Electrical Essays

A Nonlinear Problem

The ordinary rectifier circuit is used in many applications. Single-phase excitation is the most common for low power requirements. In order to increase the ripple frequency and thus simplify the filter design, it is necessary to introduce some phase inverter device, usually a transformer with a center tapped secondary, and full-wave rectification is obtained. Under such condition of operation the average currents through each half of the secondary are equal and their directions are such as to produce a net zero average magnetomotive force on the core of the transformer. As the magnetomotive force due to the primary

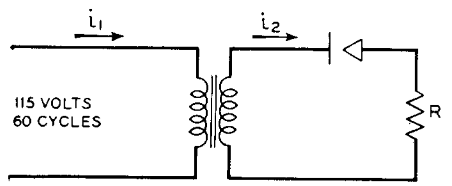

Figure 1

current must oppose that due to the secondary currents, and as the net secondary magnetomotive force has zero average, the primary current will have zero average.

In many applications requiring high constant potential a single rectifier is used in the secondary circuit of a stepup transformer. The secondary current, and therefore the magnetomotive force due to it, certainly will have an average value different from zero. Does the primary current also have an average value differing from zero?

In order to answer this question consider a half-wave rectifier without a filter, Figure 1, and assume the coefficient of coupling of the transformer is practically unity and that under no condition does the core saturate. The secondary current i_2 then has, for all practical purposes, the wave shape

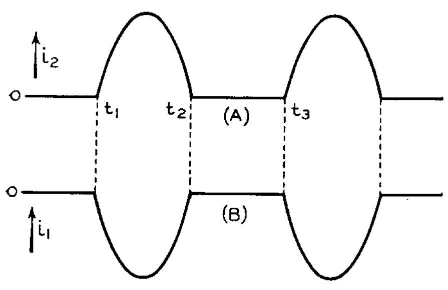

Figure 2

shown in Figure 2*A* and the magnetomotive force due to it will be of the same shape. The magnetomotive force due to the primary current (neglecting exciting current) must be the negative of the secondary magnetomotive force and therefore takes the shape shown in Figure 2*B*. Because i_2 is zero during the period from t_2 to t_3, i_1 must be zero also during the same time, and it follows that the primary current has an average value different from zero.

If a d'Arsonoval ammeter were placed in the primary line, the average value would be indicated. When this is attempted in an actual experiment there is no measurable value, even with core saturation, although the theory outlined indicates that there should be an appreciable average primary current. What is wrong?

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Which Is the Primary?

A TRANSFORMER was constructed very accurately in the form of a toroid. The doughnut-shaped core was wound uniformly with the toroidal primary winding. Then the toroidal secondary winding was wound outside the primary winding. Of course there was a slight blemish upon the uniformity of the toroidal quality, because the terminal leads of the primary needed to pass between the turns of the secondary. The effect of this asymmetry is negligible however for the problem to be presented in this essay.

In many textbooks, the primary leakage reactance is defined as arising from that flux which links the primary winding and which does not link the secondary. Similarly, the secondary leakage reactance is said to result from that flux which links the secondary and does not link the primary.

For this particular transformer, the magnetic field has a very simple geometric configuration. The lines of magnetic force are all circles with centers on the straight line axis of circular symmetry of the toroid. All the lines of force which link any one turn of the primary link all the other turns of the primary and also all the turns of the secondary. A line of force which passes through a point outside the primary but inside the secondary links all the turns of the secondary, but links no turn of the primary. Outside the secondary, however, there are no lines of magnetic force.

Since all the flux which links the primary also links the secondary, so that there is no flux which links the primary and does not link the secondary, it follows that the primary leakage reactance is zero. On the other hand there are lines of magnetic force which lie between the primary and secondary. There is therefore flux which links the secondary and does not link the primary. The secondary leakage reactance is therefore not zero.

The transformer was placed inside a case or box (carefully painted black) and connected to two sets of terminals. The case was locked securely. All information concerning the transformer subsequently was lost, in the 1936 flood, except that the transformer was so designed as to make primary leakage reactance zero. An engineer now is given the task of determining by 60-cycle measurements at the terminals which is the winding with zero leakage reactance. How shall he proceed? Surely with such a large difference in leakage reactance there must be some simple way of determining which is the primary and which the secondary.

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Answer to Previous Essay

The following is the author's answer to his previously published essay, "Turn Ratio of Transformers" (*EE, May '49, p 449*).

The number of physical turns put into the coils of the transformer is certainly important to the transformer designer and coil winder, but the turn ratio itself appears only as a somewhat nebulous and uncertain constant in the electrical performance of the transformer.

