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VIII. TEACHING AIDS

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Helmholtz coils revisited

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Two circular coaxial current rings of radius a, separated by a distance b, are usually called Helmholtz coils if $b = a$. That is the well-known prescription for greatest uniformity of field in the neighborhood of the center of symmetry. Examined here is the magnetic field of a pair of current rings at a distance r from the center large compared to the ring radius a. Expanding the distant field in powers of a/r, each term may be associated with a magnetic 2^k-pole source and a field falling off as $(a/r)^{k+2}$. The dipole field $k = 1$, falling as $(a/r)^3$, has as its source the total dipole moment of the two rings. It eventually swamps the contribution of higher multipoles. The term $(a/r)^5$ with $k = 3$ has for its source the octupole (2^3-pole) moment of the pair of rings. Its coefficient vanishes if $b = a$. Thus the Helmholtz pair, thanks to its seemingly irrelevant specialization for uniform central field, is also endowed with zero octupole moment. The term-by-term connection between the expansion of the distant field and the expansion of the central field holds for currents entirely confined to the surface of a sphere, which is the case for the Helmholtz pair of thin current rings. The dipole field can be nullified by nesting the Helmholtz pair within a larger Helmholtz pair designed to have equal area × ampere–turns but carrying reversed current. At the cost of a minor reduction in central field and without compromising the central field's uniformity, the total dipole moment of the four rings can thus be made zero. The current system now has zero dipole moment *and* zero octupole moment. As 2^k-poles with k even are ruled out by symmetry, the lowest surviving multipole is the 2^5-pole, the "32-pole." The residual distant field, of which it is now the dominant source, falls as $(a/r)^7$. An exact calculation of the ratio of field magnitude B to central field B_0 shows that B/B_0 is closely proportional to $(a/r)^7$ everywhere in the field beyond $r \gtrsim 5a$. Already at $r \gtrsim 6a$, B/B_0 has fallen to 10^{-4}. It is shown how equivalent results can be achieved with coils of finite cross section. This scheme can drastically reduce the "stray field" around a superconducting magnet. In effect, it closely confines most of the return flux. Another application involves the induction of alternating currents by precessing magnetic moments. The four coils connected in series constitute a receiving coil tightly coupled to any oscillating dipole at its center, but virtually immune to disturbance by external sources.

The combination of two similar circular coils, coaxial, connected in series, and separated axially by a distance comparable to the coil radius, is generally called a Helmholtz coil. Its well-known virtue is the homogeneity of the central magnetic field that can be achieved by adjusting the separation of the two component coils. To be specific, let the coils encircle the z axis and locate the origin midway between them. Thanks to the symmetry of the current distribution, the magnetic field on the axis B_z must be an even function of z,

$$B_z = B_0 + c_2 z^2 + c_4 z^4 + \cdots . \tag{1}$$

The quadratic term is surely positive when the coils are far apart, the origin being then at a minimum in B_z. On the other hand, if the coils are drawn together to form a single ring, B_z will be maximum at the origin and c_2 will be negative. So there must be, for a given pair of coils, a certain distance of separation for which c_2 vanishes, leaving $B_z - B_0$ proportional to z^4 rather than z^2, and thus, in a certain sense, maximizing the homogeneity of the field in the neighborhood of the center of symmetry. Of course, the field in the central region could be made still more uniform through the addition of auxiliary coils.[1] Further coefficients in Eq. (1) could thus be adjusted, or nullified. However, since central field uniformity will not be our only, or even our main, concern, we shall not go beyond the requirement $c_2 = 0$.

Determining the separation for which c_2 vanishes is easy if the two "coils" are simply two circular loops of current of infinitesimal cross section. We shall refer to that idealization as the "thin coil" case. Let a be the radius of the thin rings and locate their centers at $z = b/2$ and $z = -b/2$. It turns out, as generations of students have learned by solving this venerable problem, that the coefficient c_2 of the quadratic term vanishes if and only if $b = a$. From now on, the term *Helmholtz pair* will designate a pair of similar thin current rings that satisfy this condition. Coils of finite cross section, "thick" coils, will be discussed later.

We shall be concerned here not with the strong field in the central region, but with the *external* field, the field at distances from the coil large compared to the coil radius a. How does field strength vary with distance? How does the flux return? How can the external field be suppressed? Pursuing such questions recently, I stumbled on to a second special property of the Helmholtz pair: Its *octupole moment* is zero.

