

(1) *Electrical Oscillations in Wires.* By Mr H. C. POCKLINGTON, St John's College.

1. In this paper are discussed some problems relating to the propagation of electrical oscillations along wires. The wire is always supposed to be a perfect conductor, and to have a circular cross-section, the diameter of which is small compared with the other dimensions of the system. We have therefore to solve the equations  $\nabla^2(P, Q, R) = V^2 \frac{d^2}{dz^2}(P, Q, R)$ , conv.  $(P, Q, R) = 0$ , with the further condition that at the surface of the wire the vector  $(P, Q, R)$  is perpendicular to the surface. The method of solution used is to start with the simplest solution of the general equations and by adding an infinite number of such solutions together to obtain one of sufficient generality. The arbitrary function which represents the infinite number of arbitrary constants introduced into this last solution is then found from an equation deduced from the surface condition. This last part of the work is conducted by means of approximations.

2. The simplest solution of the general equations, that corresponding to the solution  $\phi = 1/r$  of the equation  $\nabla^2\phi = 0$ , is given by the formulæ\*

$$P = \frac{d^2\Pi}{dx dz}, \quad Q = \frac{d^2\Pi}{dy dz}, \quad R = \frac{d^2\Pi}{dz^2} + \alpha^2\Pi, \quad \text{where } \Pi = e^{er} e^{pt}/r,$$

in which  $2\pi/p$  is the period of the disturbance, and  $2\pi/\alpha (= 2\pi V/p)$  the wave-length corresponding in free ether to this period. This result can be expressed in words as follows. The electric force due to an elementary Hertzian oscillation with the element of length  $ds$  as axis, is compounded of two forces; the first of these is derived from a potential function  $-\frac{d\Pi}{ds}$ , and the second is a force  $\alpha^2\Pi$  parallel to  $ds$ . This system of forces satisfies the equations of propagation of electric force everywhere excepting at the element  $ds$ . If we place an infinite number of such elements consecutively so as to form a curve, of which  $ds$  will then be an elementary arc, and attribute varying strengths  $\lambda$  to them, we shall obtain a system of forces which satisfies the equations of propagation everywhere except on the curve. The resulting system of forces is

$$(P, Q, R) = -\left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right) \int ds \lambda \frac{d\Pi}{ds} + \alpha^2 \int ds (l, m, n) \lambda \Pi.$$

\* Hertz, *Wied. Ann.* 1889, vol. 36, p. 4; *Electrical Waves* (tr. Jones), p. 140.

If the curve is either closed or has its extremities at infinity, this is equivalent to

$$(P, Q, R) = \left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right) \int ds \frac{d\lambda}{ds} \Pi + \alpha^2 \int ds (l, m, n) \lambda \Pi \dots (1).$$

This is a general solution containing an arbitrary function  $\lambda$ .

3. It now remains to consider the equation derived from the surface conditions. At a point at a small distance  $\epsilon$  from the curve we have, neglecting all terms that are not large,

$$\int ds \Pi \frac{d\lambda}{ds} = -2 \frac{d\lambda}{ds} \log \epsilon \cdot e^{pt},$$

and similarly for  $\int ds \lambda \Pi$ , etc., so that, to this order of approximation,

$$P = -\left\{2 \frac{d}{dx} \frac{d\lambda}{ds} \log \epsilon - 2\alpha^2/\lambda \log \epsilon\right\} e^{pt},$$

and similarly for  $Q$  and  $R$ .

The component of force along the wire therefore is, to this order,

$$-2 \left(\frac{d^2\lambda}{ds^2} + \alpha^2\lambda\right) \log \epsilon \cdot e^{pt}.$$

The force tangential to the cross-section of the wire = 0 to this order. Hence the system of forces given by (1) is a solution of the problem (to this order) provided that

$$\frac{d^2\lambda}{ds^2} + \alpha^2\lambda = 0 \quad \text{or} \quad \lambda = e^{i\alpha s},$$

and the disturbance is propagated along the wire with velocity  $V$  and without diminution of amplitude. This is only what might have been expected from a knowledge of what happens in the case of a straight wire; for if in our case we take the electrical forces to be finite near the wire, at a finite distance they are zero.

4. It is clear that in order to obtain results of much interest we must approximate more closely. We will now consider the equations obtained by neglecting only small quantities of the first and higher orders.

As given by (1) the force at any point on the wire tangential to the axis is the same for all points on the same cross-section, and contains two terms, one containing  $\log \epsilon$ , the other finite. The force tangential to the cross-section is finite and varies for a given value of  $s$  as the cosine of some azimuth angle.

