma = 2P = 2P_0 + 2P_d \qquad (24)$$

Dividing the expressions (21) and (22) by I_0^2 gives the so-called "radiation resistance." Obviously we may speak of this quantity only when the currents in both wires are equal.

3. Application to Beam Antennas. Synphase System. We shall now apply these results to the computation of the radiation resistance for some types of beam antennas. We shall consider

first the so-called synphase antenna, composed of single parallel vibrators (Fig. 4). The vibrators are situated along a straight line at a distance of a half-wavelength from each other. Their currents are equal and in phase. In this case the beam has a direction perpendicular to the line of the wires.

The power radiated by any individual antenna wire is composed of the power due to its own current and that due to the electric force induced in it by other vibrators. We shall denote

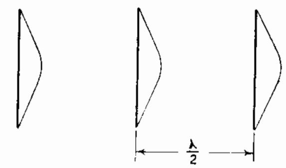

Fig. 4

by P_d the power corresponding to induction by a vibrator at a distance d; in the case of the antenna considered d will be a multiple of a half-wavelength. As stated above in (21),

$$P_d = 30 I_0^2 M_d$$

As the currents in all wires are equal the component of radiation resistance due to another wire will be

$$R_d = 30 M_d .$$

The values of these components are given in Table A (line 1) for distances d which are multiples of the half-wavelength.[9]

Using this table let us now compute the radiation resistance for the antenna composed of three vibrators.

For each of the extreme wires we shall have:

$$R_1 = R_3 = R_0 + R_{\lambda/2} + R_\lambda = 73.3 - 12.4 + 4.1 = 65.0\ \Omega$$

and for the middle wire:

$$R_2 = R_0 + 2R_{\lambda/2} = 73.3 - 2 \times 12.4 = 48.5\ \Omega .$$

The radiation resistance of the whole antenna will be:

$$R_3 = 3R_0 + 4R_{\lambda/2} + 2R_\lambda = 178.5\ \Omega .$$

[9] For these calculations I employed the integral-function curves especially plotted by Mr. E. D. Milovidoff of the staff of the Nijni-Novgorod Radiolaboratory. Interpolation from commonly used tables (Jahnke u. Emde) is rather misleading.

For the four-wire antenna we obtain by analogous calculations:

$$R_2 = 4R_0 + 6R_{\lambda/2} + 4R_\lambda + 2R_{3\lambda/2}$$

and generally for an antenna composed of n wires:

$$R_n = nR_0 + 2(n-1)R_{\lambda/2} + 2(n-2)R_{2\lambda/2} + \cdots + 2R_{(n-1)\lambda/2} \quad (25)$$

Table I contains values of the radiation resistance (a) for each vibrator, (b) for the whole antenna, and (c) the mean value for one vibrator. It might be noticed that when increasing the number of vibrators the last quantity is very rapidly approaching the limit (near 56 ohms), which was obtained for the case of an infinitely great number of vibrators by using the "Poynting vector method" of radiation resistance calculation.[10]

TABLE I

Values of radiation resistance in ohms for synphase beam antenna. n = number of wires; R_n = resistance of nth wire; R = total resistance. R_m = average resistance per wire.

n	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R	R_m
2	60.9	60.9	—	—	—	—	—	121.8	60.9
3	65.0	48.5	65.0	—	—	—	—	178.5	59.5
4	63.2	52.6	52.6	63.2	—	—	—	231.8	58.0
5	64.4	50.9	56.7	50.9	64.4	—	—	287.3	57.4
6	63.6	52.0	55.0	55.0	52.0	63.6	—	341.4	56.9
7	64.0	51.2	56.1	53.2	56.1	51.2	64.0	395.6	56.5

4. Continuation. Antiphase Beam Antenna. We shall now pass on to the antiphase antenna (Fig. 5) which radiates a beam directed in the plane of antenna. It differs from the synphase

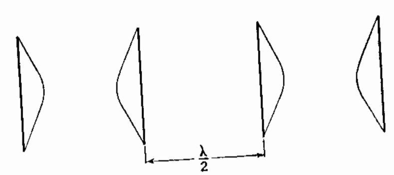

Fig. 5

antenna only in the fact that the currents in the adjacent wires have a phase difference of 180 deg. All the radiation resistance components having d equal to an odd number times the half-wavelength must therefore be multiplied by -1. The other components are the same as before because vibrators spaced by an integral number of wavelengths are in phase.