No doubt others have encountered this fact. It was new to me. I believe it deserves attention for reasons both practical and pedagogical. It opens up a promising way of reducing the external fields of superconducting magnets, and, in a more general sense, decoupling an inductive coil from its environment. That will be the main theme of this article. The relation between the vanishing c_2 in the expansion of the central field, and the vanishing octupole moment of the Helmholtz pair will be explained briefly, with an appropriate reference. Perhaps our present concern for the external fields of strong magnets can rejuvenate the Helmholtz coil as an instructive classroom exercise.

To describe the behavior of the distant field of a stationary current distribution we may view the source as a superposition of magnetic multipoles—dipole, quadrupole, octupole, and so on. We examine first the distant field of a pair of thin current rings of radius a, located in the planes $z = b/2$ and $z = -b/2$, with their centers on the z axis, and with their separation b *not* necessarily equal to a. A current I flows in the same sense in each ring. The symmetry of this current distribution rules out all 2^k-poles with k even. Thus reversing the current in one of the two rings would create a quadrupole ($k = 2$) along with higher even multipoles, while causing the dipole moment and all higher odd multipole moments to vanish. A 2^k-pole source is characterized by a field that falls off with distance r from the source as $r^{-(k+2)}$.

To make the point about the octupole moment it will suffice to consider the field of the two rings on the symmetry axis. An elementary application of the Biot–Savart law gives

$$B_z = \frac{\mu_0 I a^2}{2}\left[\left(a^2 + z^2 - bz + \frac{b^2}{4}\right)^{-3/2} + \left(a^2 + z^2 + bz + \frac{b^2}{4}\right)^{-3/2}\right]. \tag{2}$$

(The units are obvious and, in any case, irrelevant.) To show the asymptotic behavior of B_z we may expand it in inverse powers of z. Replacing z by a/u, Eq. (2) becomes

$$B_z = \frac{\mu_0 I}{2a}u^3\left[\left(1 + u^2 + \frac{b^2u^2}{4a^2} - \frac{b}{a}u\right)^{-3/2} + \left(1 + u^2 + \frac{b^2u^2}{4a^2} + \frac{b}{a}u\right)^{-3/2}\right]. \tag{3}$$

The Taylor series for the function of u in square brackets in Eq. (3) begins $[2 + 3(b^2/a^2 - 1)u^2 + \cdots]$, so the leading terms in expansion of the distant axial field in powers of z^{-1} are

$$B_z = \frac{\mu_0 I a^2}{z^3} + \frac{3\mu_0 I a^2(b^2 - a^2)}{2z^5} + \cdots . \tag{4}$$

The first term is recognizable at once as the axial field from a dipole at the origin. The second term comes from the octupole moment of the current distribution. We see that this component of the field on axis B_z vanishes if $b = a$. Now the field of any axially symmetrical multipole cannot be zero along the whole symmetry axis unless it is zero off the axis as well. Otherwise, it would be possible to draw on a plane through the axis a contour enclosing no current yet having a nonzero line integral of **B**. It follows that the entire octupole field, not just B_z on the axis, will vanish if $b = a$. But that is precisely the condition under which the quadratic term $c_2 z^2$ in the expansion of the central field [Eq. (1)] also vanishes as the reader can verify by expanding Eq. (2) near $z = 0$ and finding that the coefficient of z^2 is proportional to $b^2 - a^2$. The result can be stated simply: *Every Helmholtz pair has zero octupole moment.*

This result remains academic as long as the field at large distance is dominated by the dipolar component. A magnetic octupole field, falling as r^{-5}, is inexorably swamped by a dipole field falling only as r^{-3}; the vanishing of the octupole component in a special case is then inconsequential.

