by oblique light (the shading being produced by penumbræ). Or a series of wedge-shaped strips of tinted glass or gelatine in transmitted light might yield results. The gelatine films would have to receive the impression through the glass, I imagine, if variations in the thickness were necessary.

If this could be accomplished, the same principle could be applied to the zone-plate, all the light being brought to a single focus.

Experiments in this direction are now in progress.

Physical Laboratory of the University of Wisconsin.
Madison, February 1898.

LIV. *Note on the Pressure of Radiation, showing an Apparent Failure of the usual Electromagnetic Equations.* By Lord RAYLEIGH, *F.R.S.**

FOLLOWING a suggestion of Bartoli, Boltzmann† and W. Wien‡ have arrived at the remarkable conclusion that that part of the energy of radiation from a black body at absolute temperature θ, which lies between wave-lengths λ and $\lambda + d\lambda$, has the expression

$$\theta^5 \phi(\theta\lambda)\, d\lambda, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where ϕ is an arbitrary function of the *single* variable $\theta\lambda$. The law of Stefan, according to which the total radiation is as θ^4, is thererein included. The argument employed by these authors is very ingenious, and I think convincing when the postulates are once admitted. The most important of them relates to the *pressure* of radiation, supposed to be operative upon the walls within which the radiation is confined, and estimated at one-third of the *density* of the energy in the case when the radiation is alike in all directions. The argument by which Maxwell originally deduced the pressure of radiation not being clear to me, I was led to look into the question a little more closely, with the result that certain discrepancies have presented themselves which I desire to lay before those who have made a special study of the electric equations. The criticism which appears to be called for extends indeed much beyond the occasion which gave rise to it.

A straightforward calculation of the pressure exercised by plane electric waves incident perpendicularly upon a metallic reflector is given by Prof. J. J. Thomson §. The face of the reflector coincides with $x=0$, and in the vibrations under

* Communicated by the Author.

† Wied. *Ann.* vol. xxii. pp. 31, 291 (1884).

‡ *Berlin. Sitzungsber.* Feb. 1893.

§ 'Elements of Electricity and Magnetism,' Cambridge, 1895, § 241.

consideration the magnetic force reduces itself to the component (β) parallel to y, and the current to the component (w) parallel to z. The waves which penetrate the conducting mass die out more or less quickly according to the conductivity. If the conductivity is great, most of the energy is reflected, and such part as is propagated into the conductor is limited to a thin skin at $x=0$. According to the usual equations the mechanical force exercised upon unit of area of the slice dx of the conductor is $-wb\,dx$, or altogether

$$\int_0^\infty wb\,dx. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Here b denotes the magnetic induction, and is equal to $\mu\beta$, if μ be the permeability and β the magnetic force. Now

$$4\pi w = d\beta/dx,$$

so that the integral becomes

$$\frac{\mu}{8\pi}\{\beta_0^2-\beta_\infty^2\}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where β_0 is the value of β within the conductor at $x=0$, and $\beta_\infty=0$, if the conducting slab be sufficiently thick. Since there is no discontinuity of magnetic force at $x=0$, β_0 may be taken also to refer to the value at $x=0$ just *outside* the metallic surface.

The expression (3) gives the force at any moment; but we are concerned only with the mean value. Since the mean value of β_0^2 is one-half the maximum value, we have for the pressure

$$p=\frac{\mu}{16\pi}\beta^2_{\text{max.}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

It only remains to compare with the density of the energy outside the metal, and we may limit ourselves to the case of complete reflexion. The constant energy of the stationary waves passes alternately between the electric and magnetic forms. If we estimate it at the moment of maximum magnetic force, we have

$$\text{energy} = \frac{1}{8\pi}\iiint\beta^2 dx\,dy\,dz. \quad . \quad . \quad . \quad . \quad (5)$$

In (5) β is variable with x. If $\beta_{\text{max.}}$ denote the maximum value which occurs at $x=0$, the mean of $\beta^2=\frac{1}{2}\beta^2_{\text{max.}}$ Thus

$$\text{density of energy} = \frac{\text{energy}}{\text{volume}} = \frac{1}{16\pi}\beta^2_{\text{max.}} \quad . \quad . \quad . \quad (6)$$

Thus, if the permeability μ of the *metal* be unity, (4) and (6) coincide; and we conclude that in this case the pressure is equal to the density of the energy in the neighbourhood of the metal. This is Maxwell's result. When we consider radiation in all directions, the pressure is expressed as *one-third* of the density of energy.

