On Relativistic Theories

F. Rohrlich
Department of Physics, Syracuse University, Syracuse, New York

THE relativistic age started well over half a century ago. What is perhaps its most important message, however, still does not seem to have been fully appreciated by a number of physicists. It has to do with the relationship between observers and the realization that a physical observable must have definite transformation properties.

The special theory of relativity teaches us that inertial observers must relate their findings by a certain transformation which belongs to the group of Lorentz transformations. It made us aware of the fact that in "nonrelativistic" physics two inertial observers relate their findings by means of a different transformation which belongs to the group of Galilean transformations, thereby characterizing the relativity of that theory. But in any case, a physical quantity has meaning independent of the subjective choice of a coordinate system only when its transformation properties are of a very special nature.

Relativistic physics is not as intuitive as "nonrelativistic" physics which draws from everyday life experiences. This is exemplified by the many relativistic "paradoxes" which seem to contradict common sense. One should learn from these that physical arguments can be easily misleading when applied in an area where our intuition is insufficiently trained. (By the way, this applies also to quantum mechanics). The check on the transformation properties of the quantities involved is therefore a necessary (though not sufficient) test of any such argument.

It follows that a relativistic theory is more than an algebraic complication of a nonrelativistic one, beset by many factors of $(1-v^2/c^2)^{\frac{1}{2}}$. It also follows that the physical interpretation of certain quantities in nonrelativistic physics should not be carried over uncritically to relativistic physics. Such interpretation depends rather on the transformation properties.

The importance of these remarks are illustrated by two examples of the recent physics literature.

In a paper[1] on the *Electromagnetic Mass of the Classical Electron* a distinction is made "between the electrostatic mass of the classical electron m_0, and the electrodynamic mass $(4/3)m_0$", the related problem that has been in the literature for many years was then "resolved." The argument involves an expression for the energy of a freely moving charged particle which differs from the usual one and which is

$$\epsilon=(m_0-I_0)c^2+I_0c^2/(1-v^2/c^2)^{\frac{1}{2}}(I_0\neq m_0). \qquad (1)$$

It is easily seen that this quantity is not the fourth component of a four-vector. Therefore, it does not combine with the momentum three-vector to form a four-vector; it has no meaning independent of the reference system. An observer at $v=0$ finds $\epsilon=m_0c^2$, $\mathbf{P}=0$; a Lorentz transformation with $(\mathbf{P},\epsilon)$ a four-vector would yield $m_0c^2(1-v^2/c^2)^{\frac{1}{2}}$ which is the usual expression but differs from (1). In this way one can show that the argument presented in that paper cannot be considered meaningful in a relativistic theory. Furthermore, it should be pointed out that the identification of the electromagnetic momentum three-vector with the three-space integral over the Poynting vector is only correct for radiation fields and not for static fields.[2] The problem of the factor 4/3 was first resolved quite some time ago.[3] But its resolution has not found its way into the text books until very recently.

The second example is a problem in relativistic thermodynamics. This field was founded by Planck, Einstein, and others, but has not received much attention. Last year several papers claimed that the relativistic transformation law of the quantity of heat Q as specified in the older papers is incorrect.[4,5] This discussion was renewed after an objection was raised.[6–9] Without going into details here, suffice it to say that the recent claims against the older papers are invalid despite their plausibility. The reason for this failure lies in that the authors claimed Q to be a component of a four-vector without considering the other components. Had they done so, their argument would not have gone astray. The environment of the thermodynamic system thereby plays an important role. These matters were clarified by a covariant discussion of such a system.[10,11]

These two examples illustrate the fact which is sometimes ignored, that in a relativistic theory the transformation properties of the physical quantities must be carefully stated and the covariance of the equations must be ensured. This can be done with the aid of a manifestly covariant notation, but such a formalism is not essential for this purpose. In fact, there are occasions where such a notation can be misleading. The only safe way is, therefore, a thorough understanding of the mathematical meaning of invariance in relativity.

[1] J. W. Zink, Am. J. Phys. 34, 211 (1966).
[2] See, e.g., F. Rohrlich, *Classical Charged Particles* (Addison-Wesley Publ. Co., Reading, Mass., 1965).
[3] E. Fermi, Physik. Z. 23, 340 (1922). See also, Ref. 2.
[4] H. Arzeliés, Nuovo Cimento 35, 792 (1965).
[5] A. Gamba, Nuovo Cimento 37, 1792 (1965).
[6] T. W. B. Kibble, Nuovo Cimento 41, 72 (1966).
[7] A. Gamba, Nuovo Cimento 41, 79 (1966).
[8] H. Arzeliés, Nuovo Cimento 41, 81 (1966).
[9] T. W. B. Kibble, Nuovo Cimento 41, 83 (1966).
[10] A. Staruszkiewicz, Acta Phys. Polon. 29, 249 (1966).
[11] F. Rohrlich, Nuovo Cimento (to be published).

Laws of Motion for Variable Mass Systems

Christopher P. Gadsden
5109 Danneel Street, New Orleans, Louisiana 70115

RECENT discussions in this JOURNAL, stimulated initially by the present writer, have taken up the mechanics of systems which lose or gain matter.[1–5] The momentum law for such systems was stated by the writer as follows: *the rate of change of momentum is equal to the acting force plus the rate at which momentum is carried into or away from the system* (i.e., by the interchanged matter), but, unknown to him, had previously been formulated by Thorpe in a general equation for the nonrelativistic case as

$$d\mathbf{P}/dt=\mathbf{F}+\textstyle\int_S\rho\mathbf{U}(\mathbf{V}-\mathbf{U})\cdot\mathbf{n}ds, \qquad (1)$$

wherein the integral gives the momentum flux rate.[6]

The latest letter at the time of this writing is by Trigg,[5] who reverts to the equation $d\mathbf{P}/dt=\mathbf{F}$, leaving out the momentum flux entirely.