But what if the dipole moment itself could be suppressed? The dipole moment of a Helmholtz pair, which is proportional to area × ampere–turns [see Eq. (4)], can be exactly canceled by that of another pair, of larger size and smaller current, concentric with the first pair, but with current reversed. Figure 1 shows an example of such *compensating Helmholtz pairs.* The outer pair in this example has twice the radius of the inner pair and one-fourth of the ampere–turns. The coils, although shown with some thickness in the diagram, will be treated first as four rings of zero thickness—the "thin coil" limiting case. Adding the outer Helmholtz pair has annulled the total dipole moment while reducing the central field by only 12.5%, which is the ratio of the central field strength of the outer pair to that of the inner pair in this case. Homogeneity of the central field is not impaired; c_2 remains zero, being zero for each pair. We have here a set of coils that has zero dipole moment *and*

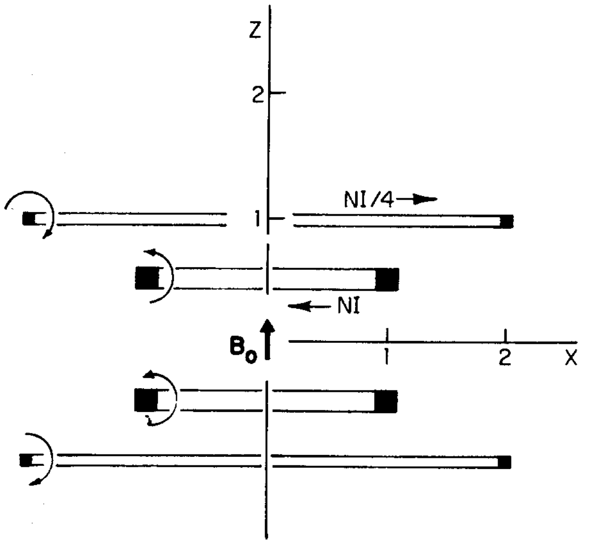
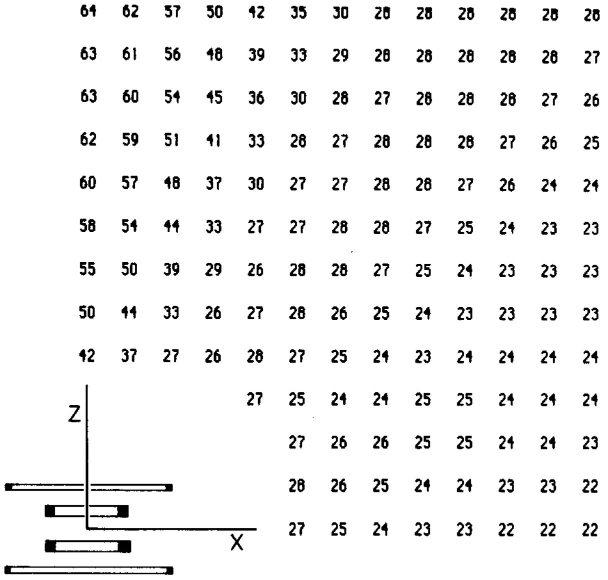


Fig. 1. One example of compensating Helmholtz pairs. The outer pair has twice the radius of the inner pair, and one-fourth the ampere–turns, with direction reversed. This reduces the central field by only 12.5%, without spoiling its homogeneity. The dipole moment is exactly annulled, and with it the z^{-3} term in the Taylor expansion of the distant field. Now the z^{-5} term vanishes also because each pair of coils, and hence their combination, has zero octupole moment. The lowest surviving term falls with distance as z^{-7}.

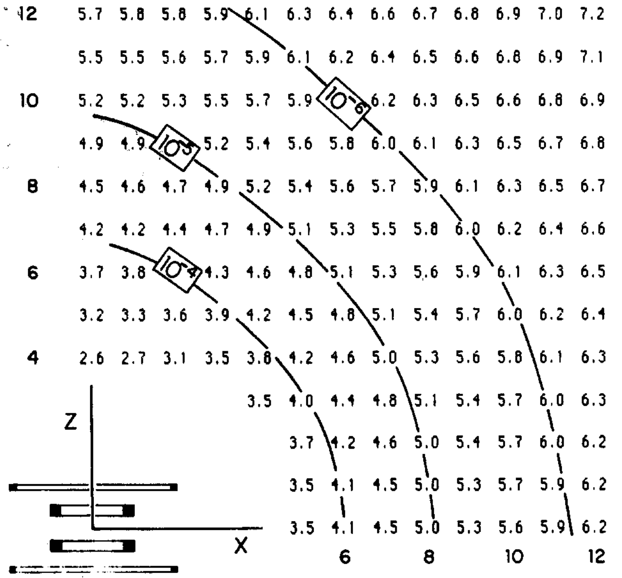


64	62	57	50	42	35	30	28	28	28	28	28	28
63	61	56	48	39	33	29	28	28	28	28	28	27
63	60	54	45	36	30	28	27	28	28	28	27	26
62	59	51	41	33	28	27	28	28	28	27	26	25
60	57	48	37	30	27	27	28	28	27	26	24	24
58	54	44	33	27	27	28	28	27	25	24	23	23
55	50	39	29	26	28	28	27	25	24	23	23	23
50	44	33	26	27	28	26	25	24	23	23	23	23
42	37	27	26	28	27	25	24	23	24	24	24	24
				27	25	24	24	25	25	24	24	24
					27	26	26	25	25	24	24	23
					28	26	25	24	24	23	23	22
					27	25	24	23	23	22	22	22