[10] M. A. Bontsch-Bruewitsch, l. c., p. 434.

The radiation resistance for this antenna is expressed by the following general formula:

$$R_n = nR_0 - 2(n-1)R_{\lambda/2} + 2(n-2)R_\lambda - \cdots + 2R_{(n-1)\lambda/2} \quad (26)$$

where the quantities R_d may be taken from Table A (line 1). The results of calculations for the antiphase antenna are given in Table II.

From Tables I and II we may see that the radiation resistance is different for the various wires in the antenna and this difference is unequal for the two types of antennas. As the number of vibrators is increased the difference diminishes.

TABLE II

Value of radiation resistance in ohms for antiphase beam antenna (for explanation see legend of Table I).

n	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R	R_m
2	85.7	85.7	—	—	—	—	—	171.4	85.7
3	89.8	98.1	89.8	—	—	—	—	277.6	92.5
4	91.5	102.2	102.2	91.5	—	—	—	387.2	96.8
5	92.7	103.9	106.2	103.9	92.7	—	—	499.2	99.8
6	93.4	105.1	108.0	108.0	105.1	93.4	—	612.8	102.1
7	93.9	105.8	109.1	109.7	109.1	105.8	93.9	727.2	103.9

5. Parallel Wires Displaced in Height. As a next step in the development of this method, the radiation resistance of parallel vibrators displaced in height may be calculated, (Fig. 6). The investigation of this case will enable us to study the radiation resistance in the presence of a perfectly conducting plane and to calculate the radiation resistance of multistage antennas.

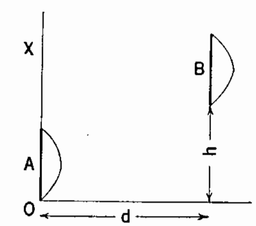

Fig. 6

We will deduce a formula for the radiation power due to the emf induced in the vibrator B by another vibrator A. The wire B is placed a distance d and elevated on a height h with respect to A. We shall denote this power by $P(d, h)$ and the corresponding radiation resistance component by $R(d, h)$.

The electric force near B due to the first vibrator is defined by (12).

The law of current distribution in the wire B is as follows:

$$I_x = I_0 \sin m(x-h) \tag{27}$$

Proceeding exactly as in Sect. 2 we shall obtain for the radiation power due to the induction the following expression:

$$P(d,h) = 30 I_0^2 \int_h^{h+\lambda/2} \left(\frac{\sin mr_{l-x}}{r_{l-x}} + \frac{\sin mr_x}{r_x} \right) \sin m(x-h)dx. \tag{28}$$

The integration gives for the corresponding radiation resistance component a rather complicated expression, as follows:

$$\begin{aligned} R(d,h) = &-15 \sin mh \cdot \left[S\left(d, h-\frac{\lambda}{2}\right) - 2S(d,h) + S\left(d, h+\frac{\lambda}{2}\right)\right] \\ &-15 \cos mh \cdot \left[C\left(d, h-\frac{\lambda}{2}\right) - 2C(d,h) + C\left(d, h+\frac{\lambda}{2}\right)\right] \end{aligned} \tag{29}$$

where $S(x, y)$ and $C(x, y)$ are the functions:

$$S(x,y) = Si\, m(\sqrt{x^2+y^2}+y) - Si\, m(\sqrt{x^2+y^2}-y) \tag{30}$$

$$C(x,y) = Ci\, m(\sqrt{x^2+y^2}+y) + Ci\, m(\sqrt{x^2+y^2}-y).$$

Fig. 7

This formula is the most general one for the case of two parallel vibrators. The expression (21), obtained for the vibrators placed at the same height, is a particular case of it for $h=0$. For the other particular case, when $d=0$ (Fig. 7) we find:

