can give longer trouble-free service than can many simple mechanisms—such as switches. New types of automatic pilots are electronic, and they can do many more things than could the previous mechanical types. Fear of complicated designs should not void the opportunities in such equipment.

The need for proper maintenance was noted often during the conference. This is a customer's problem and responsibility. The aviation industry can be very proud of the maintenance methods and standards which have been developed for engines and aircraft. Electric equipment has been overshadowed by the more important mechanical equipment. Suddenly the electric system has become important, and the maintenance problem has become acute. Without doubt this need will be satisfied in excellent fashion. In the meantime, manufacturers of apparatus can make the task easier by considering maintenance in the original design and by preparing maintenance bulletins or handbooks.

Electrical Essay

Fourier Series Representation of Square Top Wave

The square-top curve, C, shown in Figure 1A is said to be represented by the Fourier series

$$y=\frac{4}{\pi}[\cos\theta-1/3\cos 3\theta+1/5\cos 5\theta-1/7\cos 7\theta\ldots.]$$

Consider the family of curves,

$$S_1;\ y=\frac{4}{\pi}[\cos\theta]$$

$$S_2;\ y=\frac{4}{\pi}[\cos\theta-1/3\cos 3\theta]$$

$$S_3;\ y=\frac{4}{\pi}[\cos\theta-1/3\cos 3\theta+1/5\cos 5\theta]$$

$$S_4;\ y=\frac{4}{\pi}[\cos\theta-1/3\cos 3I+1/5\cos 5\theta-1/7\cos 7\theta]$$

S_1, S_2, S_3, and S_4 are shown in relation to C in Figures 1B, 1C, 1D, and 1E, respectively.

The family of curves, S_1, S_2, S_3, S_4, approaches as its limit, the square-top curve C.

True or *false?* (*Author's answer: False!*)

Answer to Previous Essay

The following is the author's reply to his previously published essay (*EE, Aug '48, p 771*).

The pessimist is wrong. There can be no diamagnetic hysteresis with diamagnetic moment I_d lagging the magnetic field H. Such a phenomenon would violate the laws of thermodynamics and permit perpetual motion. Lord Kelvin first pointed this out.

· For iron, the loss per cycle is given by

$$W=\int HdB=\int HdH+4\pi\int HdI_m=4\pi\int HdI_m$$

where the integrals are taken round the loop in the counterclockwise sense as indicated. It readily is seen that the last integral is merely the area enclosed by the loop, taken with a positive sign.

However, for a diamagnetic material, with diamagnetic moment I lagging H, a hysteresis loop would be as shown in Figure 2, so that the integrals in

$$W=\int HdB=\int HdH+4\pi\int HdI_d=4\pi\int HdI_d$$

would be taken in a clockwise sense.

Hence the last integral would equal the area of the loop, but with a negative sign. That is, the loss per cycle would be negative, and energy would be continuously drawn out from the diamagnetic material.

Any imperfection in the diamagnetism will make the

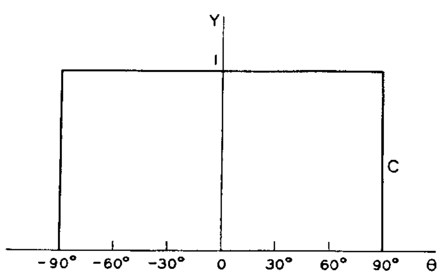

Figure 1A

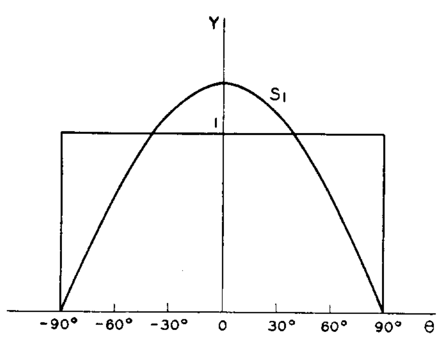

Figure 1B

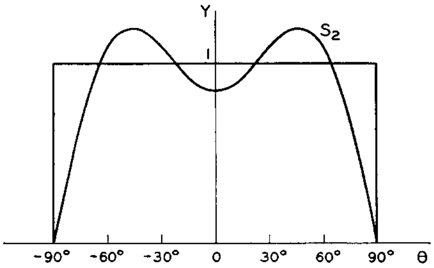

Figure 1C

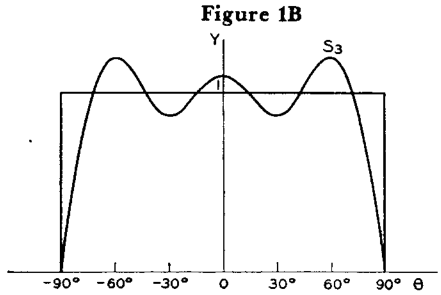

Figure 1D

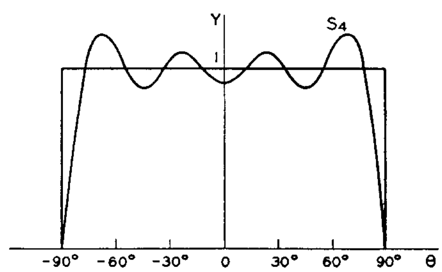

Figure 1E

Electrical Essay

Electrical Heat Generation in Metal Bar

A STRAIGHT metallic bar, of uniform section, maintained at a constant uniform temperature, has flowing through it at constant density a steady, continuous current of magnitude I. The electric equipotential surfaces are the plane sections of the bar, perpendicular to its edges.