Fig. 3. Showing the r^{-7} dependence of the residual distant field. The number at each location is the product of B/B_0 and $(x^2+z^2)^{7/2}$. This product is remarkably constant over the field. To within a factor 2 either way, the field at any point with $r>5$ is given approximately by $B/B_0 = 30/r^7$, the unit of distance being the radius of the inner Helmholtz pair. Thus the product at location (6,7) is $(5\times10^{-6})\ (36+49)^{7/2}=28$.

zero octupole moment. The lowest surviving multipole is the 2^5-pole, the field of which is expected to fall off with distance as r^{-7}.

To confirm this and to explore the distant field of the coils in Fig. 1 away from the symmetry axis, I calculated the field intensity $B=(B_x^2+B_z^2)^{1/2}$ at 150 points on the x-z plane. This was done without invoking elliptic integrals, by directly summing, over the four rings, the Biot–Savart integrand for the field point in question. Double precision is called for, the small residual field being the difference between large off-setting contributions from the two pairs of current rings. The results, expressed by the ratio of B to B_0, the field at the origin, are shown in Fig. 2. Printed there at each grid point is the negative two-place $\log_{10}(B/B_0)$. The unit of distance is the radius a of the inner (or "main") Helmholtz pair. Notice that B/B_0 has fallen below 10^{-4} when r exceeds $6a$ and below 10^{-6} when $r>12a$. Figure 3 shows how closely the predicted r^{-7} dependence is followed. To within a factor 2 either way, B/B_0 over this grid can be approximated by $B/B_0\approx30(a/r)^7$.



12	5.7	5.8	5.8	5.9	6.1	6.3	6.4	6.6	6.7	6.8	6.9	7.0	7.2
	5.5	5.5	5.6	5.7	5.9	6.1	6.2	6.4	6.5	6.6	6.8	6.9	7.1
10	5.2	5.2	5.3	5.5	5.7	5.9		6.2	6.3	6.5	6.6	6.8	6.9
	4.9	4.9		5.2	5.4	5.6	5.8	6.0	6.1	6.3	6.5	6.7	6.8
8	4.5	4.6	4.7	4.9	5.2	5.4	5.6	5.7	5.9	6.1	6.3	6.5	6.7
	4.2	4.2	4.4	4.7	4.9	5.1	5.3	5.5	5.8	6.0	6.2	6.4	6.6
6	3.7	3.8		4.3	4.6	4.8	5.1	5.3	5.6	5.9	6.1	6.3	6.5
	3.2	3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.0	6.2	6.4
4	2.6	2.7	3.1	3.5	3.8	4.2	4.6	5.0	5.3	5.6	5.8	6.1	6.3
					3.5	4.0	4.4	4.8	5.1	5.4	5.7	6.0	6.3
						3.7	4.2	4.6	5.0	5.4	5.7	6.0	6.2
						3.5	4.1	4.5	5.0	5.3	5.7	5.9	6.2
						3.5	4.1	4.5	5.0	5.3	5.6	5.9	6.2
							6		8		10		12

Fig. 2. Relative field strengths outside the coils of Fig. 1 for 150 locations in the x-z plane. The field intensity $B=(B_x^2+B_z^2)^{1/2}$ was calculated accurately. Its value relative to the external field B_0 is indicated by a number at that location that is $-\log_{10}(B/B_0)$. Thus the number 5.3 at location (6,7) indicates that B at that spot is approximately $5\times10^{-6}\ B_0$.