Suppose now that we shift the curve formed by the Hertzian elements through a small distance of the second order. The effect is to change the value of the component tangential to the cross-section by a finite amount which varies as the cosine of some azimuth angle, and that parallel to the axis of the wire by an amount of the first order of small quantities.

By making such a shift of appropriate magnitude and direction at every point, we can therefore eliminate the component tangential to the cross-section. At the same time, the component parallel to the wire is unaltered (to our order). Hence we may still derive the surface condition from (1) by taking the integration along the axis of the wire.

The condition thus obtained is

$$0 = \frac{d}{ds} \int ds \frac{d\lambda}{ds} \Pi + \alpha^2 \{ l \int ds \lambda \Pi + m \int ds m \lambda \Pi + n \int ds n \lambda \Pi \} \dots (2).$$

5. *Circular Ring.* The simplest case that we can consider is that of a circular ring. Let the radius of the ring be  $a$ , the chosen as the axis of  $z$ . We shall assume  $\lambda = A \cos r\phi$ , where  $\phi$  defines a point on the axis of the wire. This assumption will be justified later.

At the point  $(\varpi, \theta, z)$

$$\begin{aligned} \int ds \frac{d\lambda}{ds} \Pi &= -rA \int_0^{2\pi} d\phi \Pi \sin r\phi \\ &= -rA \int_0^{2\pi} d\phi \Pi_0 (\sin r\theta \cos r\phi + \cos r\theta \sin r\phi) e^{i\psi}, \end{aligned}$$

where  $\Pi_0$  is the value that  $\Pi$  takes when  $\phi$  is put for  $(\theta - \phi)$  and 0 for  $t$ , and is thus a function of  $\phi, \varpi, z$  only,

$$= -2rA e^{i\psi} \sin r\theta \int_0^\pi d\phi \Pi_0 \cos r\phi;$$

$$\int ds \lambda \Pi = -A \int_0^{2\pi} a d\phi \Pi \cos \phi \cos r\phi$$

$$\begin{aligned} &= A a e^{i\psi} \left[ \cos(r+1)\theta \int_0^\pi d\phi \Pi_0 \cos(r+1)\phi \right. \\ &\quad \left. + \cos(r-1)\theta \int_0^\pi d\phi \Pi_0 \cos(r-1)\phi \right], \end{aligned}$$

as above; and  $\int ds n \lambda \Pi = 0$ .

Hence (2) becomes

$$0 = \left[ -2rA \frac{r \cos r\theta}{a} \int_0^\pi d\phi \Pi_0 \cos r\phi + \alpha^2 A a \cos r\theta \int_0^\pi d\phi \Pi_0 \{ \cos(r+1)\phi + \cos(r-1)\phi \} e^{i\psi} \right] \dots (3),$$

or  $\int_0^\pi d\phi \Pi_0 \cos r\phi (r^2 - \alpha^2 a^2 \cos \phi) = 0 \dots \dots \dots (3),$

the disappearance of  $\theta$  justifying the assumption made as to the form of  $\lambda$ . In this equation for finding  $a$  we may give to  $\varpi$  and  $z$  in  $\Pi_0$  any values which correspond to a point on the surface of the wire. The simplest values are  $\varpi = a + \epsilon, z = 0$ , and these should therefore be chosen.

6. The special case  $r = 1$ , which corresponds to the fundamental node of the wire, is that of the most interest and will be investigated in detail.

In this case, (3) becomes on substituting for  $\Pi_0$  its value (when  $z = 0$ ),

$$\int_0^\pi d\phi \frac{e^{i\alpha\sqrt{a^2 - 2a\varpi \cos \phi + \varpi^2}}}{\sqrt{a^2 - 2a\varpi \cos \phi + \varpi^2}} \{ \cos \phi - \frac{1}{2} \alpha^2 a^2 (1 + \cos 2\phi) \} = 0,$$

or, putting  $\varpi = a + \epsilon$ ,

$$\begin{aligned} \int_0^\pi d\phi \frac{\frac{1}{2} \alpha^2 a^2 - \cos \phi + \frac{1}{2} \alpha^2 a^2 \cos 2\phi}{\sqrt{2a(a+\epsilon)(1-\cos \phi)} + \epsilon^2} \\ + \int_0^\pi d\phi \frac{e^{2i\alpha a \sin \frac{1}{2}\phi} - 1}{2a \sin \frac{1}{2}\phi} \left( \frac{1}{2} \alpha^2 a^2 - \cos \phi + \frac{1}{2} \alpha^2 a^2 \cos 2\phi \right) = 0 \dots (4), \end{aligned}$$

where in the second integral  $\epsilon$  has been put  $= 0$ , since we are neglecting small quantities.