$$\begin{aligned} R(0,h) = &-15 \sin mh \left[Si\, 2m\left(h-\frac{\lambda}{2}\right) - 2Si\, 2mh + Si\, 2m\left(h+\frac{\lambda}{2}\right)\right] \\ &-15 \cos mh \left[\log \frac{h^2}{h^2-\frac{\lambda^2}{4}} + Ci\, 2m\left(h-\frac{\lambda}{2}\right) - 2Ci\, 2mh + Ci\, 2m\left(h+\frac{\lambda}{2}\right)\right] \end{aligned} \tag{31}$$

which is in agreement with the analogous formula obtained by M. A. Bontsch-Bruewitsch.

Having any given complex antenna system comprised of n synphase parallel vibrators we can by means of expressions (29) calculate the radiation resistance for every one of them. This resistance will be:

$$R_\Sigma = R(0,\ 0) + R(d_1\ h_1) + R(d_2,\ h_2) + \cdots + R(d_{n-1},\ h_{n-1}) \tag{32}$$

where h and d denote the height difference and the distance between the first vibrator and each other one; $R(O,O) = 73.3\Omega$.

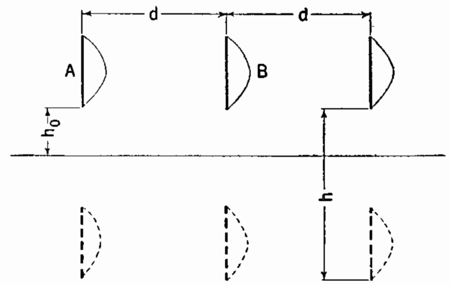

Fig. 8

6. Antenna Over Perfectly Conducting Plane. The expression (29) may also be used for the calculation of the radiation resistance of an antenna erected over a perfectly conducting plane by application of the simple image theory. We shall treat the case of an antenna of which the vibrators are placed at the same height over the plane and at equal distances d from each other.

Let h_0 be the height of antenna over the conducting plane (Fig. 8). Introducing the correction due to the images we shall obtain:

For each of the extreme wires:

$$R_{\Sigma A}=73.3+R(0,h)+R(d,0)+R(d,h)+R(2d,0)+R(2d,h)+\cdots+R[(n-1)d,0]+R[(n-1)d,h] \quad (33)$$

where $h=2h_0+\frac{\lambda}{2}$.

For each wire second from the edge

$$R_{\Sigma B}=73.3+R(0,h)+2[R(d,0)+R(d,h)]+\cdots+R[(n-2)d,0]+R[(n-2)d,h]. \quad (34)$$

If the radiation resistance of the whole antenna is to be found we can use a formula analogous to the formula (25)

$$R_\Sigma=nR_0+2(n-1)R_1+2(n-2)R_2+\cdots+2R_{n-1} \quad (35)$$

where $$R_k=R\left(\frac{k\lambda}{2},\ 0\right)+R\left(\frac{k\lambda}{2},\ 2h_0+\frac{\lambda}{2}\right).$$

The calculations were carried through by the author for a synphase 7-wire antenna elevated 0, $\frac{\lambda}{8}$, $\frac{\lambda}{4}$, $\frac{3\lambda}{8}$, and $\frac{\lambda}{2}$ over the plane. The results are shown in Table III. We may notice that with increasing height the total radiation resistance rapidly approaches the value obtained for free space, but the energy distribution between the individual wires is different. As expected the radiation resistance increases near the plane.

TABLE III

h_0	$r_1=r_7$	$r_2=r_6$	$r_3=r_5$	r_4	R	r_m
0	84.7	58.2	74.7	62.0	497.3	71.0
$\frac{\lambda}{8}$	65.0	41.8	59.8	43.7	376.8	53.8
$\frac{\lambda}{4}$	62.8	44.7	58.8	42.2	374.8	53.5
$\frac{3\lambda}{8}$	65.8	50.5	59.0	51.2	401.8	57.4
$\frac{\lambda}{2}$	66.4	51.2	55.5	54.4	400.6	57.2
∞	64.0	51.2	56.1	53.2	395.6	56.5

7. Multistage Antenna. The method of induced emfs may be applied to more complicated systems, particularly multistage antennas. Let us consider for example a three-stage antenna of the type employed by Marconi.