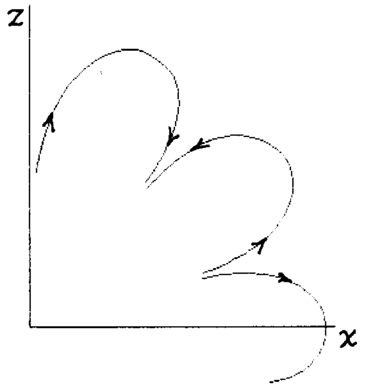


Fig. 4. A few field lines, accurately traced, in the distant field surrounding the coils of Fig. 1. The region covered, out to $x=12$ and $z=12$, is the same as in Figs. 2 and 3. In any half-plane with its edge on the symmetry axis, the distant field shows five lobes, characteristic of a 32-pole (2^5-pole) source. A dipole (2^1-pole) source would have one lobe; an octupole (2^3-pole) source would have three lobes. Symmetry of the current system about the x-y plane (and the nonexistence of magnetic monopoles) excludes 2^k-pole sources with k even.

Evidently, the residual field in this region is already dominated by the 32-pole component of the source. Of course, a discrepancy, however small, in the cancellation of the dipole moments would lead to a recovery of dipolar dominance at a sufficiently great distance from the source.

One might have expected to see in the field of a 32-pole a conspicuous multilobed pattern, not the featureless landscape of Fig. 2. But we are viewing in Fig. 2 only the field magnitude $(B_x^2 + B_z^2)^{1/2}$. Lobes will become evident, even at this coarse resolution, if we look at the vector field itself. I have traced accurately in Fig. 4 a few field lines of the residual field of the compensating Helmholtz coils of Fig. 1. The fidelity with which these follow the pattern to be expected for a 2^5-pole source implies that multipole components both lower and higher than 2^5-pole are relatively minor over the region here viewed. It provides assurance too that the computation of the residual field was not compromised by round-off errors, or by my replacement of each circular ring by a "200-gon." I emphasize that the field described in Figs. 2–4 was computed directly from the current distribution in Fig. 1 using nothing but the Biot–Savart law and double-precision arithmetic, with no reference to a multipole decomposition.

Why should the octupole moment of the two current rings vanish along with the coefficient c_z in Eq. (1), under the Helmholtz condition $b = a$? A basis for the understanding of this is provided in Smythe's classic text.[2] Under the heading "Field of Currents in a Spherical Shell" (p. 292), Smythe develops the vector potential **A** for both the interior and exterior field of currents confined to the surface of a sphere of radius a. The current distribution is described as a superposition of surface harmonics with each of which is associated a vector potential **A**. A particular harmonic in the shell of current gives rise to a vector potential proportional to $(r/a)^n$ for $r < a$ and to $(a/r)^{n+1}$ for $r > a$, with continuity at $r = a$. For the axial field component, $B_z = (\nabla \times \mathbf{A})_z$, this implies that an internal field with B_z proportional to z^{n-1} will continue externally with $B_z \propto z^{-(n+2)}$. The case $n = 3$ associates an internal field proportional to r^2, the second term in our Eq. (1), with an external field proportional to r^{-5}, that is, an octupole field. They arise from the same surface harmonic and appear or vanish together.

The relation can be seen as a symmetry associated with inversion in the sphere, a symmetry expressed in terms of a potential function in this way: If $f(r)\Psi(\theta,\varphi)$ satisfies Laplace's equation, then so does $(1/r)f(1/r)\Psi(\theta,\varphi)$. From this follows at once the connection between terms r^l and $r^{-(l+1)}$ in the potential, hence between terms r^n and $r^{-(n+3)}$ in the field.

All of this applies to our thin coil Helmholtz pair because the two current rings do lie on one spherical surface of radius $(a^2 + b^2/4)^{1/2}$.

The case $n = 1$ is instructive. We get it with a spherical shell of azimuthal current with density J_ϕ proportional to $\sin\theta$. (This happens to be the current distribution you would create by rotating a charged spherical conductor.) The external magnetic field is a pure dipole field. The internal field is uniform throughout the sphere. All terms but c_0 in Eq. (1), and all multipole moments except the dipole moment, are zero. Suppose we enclose this shell within a similar, larger shell that has equal but opposite dipole moment, as in Fig. 5. The magnetic field is zero outside the larger shell, and remains exactly uniform (although somewhat reduced in strength) inside the smaller shell. The returning flux is *entirely* confined to the space between the shells. Here is the ultimate combination of isolation and uniformity. But concentric spherical shells are not attractive as a design for a magnet—for some obvious reasons including inaccessiblity of the interior. Our set of compensating Helmholtz pairs offers a practical compromise. It, too, can be described as confining the main coils' returning flux. This will be clear from Fig. 6, which shows the configuration of the field around the four current rings of Fig. 1.