The first integral in (4) is, calling  $2\alpha a = x, \log \frac{8a}{\epsilon} = L$ , and neglecting small quantities,

$$\frac{1}{a} \left\{ 2 - \frac{x^2}{3} + \left( \frac{x^2}{4} - 1 \right) L \right\}.$$

The second integral is, putting  $\phi/2 = \psi$ ,

$$\begin{aligned} \int_0^{\frac{\pi}{2}} d\psi \frac{e^{ix \sin \psi} - 1}{\sin \psi} \left\{ 8a (1 + \cos 4\psi) - \frac{1}{a} \cos 2\psi \right\} \\ = \int_0^{\frac{\pi}{2}} d\psi \frac{\sin(x \sin \psi)}{\sin \psi} \left\{ \frac{x^2}{8a} (1 + \cos 4\psi) - \frac{1}{a} \cos 2\psi \right\} \\ + \int_0^{\frac{\pi}{2}} d\psi \frac{\cos(x \sin \psi) - 1}{\sin \psi} \left\{ \frac{8a}{x} (1 + \cos 4\psi) - \frac{1}{a} \cos 2\psi \right\} \dots (5). \end{aligned}$$

Now,  $n$  being an integer,

$$\begin{aligned} \frac{2}{\pi} \int_0^{\frac{\pi}{2}} d\psi \cos 2n\psi \cos(x \sin \psi) &= J_{2n}(x), \\ \frac{2}{\pi} \int_0^{\frac{\pi}{2}} d\psi \cos 2n\psi \frac{\sin(x \sin \psi)}{\sin \psi} &= \int_0^x dx J_{2n}(x), \\ &= -2[J_{2n-1}(x) + J_{2n-3}(x) + \dots + J_1(x)] + \int_0^x dx J_0(x). \end{aligned}$$

Therefore the first integral in (5) is

$$\frac{\epsilon\pi}{2a} \left\{ -\frac{x^2}{4} J_3(x) + \left(2 - \frac{x^2}{4}\right) J_1(x) + \left(\frac{x^2}{4} - 1\right) \int_0^x dx J_0(x) \right\},$$

and (4) becomes, on re-arrangement and multiplication by  $a$ ,

$$\begin{aligned} \left(\frac{x^2}{4} - 1\right) L &= \frac{x^2}{3} - 2 - \frac{\epsilon\pi}{2} \left\{ -\frac{x^2}{4} J_3(x) \right. \\ &\quad \left. + \left(2 - \frac{x^2}{4}\right) J_1(x) + \left(\frac{x^2}{4} - 1\right) \int_0^x dx J_0(x) \right\} \end{aligned}$$

$$\begin{aligned} - \left\{ \frac{x^2}{8} \int_0^{\frac{\pi}{2}} d\psi \frac{\cos(x \sin \psi) - 1}{\sin \psi} (1 - 2 \cos 2\psi + \cos 4\psi) \right. \\ \left. + \left(\frac{x^2}{4} - 1\right) \int_0^{\frac{\pi}{2}} d\psi \frac{\cos(x \sin \psi) - 1}{\sin \psi} \cos 2\psi \right\} \dots (6). \end{aligned}$$

This equation gives  $x$  and thence  $\alpha$  with an error of the order of  $\epsilon/a$ . It can only be solved by trial. If however  $\epsilon$  is so small that errors of the order of  $1/L^2$  can be neglected, we may use an approximate solution of the above. A first approximation is  $x^2 = 4$  or  $x = 2$ . A second is obtained by putting  $x = 2$  on the right-hand side of (6). This gives

$$\left(\frac{x^2}{4} - 1\right) L = 485 - 703\epsilon,$$

so that

$$\alpha = \frac{1}{a} \{1 + (243 - 351\epsilon)/L\}.$$

Hence the period of the oscillation is equal to the time required for a free wave to traverse a distance equal to the circumference of the circle multiplied by  $1 - 243/L$ , and the ratio of the amplitudes of consecutive vibrations is  $1 : e^{-2x/L}$  or  $1 : 1 - 221/L$ . It is easy to verify from first principles that the decrease in amplitude of the vibrations is of this order.

7. *Induced Vibrations in a Ring.* We will now consider the case of a ring upon which plane waves are incident in a direction parallel to the axis of the ring.