A unit of such an antenna is a system of three synphase vibrators, spaced along a straight line. These vibrators are connected through antiresonant coils. Let us compute the power due to the emf induced by such an antenna unit in another one spaced at a distance d (Fig. 9). We assume the currents to be equal and in phase.

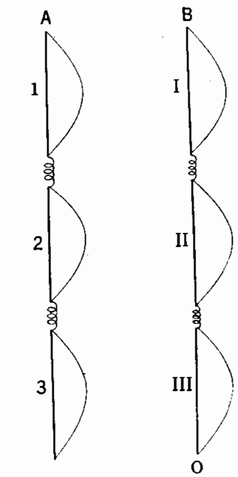

Fig. 9

The power induced in vibrator I of the wire B may be resolved into three parts due to the vibrators 1—3, respectively, of the wire A. Using our notation we may write:

$$P_{AI}=P(d,0)+P\left(d,\frac{\lambda}{2}\right)+(P,\lambda) \quad (36)$$

Similarly for the II and III vibrators:

$$P_{AII}=P(d,0)+2P\left(d,\frac{\lambda}{2}\right)$$

$$P_{III}=P_{AI}. \quad (37)$$

The whole power induced in the wire B will be

$$P_{AB}=3P(d,\lambda)+4P\left(d,\frac{\lambda}{2}\right)+2P(d,\lambda) \qquad (38)$$

and generally in the case of n-stage wire

$$P_{AB}=nP(d,0)+2(n-1)P\left(d,\frac{\lambda}{2}\right)+2(n-2)P(d,\lambda)+\cdots+2P\left(d,(n-1)\frac{\lambda}{2}\right). \qquad (39)$$

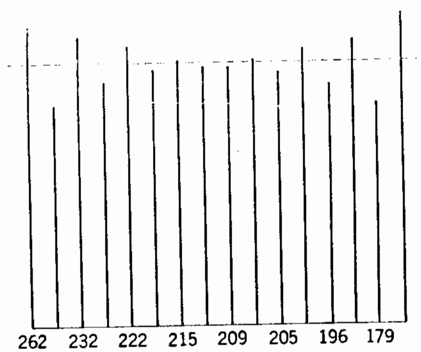

Fig. 10

The radiation resistance of single three-stage wire is obtained by substituting $d=0$.

$$P_0=3P(0,0)+4P\left(0,\frac{\lambda}{2}\right)+2P(0,\lambda)=317.1\,\Omega \qquad (40)$$

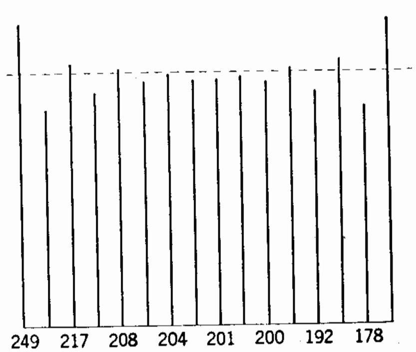

Fig. 11

In order to calculate the radiation resistance of different wires and of the whole antenna, formulas may be used analogous to those obtained above.

The author has performed these calculations for the case of a 16-wire antenna, the distance between the wires being assumed to be $\frac{\lambda}{2}$. The results are shown graphically in Fig. 10; the numbers below give the radiation resistance in ohms. The mean value of this resistance (214 ohms) is marked by a dotted line. Analogous calculations were also made for this antenna elevated at $\frac{\lambda}{4}$ over a perfectly conducting earth. The mean value of the radiation resistance for one wire is then 206 Ω. The energy distribution is shown in Fig. 11. In both cases this distribution

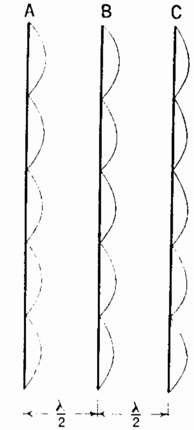

Fig. 12

is very nonuniform in the extreme wires. This means that the design of the feeding devices for the several wires must be quite different, if we wish to obtain equal currents in all vibrators.