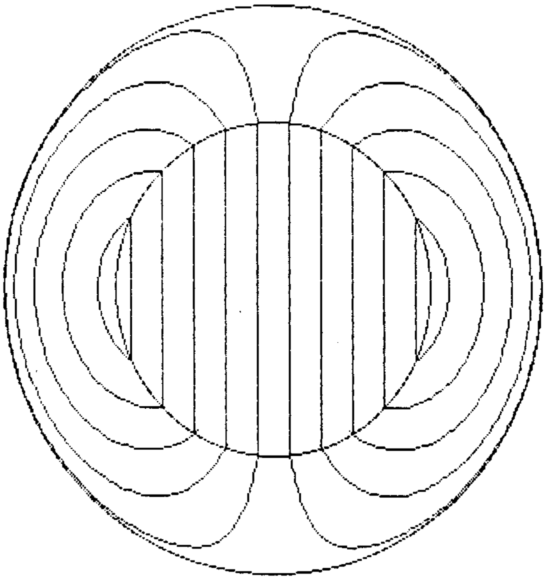


Fig. 5. Two spherical current shells with total dipole moment zero. On each shell, the surface current is azimuthal, with density in amp/m proportional to $\sin\theta$, making all higher moments zero, for each shell. The central field is perfectly uniform. Its strength is 0.8 of what it would be with the outer shell's current turned off, the ratio of radii being here $5^{1/3}/1$. Outside the outer shell there is no field whatever.

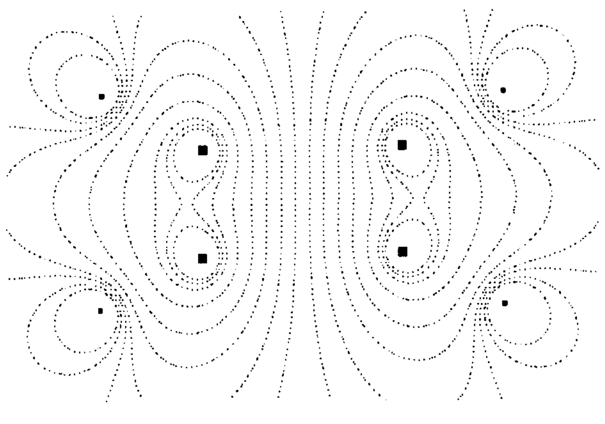


Fig. 6. The field close to the coils of Fig. 1. The vector potential $\mathbf{A}_y$ was calculated for a lattice of 2400 points in the x-z quadrant. Thanks to the axial symmetry, field lines can then be easily mapped as level contours of the function $x\,\mathbf{A}_y(x,z)$. Compare the shapes with the lobes in the distant field shown in Fig. 4. Here, we see what a 32-pole looks like! The special property of our exactly compensating Helmholtz pairs is manifested in the fact that any field line (other than the z axis itself), which passes through the inner pair of current rings returns by passing *inside* both outer rings. The outer current rings have "caged" the field as well as two thin coils can.

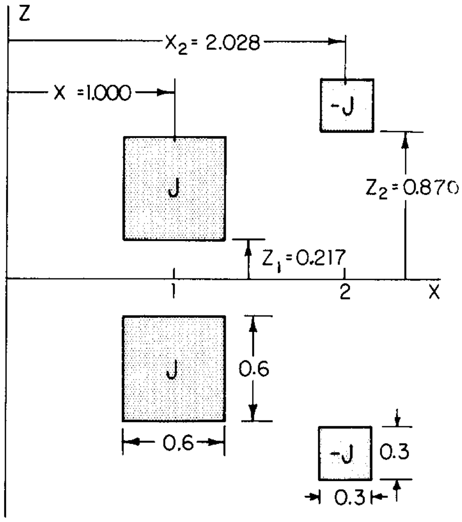


Fig. 7. An exactly compensating set of "thick" coils. Current density J is uniform over the coil cross section and the same in all four coils. Dimensions are in units of the mean radius of the inner coils. Dimension x_2 was set for zero dipole moment. Dimensions z_1 and z_2 were adjusted to annul both the quadratic term in the central field expansion and the total octupole moment. Except close to the conductors, the resulting field, both central and distant, is hardly distinguishable from that of the "thin" coils of Fig. 1.