By some suitable method, the difference in potential $V_A - V_B$ between two such sections is determined. Also by some suitable method, the rate of removal of heat Q_{AB}, from the volume of the bar between the sections A and B, which is necessary to keep the bar temperature constant, is determined.

Then always $(V_A - V_B)I$ and Q_{AB} are equal in magnitude.

True or *false?*

Answer to Previous Essay

The following is the author's reply to his previously published essay (*EE, Sep '48, p 904*).

The author's answer is *false.*

The family of curves

$$S_1;\ y=\frac{4}{\pi}\left[\cos\theta\right]$$

$$S_2;\ y=\frac{4}{\pi}\left[\cos\theta-1/3\cos 3\theta\right]$$

$$S_3;\ y=\frac{4}{\pi}\left[\cos\theta-1/3\cos 3\theta+1/5\cos 5\theta\right]$$

$$S_4;\ y=\frac{4}{\pi}\left[\cos\theta-1/3\cos 3\theta+1/5\cos 5\theta-1/7\cos 7\theta\right]$$

and so forth, approaches the curve shown by the heavy line

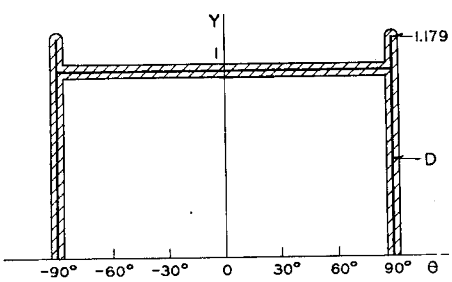

Figure 1

D in Figure 1 of this reply and not the curve C shown in Figure 1*A* of the essay.

To see this, we first must define what we mean by saying that the curve D is the limit of a sequence of curve S_1, S_2, S_3, ... and so forth. This we do as follows. Surround the ostensible limit curve D by a strip or belt of width ϵ (shaded strip in Figure 1). Then D is the limit curve of the sequence if, no matter how small the belt width ϵ is made, we always may go far enough along in the sequence, S_1, S_2, S_3, ... So that all the curves in the sequence beyond this point will be each respectively entirely within the belt of width ϵ.

The curve C of Figure 1*A* of the essay is not the limit curve of the sequence S_1, S_2. . . according to this definition. The last arches to the right of S_1, S_2, S_3, S_4, are shown in Figure 2 of this reply. The heights of these arches do not approach the value 1.000 as we proceed in the sequence, but approach the value 1.179. Hence, if we draw a narrow

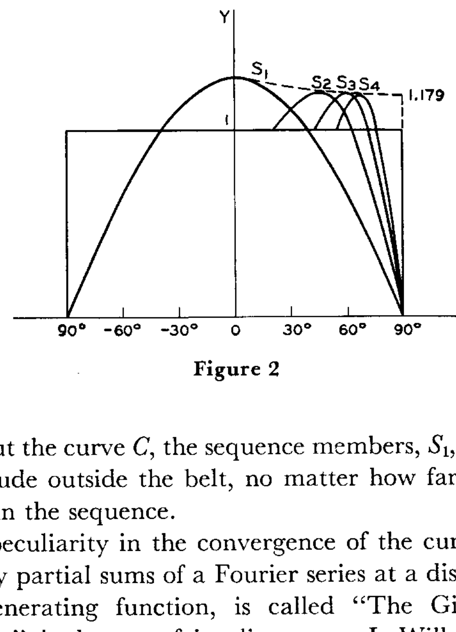

Figure 2

belt about the curve C, the sequence members, S_1, S_2, S_3. . . . all protrude outside the belt, no matter how far along we may go in the sequence.

This peculiarity in the convergence of the curves represented by partial sums of a Fourier series at a discontinuity in its generating function, is called "The Gibb's Phenomenon," in honor of its discoverer, J. Willard Gibbs, America's greatest native-born scientist.

The sequence of functions given by

$$F_1(\theta),\ y=\frac{4}{\pi}\left[\cos\theta\right]$$

$$F_2(\theta),\ y=\frac{4}{\pi}\left[\cos\theta-1/3\cos 3\theta\right]$$

$$F_3(\theta),\ y=\frac{4}{\pi}\left[\cos\theta-1/3\cos 3\theta+1/5\cos 5\theta\right]$$

$$F_4(\theta),\ y=\frac{4}{\pi}\left[\cos\theta-1/3\cos 3\theta+1/5\cos 5\theta-1/7\cos 7\theta\right]$$

and so forth, has as its limiting function, the function $G(\theta)$ defined as follows:

For θ between -90 degrees and $+90$ degrees,	$G(\theta)=1$	-90 degrees $< \theta <$ $+90$ degrees
For $\theta=-90$ degrees	$G(\theta)=0$	
For $\theta=+90$ degrees	$G(\theta)=0$	

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