8. Table A. Various other types of antennas may be computed in the same manner. To simplify the calculations a table is appended (Table A), containing the functions $R(d, h)$ for values of d and h, which are multiples of a half-wavelength. This table will be found useful in the calculation of many practical types of directive antennas.

As an example let us calculate the radiation resistance of an antenna formed by three five-stage wires (Fig. 12) spaced at a distance of a half-wavelength from each other. Proceeding exactly as in Sect. 7 for the three-stage antenna we shall obtain for the radiation resistance component R_1 of the wire A due to the wire B the following expression:

$$R_1 = 5R\left(\frac{\lambda}{2}, 0\right) + 8R\left(\frac{\lambda}{2}, \frac{\lambda}{2}\right) + 6R\left(\frac{\lambda}{2}, \lambda\right) + 4R\left(\frac{\lambda}{2}, \frac{3}{2}\lambda\right) + 2R\left(\frac{\lambda}{2}, 2\lambda\right) \quad (41)$$

The values of $R(d, h)$ may be taken from the table. Thus we obtain

$$R_1 = 5\cdot(-12.36) + 8\cdot(-11.80) + 6\cdot(-0.78) + 4\cdot(+0.80) + 2\cdot(-1.00) = -159.7\,\Omega \quad (42)$$

TABLE A

h \ d	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
0	+73.29	−12.36	+4.08	−1.77	+1.18	−0.75	+0.42	−0.33
0.5	+26.40	−11.80	+8.83	−5.75	+3.76	−2.79	+1.86	−1.54
1.0	− 4.065	− 0.78	+3.56	−6.26	+6.05	−5.67	+4.51	−3.94
1.5	+ 1.78	+ 0.80	−2.92	+1.96	+0.16	−2.40	+3.24	−3.76
2.0	− 0.96	− 1.00	+1.13	+0.56	−2.55	+2.74	−2.07	+0.74
2.5	+ 0.58	+ 0.45	−0.42	−0.96	+1.59	−0.28	−1.59	+2.66
3.0	− 0.43	− 0.30	+0.13	+0.85	−0.45	−0.10	+1.74	−1.03

TABLE A (Cont'd)

h \ d	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5
0	+0.21	−0.18	+0.15	−0.12	+0.12	−0.10	+0.06	−0.03
0.5	+1.08	−0.85	+0.69	−0.57	+0.51	−0.45	+0.36	−0.30
1.0	+3.08	−2.50	+2.10	−1.80	+1.56	−1.18	+1.14	−1.00
1.5	+3.68	−3.40	+3.14	−2.90	+2.61	−2.31	+2.06	−1.86
2.0	+0.51	−1.30	+1.82	−2.24	+2.28	−2.29	+2.26	−2.14
2.5	−2.49	+2.00	−1.35	+0.49	−0.06	−0.45	+0.85	−1.03
3.0	−0.09	+1.12	−1.87	+1.77	−2.02	+1.71	−1.32	+0.66

The values of d and h are given here in parts of the wavelength; those of radiation resistance in ohms.

To find the radiation resistance component R_2 due to the wire C or the radiation resistance of the single five-stage wire R_0 we must substitute in the above expression $d = \lambda$ or $d = 0$ instead $d = \frac{\lambda}{2}$. Therefore

$$R_2 = +103.0\,\Omega \quad \text{and} \quad R_0 = 558.5\,\Omega$$

The radiation resistance of the wires A and C will then be:

$$R_A = R_C = R_0 + R_1 + R_2 = 501.8\,\Omega$$

and of the wire B

$$R_B = R_0 + 2R_1 = 239.1\,\Omega\ .$$

The whole radiation resistance of the antenna will be 1242.7 Ω, the mean value for 1 wire = 416.2 Ω and for 1 vibrator = 83.0 Ω.