For simplicity, let us first in our discussion neglect any saturation or other effects in the iron which would introduce harmonic currents or voltages. In other words, let us suppose that the terminal voltages and currents are related linearly. Also, let us suppose that eddy currents are negligibly present so that in equations 1 M may be taken as real. Then all that our measurements can give us are the coefficients in these equations.

$$E_1=(r_1+j\omega L_1)I_1+j\omega MI_2 \quad (1)$$
$$E_2=j\omega MI_1+(r_2+j\omega L_2)I_2$$

Here, the E's and I's are complex numbers, representing in the usual way the sinusoidal voltages and currents. r_1, L_1, r_2, and L_2 are real and positive. M, as stated previously, is real; and we choose the positive direction for I_2 so as to make M positive. Furthermore, we always have $M^2<L_1L_2$. Whatever electrical significance the turn ratio $r=n_1/n_2$ may have, it must be expressible in terms of the coefficients of equations 1.

Now, $r=n_1/n_2$ appears in the theory of the transformer as more usually taught through the following fairy story. There is a good fairy flux, Φ, which graciously and fairly links each and every turn of each primary and each secondary coil exactly 1.000000 times. Thus each turn has induced in it a voltage

$$E_t=k_1j\omega\Phi \quad (2)$$

where k_1 is the proper numeric. Hence we have induced in the primary and secondary coils the voltages

$$E_1\text{ ind}=n_1k_1j\omega\Phi \quad (3)$$
$$E_2\text{ ind}=n_2k_1j\omega\Phi$$

However, these are not the terminal voltages. Besides the r_1I_1 and r_2I_2 drops which cannot be helped, there are certain bad fairy leakage fluxes. Bad fairy primary leakage flux links only the primary turns, skillfully dodging the secondary turns, and remains always proportional to the primary current, and indifferent to the secondary current. It therefore introduces a reactive drop in the primary, jX_1I_1.

Similarly, a secondary leakage flux introduces a secondary reactive drop, jX_2I_2. Our fairy story then becomes

$$E_1=(r_1+jX_1)I_1+n_1k_1j\omega\Phi \quad (4)$$
$$E_2=(r_2+jX_2)I_2+n_2k_1j\omega\Phi$$

Now, what is the relation of Φ to the currents in the primary and secondary. Naturally, since Φ treats all turns alike, Φ will be proportional to $n_1I_1+n_2I_2$, so that

$$\Phi=k_2(n_1I_1+n_2I_2) \quad (5)$$

where k_2 is the appropriate numeric.

Substituting equation 5 in equation 4 we get

$$E_1=(r_1+j[X_1+k_1k_2n_1^2\omega])I_1+(jk_1k_2n_1n_2\omega)I_2 \quad (6)$$
$$E_2=(jk_1k_2n_1n_2\omega)I_1+(r_2+j[X_2+k_1k_2n_2^2\omega])I_2$$

Comparing these equations with equation 1 which contains all that experiment can give, we get, to determine our electrical turn-ratio and the primary and secondary leakage reactances,

$$r_1+j\omega L_1=r_1+j[X_1+k_1k_2n_1^2\omega]$$
$$r_2+j\omega L_2=r_2+j[X_2+k_1k_2n_2^2\omega] \quad (7)$$
$$j\omega M=j\omega k_1k_2n_1n_2$$
$$r=n_1/n_2$$

These are not enough equations, but unfortunately they are all we have.

Eliminating as far as we can we get

$$X_1=\omega(L_1-Mr) \quad (8)$$
$$X_2=\omega(L_2-M/r)$$

Hence, so far as the electrical performance is concerned, we may take the electrical turn ratio to be anything we please ignoring completely anything the manufacturer tells us. He may be trying to deceive us. If we take X_1 and X_2 by equations 8, then equations 6 and fairy story, equations 4, will be satisfied.

However, we may have a quite unfounded prejudice against reactances X_1 and X_2 being negative or zero even though they are such ill-defined fairy creatures. In that case we cannot be completely free in our choice of r. W must have (L_1-Mr) and (L_2-M/r) each greater than zero. This requires that

$$L_1/M>r>M/L_2 \quad (9)$$

Since always

$$L_1L_2>M^2 \quad (10)$$

and therefore

$$L_1/M>M/L_2 \quad (11)$$

there always will be a range for r which will make both X_1 and X_2 positive.

Thus, if we have no prejudice against negative leakage reactance, we may make r anything we please, and appropriate main and leakage fluxes can be found. If, however, we insist that leakage fluxes be positive, then r is restricted to a range which will be narrow for the usual good power transformer, generally a fraction of one per cent of r. However, again, by varying the choice of r in this narrow range we may vary the distribution of leakage reactance between the two windings. At one extreme, we may put it all into the primary with zero secondary leakage reactance, or we may put it all into the secondary with zero primary leakage reactance.