The difficulty that I have to raise relates to the case where μ is not equal to unity. The conclusion in (4) that the pressure is proportional to μ would make havoc of the theory of Boltzmann and Wien and must, I think, be rejected. So long as the reflexion is complete—and it may be complete independently of μ—the radiation is similarly influenced, and (one would suppose) must exercise a similar force upon the reflector. But if the conclusion is impossible, where is the flaw in the process by which it is arrived at? Being unable to find any fault with the deduction above given (after Prof. J. J. Thomson), I was led to scrutinize more closely the fundamental equation itself; and I will now explain why it appears to me to be incorrect.

For this purpose let us apply it to the very simple case of a wire of circular section, parallel to z, moving in the direction of x across an originally uniform magnetic field (β). The uniformity of the field is disturbed in two ways: (i.) by the operation of the current (w) flowing in the various filaments of the wire, and (ii.) independently of a current, by the magnetic effect of the material composing the wire whose permeability (μ) is supposed to be great. In estimating as in (2) the mechanical force parallel to x operative upon the wire, we should have to integrate wb over the cross-section. In this w is supposed to be constant, and the local value is everywhere to be attributed to b. We may indeed, if we please, omit from b the part due to the currents in the wire, which will in the end contribute nothing to the result; but we are directed to use the actual value of b as disturbed by the presence of the magnetic material. In the particular case supposed, where μ is great, the value of b within the wire is uniform, and just twice as great as at a distance. It follows, when the integration is effected, that the force parallel to x acting upon the wire is greater (in the particular case doubly greater) than it would be if the value of μ were unity.

But this conclusion cannot be accepted. The force depends upon the number of lines of force to be crossed when the wire makes a movement parallel to x. And it is clear that the lines effectively crossed in such a movement are not the condensed lines due to the magnetic quality of the wire, but are to be reckoned from the intensity of the *undisturbed* field.

The mechanical force cannot really depend upon μ, and the formula which leads to such a result must be erroneous.

As regards the problem of the pressure of radiation, I conclude that in this case also, and in spite of the formula, the permeability of the reflector is without effect, and that the consequences deduced by Boltzmann and Wien remain undisturbed.

Another investigation to which perhaps similar considerations will apply is that of the mechanical force between parallel slabs conveying rapidly alternating electric currents. Prof. J. J. Thomson's conclusion * is that the electromagnetic repulsion is μ times the electrostatic attraction, so that a balance will occur only when $\mu=1$. It seems more probable that the factor μ should be omitted, and that balance between the two kinds of force is realized in every case.

LV. *The Reduction to normal Air-Temperatures of the Platinum-Temperatures in the Low-Temperature Researches of* Professors Dewar *and* Fleming. *By* J. D. Hamilton Dickson, *M.A., F.R.S.E.*†

THE measurement of temperature by means of platinum depends upon the two following propositions:—(1) That for a given piece of pure annealed platinum-wire the temperature is a single-valued function of the electric resistance. This proposition is due to Prof. Callendar (1886), and has been fully verified by many subsequent observers. The second proposition is:—(2) That however different specimens of pure annealed platinum-wire may vary among themselves, nevertheless they agree in giving the same normal air-temperature of any enclosure in which they may be simultaneously placed. This proposition might, at first sight, appear as a logical deduction from the first; but a little consideration will show that the two propositions are equally fundamental, and equally necessarily due to experiment. We are indebted for it to the careful researches of Mr. E. H. Griffiths.

Theory has not yet provided the formula referred to in the first proposition; meanwhile, Prof. Callendar has devised a double formula—or, rather, a formula with a correction—which amounts to the expansion of the electric resistance of the platinum-wire in powers of the temperature, and leads to a somewhat troublesome reduction before finally

* 'Recent Researches in Electricity and Magnetism,' 1893, § 277.

† Communicated by the Author.