Let the coordinate axes be chosen as before, and let the incident vibration be given by

$$P = R = 0, \quad Q = Ae^{i\alpha x} e^{i\theta t}.$$

The tangential force at the point  $\theta$  due to this wave together with a disturbance induced in the wire of the fundamental mode and magnitude  $B$ , is to be equated to zero, giving

$$0 = Ae^{i\theta t} \cos \theta - 2Be^{i\theta t} \frac{\cos \theta}{a} \int_0^\pi d\phi \Pi_0 \cos \phi (1 - \alpha^2 a^2 \cos \phi).$$

If we neglect  $\epsilon/a$  in comparison with unity, this becomes, as in §6,

$$\begin{aligned} 0 = A + \frac{2B}{a^2} \left[ \left\{ 2 - \frac{x^2}{3} + \left(\frac{x^2}{4} - 1\right) L \right\} \right. \\ \left. + \frac{\epsilon\pi}{2} \left\{ -\frac{x^2}{4} J_3(x) + \left(2 - \frac{x^2}{4}\right) J_1(x) + \left(\frac{x^2}{4} - 1\right) \int_0^\pi dx J_0(x) \right\} \right] \end{aligned}$$

$$\begin{aligned} + \frac{x^2}{8} \int_0^{\frac{\pi}{2}} d\psi \frac{\cos(x \sin \psi) - 1}{\sin \psi} (1 - 2 \cos 2\psi + \cos 4\psi) \\ + \left(\frac{x^2}{4} - 1\right) \int_0^{\frac{\pi}{2}} d\psi \frac{\cos(x \sin \psi) - 1}{\sin \psi} \cos 2\psi \end{aligned}$$

Unless  $x = 2$ , this gives

$$B = \frac{2Aa^2}{(4 - x^2)} L,$$

so that in general the induced vibration is small, and the thinner the wire the smaller the induced vibration. The phase is the same as or opposite to that of the electric displacement in the plane of the ring due to the incident wave. If, however,  $x = 2 = 2 + \xi$ , where  $\xi$  is small, we may put  $x = 2$  in all the terms not involving  $L$ , and get

$$B = -2L\xi + 970 - 1.055\epsilon.$$

The maximum amplitude of the induced vibration is obtained when  $2L\xi = 970$ , or  $\alpha = x/2a = \{1 + 243/L\}/a$ , i.e. when the period of the incident wave is the same as the free period of the ring; the amplitude then is  $.9484a^2$ , and the phase is in quadrature

with that of the wave. It is noteworthy that the amplitude of the induced vibration is independent of the thickness of the wire. If the incident wave is not proceeding in a direction perpendicular to the ring, the problem can still be solved by a method similar to the above. The vibrations induced in the ring will however not be confined to the fundamental, but will include vibrations of all modes.

8. *Helix*. We will now consider the case of vibrations propagated along an infinite wire wound into a uniform helix. Let the equations of the axis of the wire be  $x = a \cos \phi$ ,  $y = a \sin \phi$ ,  $z = a\phi \tan \omega$ . We shall assume  $\lambda = Ae^{i\phi}$ . This assumption is justified later. The value of the force tangential to the wire at the point  $(\varpi, \theta, z)$  is  $e^{i\theta}$  times that at the point  $(\varpi, \theta, z - a\theta \tan \omega)$ . Hence (2) gives

$$0 = Ae^{i\varpi t} \left[ \left( \frac{d}{ds} \right)_{\theta=0} e^{i\theta} \int_{-\infty}^{\infty} d\phi \beta e^{i\theta} \Pi_0 + \alpha^2 \left\{ \cos \omega \int_{-\infty}^{\infty} d\phi a \cos \phi e^{i\theta} \Pi_0 \right. \right. \\ \left. \left. + \sin \omega \int_{-\infty}^{\infty} d\phi a \tan \omega e^{i\theta} \Pi_0 \right\} \right],$$

where  $\Pi_0$  is the value that  $\Pi$  takes when  $t = 0$  and  $\phi$  is put for  $\phi - \theta$ .

$$\text{Hence } 0 = \int_{-\infty}^{\infty} d\phi e^{i\theta} (\alpha^2 a^2 \tan^2 \omega - \beta^2 + \alpha^2 a^2 \cos \phi) \Pi_0 \dots (7).$$

In obtaining this equation small quantities only have been neglected. If however  $\epsilon/a$  is very small, we may in this equation neglect all finite quantities in comparison with those of the order of  $\log \epsilon/a$ . In this case we may with advantage find an approximate value of the right-hand side of (7).