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The Steinmetz Network

A young distribution engineer is diligently studying a piece of apparatus recently purchased by his employer for supplying power to a series street light circuit from primary distribution lines. The equipment involved is essentially made up of two reactors and two capacitors connected as shown in Figure 1. The ohmic impedance of each of

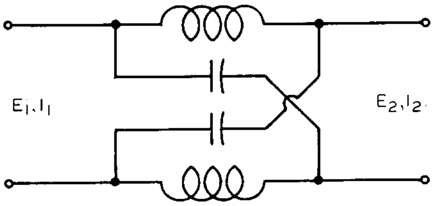

Figure 1

four elements at 60 cycles is equal to x. The network often is referred to as the Steinmetz network. The network relates the rms voltages and currents as follows:

$$E_1 = xI_2 \quad (1)$$

$$E_2 = xI_1 \quad (2)$$

(*Note:* The currents and voltages at each end of the network are not related by the network itself but depend on the external circuit constants.)

One of the engineer's coworkers suggests that time and effort can be saved if the young man only would apply Thevenin's theorem to the problem. The young engineer reads a page in a textbook and proceeds with his calculations. Assuming an applied primary voltage and the secondary terminals of the network open-circuited he calculates the open circuit voltage to be infinity. Also, with the primary voltage made zero, the impedance of the network when measured at the secondary terminals is found by him to be infinite. On the basis of these calculations the Steinmetz network must be represented by an infinitely large voltage and an infinitely large series impedance.

Is this correct?

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Answers to Previous Essays

A Nonlinear Problem. The following is the author's answer to a previously published essay of the foregoing title (*EE, Jun '49, p 518*).

The magnetic flux, ϕ_1, threading the primary winding is periodic. The counter electromotive force developed by the flux ϕ_1 is

$$e_c = -k\frac{d\phi_1}{dt} \quad (1)$$

where k is a constant factor depending on the system of units used. The counter electromotive force is almost equal to the impressed voltage. Thus for a sinusoidal applied voltage

$$e_1 = E_1 \sin \omega t = -k\frac{d\phi_1}{dt} \quad (2)$$

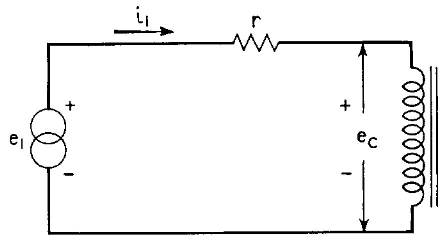

Figure 1

which does not define the flux but only its derivative.

Solving equation 2,

$$\phi_1 = \frac{E_1}{k\omega}\cos \omega t + \phi_0 \quad (3)$$

where ϕ_0 is the average value of the flux and is yet to be determined.

The flux is determined by the net magnetomotive force on the core:

$$\phi = Ai_1 + Bi_2 \quad (4)$$

where A and B are dimensional constants.

The secondary current i_2 has an average value different from zero and, therefore, it follows that if the average value of the flux, ϕ_0, is zero that the primary current, i_1, must have a nonvanishing average value. If i_1 has a zero average then ϕ_0 must exist.

Consider the primary circuit, Figure 1, with the insertion of a very small resistance, r. e_1 has zero average value. Therefore, if i_1 has an average value there must be an average value of the volt drop across r. This can be only if e_c has an average value.

$$e_c = -k\frac{d\phi_1}{dt}$$

and average $$e_c = \frac{1}{T}\int_0^T e_c dt = -\frac{k}{T}\int_0^T \frac{d\phi_1}{dt}dt$$

$$= -\frac{k}{T}\int_{\phi_1 \text{ at } t=0}^{\phi_1 \text{ at } t=T} d\phi_1$$

ϕ_1 is periodic. Therefore ϕ_1 at $t=0$ has exactly the same value as ϕ_1 at $t=T$. Thus e_c has zero average value. It follows that i_1 also has a zero average value and that the flux has an average value.

The development so far agrees with the experiment when the measurement of the average value of the primary current was attempted.

If all this is true, the power line must be delivering power, p_1, to the primary of the transformer during the time interval t_2 to t_3 that no power is delivered by the secondary to the load resistor (Figure 2). The only place this energy can go is into the magnetic field of the transformer.

The power delivered to the magnetic field (neglecting exciting power) is, therefore, the difference between the power delivered to the primary and the power delivered by the secondary to the load.