Consider now the "thick coil" case. Can the properties of the thin coil compensating pairs be duplicated with coils of large cross section? Naturally, we expect an approximation to those results if the ratio of coil thickness to coil radius is not too great. But, in the absence of a concrete problem with specific requirements, little can be said about the adequacy of the approximation. Let us inquire instead whether with thick coils we can achieve what we shall call *exact* compensation: the simultaneous vanishing of dipole moment, octupole moment, and c_2.

One might be tempted to exploit the intrinsic property of the Helmholtz pair, zero octupole moment. Any superposition of Helmholtz pairs will have total octupole moment zero and certain thick coil cross sections can be formed by superposing Helmholtz pairs of current rings. But these rings do *not* all lie in a single sphere; the connection between octupole moment and c_2 is broken. A simple illustration: Sliding a Helmholtz pair axially through a distance equal to its radius generates a solenoid of length equal to its diameter. This solenoid, being a superposition of Helmholtz pairs, is guaranteed to have zero octupole moment. But c_2 at the center of such a solenoid is certainly not zero. So, although this approach may have some advantage if one is not concerned with central field uniformity, it does not lead to our three-fold goal, exact compensation as defined above.

A straightforward method by which exact compensation can be arranged is exemplified in Fig. 7. Here, a square has been arbitrarily chosen for the cross section of both the inner coils and the outer coils. In each of the four coils, the current density J is the same and uniform over the cross section—as it naturally would be if the coils were wound with the same wire. The ampere–turns ratio, outer coils to inner coils, has been set, likewise arbitrarily, at $\frac{1}{4}$. This is a thick coil version of Fig. 1.

We now have three parameters, the dimensions x_2, z_1, and z_2, which can be adjusted to make zero, or nearly zero, each of the following integrals:

$$D = \int\int J(x,z)x^2\,dx\,dz, \tag{5}$$

$$E = \int\int J(x,z)x^2(x^2 - 4z^2)dx\,dz, \tag{6}$$

$$C = \int\int J(x,z)x^2(x^2 - 4z^2)(x^2 + z^2)^{-3/2}\,dx\,dz, \tag{7}$$

D is proportional to the dipole moment, E to the octupole moment, and C to the coefficient c_2 in Eq. (1). The integrals extend over the x-z half-plane. The value of x_2 required to make D zero is easily calculated. The integrals E and C are best done numerically. As each involves both z_1 and z_2, some iterations are needed to determine the final values for z_1 and z_2. In a practical application, it would of course be pointless to seek more accuracy in z_1 and z_2 than consistent with inaccuracies in construction. With the dimensions given in Fig. 7, integrals D, E, and C are all closer to zero than they would usually need to be. The method would work as easily for coil cross sections of any shape.

In both the central and the distant region, the field of the coils in Fig. 5 is virtually indistinguishable from that of the compensating Helmholtz pairs of Fig. 1. A direct calculation of the field near the origin shows that it fits $B_z/B_0 = 1 + 1.1\ (z/a)^4\cdots$ The distant field is quite well represented by Fig. 2. At a distance of six times the mean radius of the main coil, it has fallen below $10^{-4}\ B_0$. Whether such a drastic reduction of the external field of a superconducting magnet could be worth its several costs in a particular case, I cannot say.

There is another, less obvious, application that ought to be pointed out. Suppose that the four coils in a set of compensating Helmholtz pairs are connected in series, to be used not as a magnet but as a sensitive pickup coil for detecting alternating flux. These might be small, open coils of a few turns only, suitable for radio frequency induction, as in NMR. Indeed, imagine that an oscillating magnet moment occupies the central region. It is closely coupled, inductively, to the coil, being at a place where a relatively strong field would be produced by a current in the coil. But the coupling to the coil of any *external* source of alternating flux is reduced by the same factor by which the external field of the coil is suppressed at the location of the source. (The comparison of coupling can easily be made quantitative by invoking the reciprocity law for mutual inductance.) By cancellation of its dipole moment and octupole moment, this pickup coil has, in effect, been isolated from its environment and rendered immune—or very nearly so—to electromagnetic disturbance by external sources.

ACKNOWLEDGMENT

I am indebted to Gerald Gabrielse for stimulating discussions and especially for his elucidation of the connection between the central field and distant field expansions.

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