Assuming  $\kappa$  any finite quantity, and neglecting terms that are finite, the right-hand side of (7) is

$$\int_{-\infty}^{\infty} d\phi e^{i\theta} \frac{e^{-i\alpha a \phi} \tan \omega}{-a\phi \tan \omega} \{ \alpha^2 a^2 \tan^2 \omega - \beta^2 + \alpha^2 a^2 \cos \phi \} \\ + \int_{-\kappa}^{\kappa} d\phi \frac{\alpha^2 a^2 \sec^2 \omega - \beta^2}{\sqrt{2a(a+\epsilon)(1-\cos \phi)} + a^2 \phi^2 \tan^2 \omega + \epsilon^2} \\ + \int_{\kappa}^{\infty} d\phi e^{i\theta} \frac{e^{i\alpha a \phi} \tan \omega}{a\phi \tan \omega} \{ \alpha^2 a^2 \tan^2 \omega - \beta^2 + \alpha^2 a^2 \cos \phi \}.$$

The second integral is, neglecting finite quantities,

$$- 2 (\alpha^2 a^2 \sec^2 \omega - \beta^2) \frac{\cos \omega}{a} \log \epsilon.$$

The first and last can be reduced to sums of integrals of the form  $\int_{\kappa}^{\infty} d\phi \frac{e^{-\gamma \phi}}{\phi}$ . This integral\* is  $-\log \gamma$  to our order, and thus the first and last integrals give

$$\frac{1}{a \tan \omega} [(\alpha^2 a^2 \tan^2 \omega - \beta^2) \log (\alpha^2 a^2 \tan^2 \omega - \beta^2) \\ + \frac{1}{2} \alpha^2 a^2 \log \{ (1 + \beta + \alpha a \tan \omega)(1 - \beta + \alpha a \tan \omega)(1 + \beta - \alpha a \tan \omega) \\ (1 - \beta - \alpha a \tan \omega) \}],$$

and therefore the approximate form of (7) is

$$2 (\alpha^2 a^2 \sec^2 \omega - \beta^2) \sin \omega \log \epsilon = (\alpha^2 a^2 \tan^2 \omega - \beta^2) \log (\alpha^2 a^2 \tan^2 \omega - \beta^2) \\ + \frac{1}{2} \alpha^2 a^2 \log \{ (1 + \beta + \alpha a \tan \omega)(1 - \beta + \alpha a \tan \omega)(1 + \beta - \alpha a \tan \omega) \\ (1 - \beta - \alpha a \tan \omega) \} \dots (8).$$

Several cases may occur. (i) In general, if  $\alpha$  is not small, the only term of importance is that on the left, so that

$$\alpha^2 a^2 \sec^2 \omega - \beta^2 = 0, \text{ or } \beta = \alpha a \sec \omega,$$

and the velocity of propagation measured along the wire is

$$\frac{p}{\beta} a \sec \omega = \frac{p}{\alpha} = V,$$

the same result as that obtained in the case of a circle.

(ii) If however  $\alpha$  and  $\beta$  are small, the first term on the right is also of importance. If  $\alpha$  and  $\beta$  are so small that  $\log \epsilon$  can be neglected in comparison with  $\log (\alpha^2 a^2 \tan^2 \omega - \beta^2)$ , i.e. if the product of the wave-length of the disturbance into the radius of the wire is very large compared with the square of the radius of the helix, we have  $\alpha^2 a^2 \tan^2 \omega - \beta^2 = 0$ , or  $\beta = \alpha a \tan \omega$ , and the velocity of propagation measured along the wire is

$$\frac{p}{\beta} a \sec \omega = \frac{p}{\alpha} \operatorname{cosec} \omega = V \operatorname{cosec} \omega,$$

so that the disturbance is propagated with a velocity  $V$  measured along the axis of the helix.

If  $\alpha$  and  $\beta$  are small, but not so small that  $\log \epsilon$  can be neglected, i.e. if the product of the wave-length into the radius of the wire is comparable with the radius of the helix, the velocity of propagation has an intermediate value†. It is easy to see that a like result does not follow if we try to make  $(\alpha^2 a^2 \tan^2 \omega - \beta^2)$  small without making  $\alpha$  and  $\beta$  small.

\* J. W. L. Glaisher, *Phil. Trans.*, 1870, p. 369.

† Hertz, *Wied. Ann.*, 1889, vol. 36, p. 21; *Electrical Waves* (tr. Jones), p. 158, has proved experimentally that this is the case.