$$p_1 = e_1 i_1 = E_1 I_1\left[\sin^2 \omega t - \frac{1}{\pi}\sin \omega t\right]_{t_1}^{t_2}$$

$$= \frac{E_1 I_1}{\pi}\sin \omega t\Big|_{t_2}^{t_3}$$

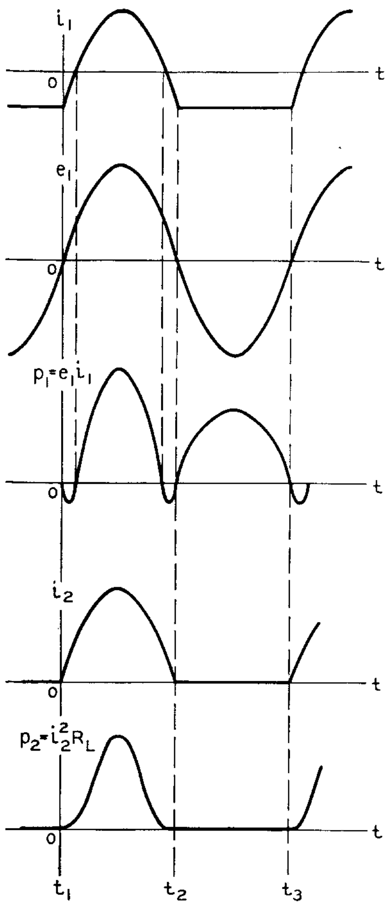

Figure 2

$$p_2 = e_2 i_2 = E_2 I_2 \sin^2 \omega t \Big|_{t_1}^{t_2} = E_1 I_1 \sin^2 \omega t \Big|_{t_1}^{t_2}$$

$$= 0 \Big|_{t_2}^{t_3}$$

$$p_1 - p_2 = -\frac{E_1 I_1}{\pi} \sin \omega t \Big|_{t_1}^{t_2} \text{(magnetic field energy decreases)}$$

$$p_1 - p_2 = \frac{E_1 I_1}{\pi} \sin \omega t \Big|_{t_2}^{t_3} \text{(magnetic field energy increases)}$$

Thus during the period t_1 to t_2 part of the energy delivered to the load is drawn from that stored in the magnetic field during the period t_2 to t_3 and the required energy balance is obtained.

Although saturation effects were not explicitly considered, the conclusion that the primary current has a zero average value still holds because the average value of the time derivative of any periodic function is zero. These conclusions cannot result if an ideal transformer were considered because the flux in an ideal transformer will increase indefinitely and therefore will not be periodic.

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Which Is the Primary? The following is the author's answer to a previously published electrical essay of the foregoing title (*EE, Jun '49, pp 518–19*).

As the author indicated in his answer to the preceding electrical essay, "Turn Ratio of Transformers," if we do not make use of the saturating or nonlinear properties of the iron core, then it is not possible to determine the turn ratio or primary and secondary leakage reactances of a transformer by purely electrical measurements. Such measurements can give us only the coefficients L_1, L_2, and M of the two equations

$$X_1 = \omega(L_1 - Mr), \qquad X_2 = \omega(L_2 - M/r) \tag{1}$$

for the three quantities, X_1, X_2, and r, the primary leakage reactance, the secondary leakage reactance, and the turn ratio respectively. (See author's answer to preceding Electrical Essay, *EE, Jun '49, p 519.*) To determine the three quantities, we need a third equation, or relationship, which can be obtained only in some arbitrary way.

We might agree for example to define an "electrical turn ratio" as given by that ratio of secondary current magnitude to primary current magnitude which for constant primary current will make the mean magnetic energy a minimum. If we take I_1 and I_2 as both real, the magnetic energy will be proportional to $L_1 I_1^2 + 2M_1 I_1 I_2 + L_2 I_2^2$; if I_2 makes this a minimum with I_2 constant, then

$$r = |I_2/I_1| = M/L_2$$

This value of r then always will make X_2, the secondary leakage reactance zero.

On the other hand, if we define r as the ratio of secondary current magnitude to primary current magnitude which makes the mean magnetic energy a minimum for constant secondary current, then we get a different value, $r = L_1/M$, and the primary leakage reactance is zero.

Of course both of these possible values of r cannot agree with the actual physical turn ratio. In general, neither will. For the particular case of the toroidal transformer, one of these r's will agree with the actual physical turn ratio, but the engineer will be unable to tell which one.

If we make use of the nonlinear properties of the iron core, then we may define r electrically so as to make it agree very closely with the physical turn ratio for usual power transformers. For example, we may define r as that ratio of secondary sinusoidal current to primary sinusoidal current which for fixed primary current will make the harmonic components of the primary voltage a minimum. For the toroidal transformer, this value of r will be precisely the physical turn ratio, and the engineer of the electrical essay may be well advised to use this idea. For a high reactance transformer such as is sometimes used in arc welding, the agreement of this electrically defined r with the physical turn ratio may be less good.

It is principally because of the saturation properties of the iron core that the analysis of the performance of a transformer in terms of turn ratio and leakage reactances is of value. In the extreme cases where they disagree, the electrical turn ratio defined in the foregoing should be used rather than the physical or geometric turn ratio. If this ratio is used then the nonlinear properties of the core will affect only the "common flux" or mutual reactance of the windings. The leakage reactances, based on the turn ratio so defined, will be nearly independent of current amplitude and free from nonlinear effects.

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