(iii) If one of the factors, e.g. the last, of the term under the second log sign be small we have, since  $\alpha$  and  $\beta$  cannot then both be small,

$$2(\alpha^2 v^2 \sec^2 \omega - \beta^2) \sin \omega \log \epsilon = \frac{1}{2} \alpha^2 v^2 \log(1 - \beta - \alpha \tan \omega).$$

In this case we must have  $\beta < \alpha \sec \omega$ , so that the velocity of propagation measured along the wire must be greater than  $V$ . There is however no superior limit to the value that it may have. There is an inferior limit to  $\alpha$ . Hence for periods which are not greater than a certain value, there are two velocities of propagation along the wire, one =  $V$  given by (i), the other,  $> V$ , given by (iii).

No other cases arise by making  $\alpha$  and  $\beta$  great, since we must then recur to (7), as (8) does not then hold; and here if  $\alpha$  and  $\beta$  are great, we simply have  $\alpha^2 v^2 \sec^2 \omega - \beta^2 = 0$ , the case considered in (i).

(2) *On Circles, Spheres and Linear Complexes.* By Mr J. H. GRACE.

This paper is printed in the *Transactions*, Vol. XVI. Part III.

(3) *Reduction of a certain Multiple Integral.* By ARTHUR BLACK. Communicated by Professor M. J. M. HILL, M.A., Sc.D., F.R.S.

This paper is printed in the *Transactions*, Vol. XVI. Part III.

(4) *On the Gamma Function.* By Mr H. F. BAKER.

The Gamma Function could be defined for real values of  $x$  by the conditions (i) that  $\Gamma(1) = 1$ , (ii) that  $\Gamma(x+1) = x\Gamma(x)$ , (iii) that, for a fixed finite  $h$ , as  $x$  tends to  $+\infty$ , the difference

$$\frac{\Gamma'(x+h)}{\Gamma(x+h)} - \frac{\Gamma'(x)}{\Gamma(x)}$$

tends always to zero. The condition (iii) was well known, being deducible from the result

$$\frac{\Gamma'(x)}{\Gamma(x)} - \log x = \int_0^\infty [e^{-xt} - (1+t)^{-x}] \frac{dt}{t},$$

and was of suitable character for a definition; it was desirable however to deduce it immediately from the equation

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt;$$

this note dealt with such a deduction.

*On the lines of striction of a hyperboloid.* By Mr H. F. BAKER.

It was a known fact that the lines of striction of a hyperboloid formed two unicursal quartic curves. The most commonly given equation shewed that the curve was an octavic curve with six double points; such a curve on a surface of the second order could not be a proper octavic curve; for a cubic surface drawn through the double points and thirteen arbitrary points of the curve would otherwise cut the curve in  $12 + 13 = 25$  points. The question considered was what are the possible forms of such octavic curves.

(5) *On the Action of Uranium rays on the Condensation of Water Vapour.* By C. T. R. WILSON, M.A., of Sidney Sussex College, Clerk Maxwell Student.

I have already (*Proc. Roy. Soc.*, Vol. 59, p. 338, 1896; *Phil. Trans.* A. 189, p. 265, 1897) described experiments upon the effect of Röntgen rays on the condensation of water vapour. These experiments proved that the rays, in traversing moist air, introduce nuclei capable of acting as centres of condensation when supersaturation exceeding a definite limit is brought about by the sudden expansion of the gas. Nuclei, requiring exactly the same degree of supersaturation to enable condensation to take place upon them, are always present in very small numbers even without the action of the X-rays; but these rays enormously increase their number. To produce the degree of supersaturation necessary to bring these nuclei into play in air originally saturated, a sudden expansion is necessary such that  $v_2/v_1$ , the ratio of the final to the initial volume, exceeds 1.25; corresponding approximately to a fourfold supersaturation.

In the absence of X-rays and other disturbing influences, no condensation is observed (after the removal of all foreign nuclei) if  $v_2/v_1$  is less than 1.252. If  $v_2/v_1$  lies anywhere between this and 1.37 a rainlike condensation results, the drops being few and scattered. Beyond this second limit dense fogs appear.

The action of the X-rays is simply to increase the number of the drops which are formed when  $v_2/v_1$  lies between these limits; the minimum expansion required for condensation not being sensibly altered.

The experiments described in the present paper show that nuclei of exactly the same nature are produced in moist air under the action of the Uranium radiation discovered by Becquerel.

The form of apparatus used is represented in the accompanying figure.