Electrical Essays

Electrostatic or Electromagnetically Induced Electric Field?

By measuring the alternating force on a *stationary* small charged probe whose position is varied it is discovered that an alternating electric field exists outside of a black box made of insulating walls. The field configuration is shown in Figure 1.

It may be that the electric field is set up by charges being supplied to metallic plates, A, by an alternating source as in Figure 2.

Or it may be that the black box contains a toroidal magnetic core B, through which an alternating magnetic flux is passing, as in Figure 3.

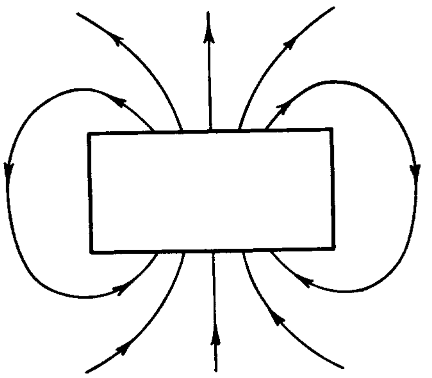

Figure 1. Electric field outside black box

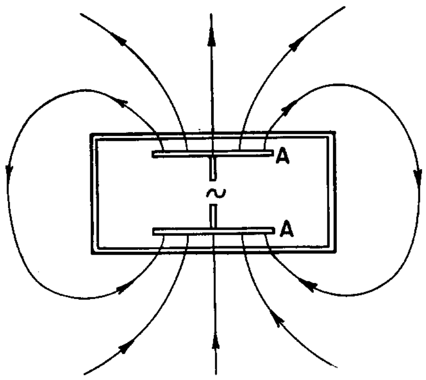

Figure 2. Electrostatic field

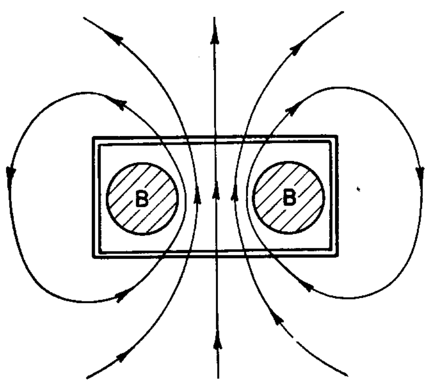

Figure 3. Electromagnetically induced electric field

In many places in electrical engineering literature these two types of electric field are said to be fundamentally different in nature. Thus, most recently, G. I. Cohn,* on page 445 of his article, "Electromagnetic Induction" (*EE, May '49, pp 441–7*) speaks of the induced field of Figure 3 as "(hypothetical)", presumably in contrast to the "real" electrostatic field of Figure 2. The electrostatic field, which is presumably not "(hypothetical)", has a scalar potential of which it is the gradient, whereas the best we can do for the electromagnetically induced "(hypothetical)" field is to make it the time derivative of the vector potential of the magnet induction. (Equations 43 and 40, page 445 of afore-mentioned article).*

Now with all this important difference between the electrostatic and electromagnetically induced electric fields, go back to Figure 1, and by suitable observations or by making the proper measurements, determine which type of electric field is present. Of course you must not peek into the black box.

J. SLEPIAN (F '27)

(Associate Director, Westinghouse Research Laboratories, East Pittsburgh, Pa.)

Another Nonlinear Circuit

Two identical transformers and a rectifier are connected in series as shown in Figure 1 and are energized from a source of rated voltage and frequency. For simplicity the following assumptions are made: unity turn ratio, no losses,

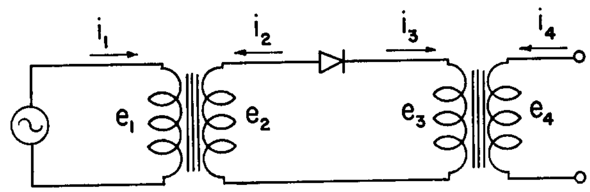

Figure 1

no saturation, and no impedance drop ($L_1 = L_2 = M$). An ideal rectifier is also assumed with zero voltage drop when conducting, and infinite resistance when blocking. Since no copper or iron loss is assumed and transient components cannot decay, the transient components are eliminated at the start.

Consider the following questions. What voltage

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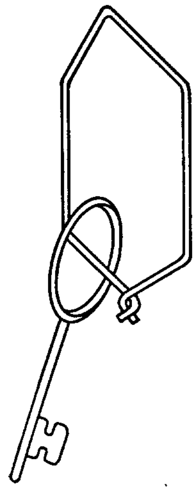
* In my essays, generally I have given no literature references, as compiling such would be burdensome, and since I make no claim to scientific novelty in these essays. However, in this essay I do refer to Cohn, firstly, because of the very high excellence of the article, secondly, because of its pertinence to the subject of this essay, and thirdly, because it has appeared so recently in *Electrical Engineering*.

it. Now, a turn on a wave winding doesn't come anywhere near closing.

I am not a member of Eta Kappa Nu, not yet anyway, but I like to wear something that reflects my interest in electricity, so I have made myself a key ring shaped like a turn on a machine. But I was smart enough to choose a



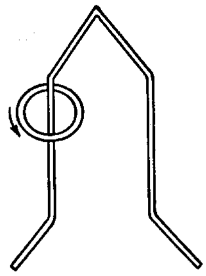


Figure 1 (left). Alter Ego's key ring

Figure 2 (right). Flux linking wave turn

lap winding, and by twisting together the turn ends, which are already close together, my keys link the ring without any question, and the ring stays linked by the keys, Figure 1. But suppose I had used a wave winding, like in Figure 2. I wouldn't know whether my keys were linked to the key ring or not.

In Figure 2 I have shown a magnetized ring of iron encircling one side of the wave turn. The engineers would say that the flux of this ring links the turn, because this flux is in exactly the same position as that flux, in the machine, which they said was linking the turn. Now as I slip the magnetized iron ring off the coil side, there must be some place where the magnetic flux will no longer be linking the turn.

Question 1. Where does the magnetized ring stop linking the turn?

Question 2. As the ring stops linking the turn, so that the flux linkages are reducing to zero, an electromotive force will be induced in the open turn, since the rate of change of the flux linkages will not be zero, isn't that right? True or false?

J. Slepian, Alter Ego

J. SLEPIAN (F '27)

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Answers to Previous Essays

Electrostatic or Electromagnetically Induced Electric Field? The following is the author's answer to a previously published essay of the foregoing title (*EE, Oct '49, p 877*).

"There is but one god, Allah, and Mohammed is his prophet!" There is but one electric field, $\boldsymbol{E}$, (for a particular frame of reference) and Maxwell, our prophet, has proclaimed its properties (in free space) in his eternal equations.

$$\text{div } \boldsymbol{E} = 4\pi\rho \tag{1}$$

$$\text{curl } \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \tag{2}$$

These equations do not talk of two electric fields, one, $\boldsymbol{E}_s$, an electrostatic field for equation 1 which is concerned with electric charge density ρ, and the other, $\boldsymbol{E}_T$, an inductively produced field, in equation 2 which is concerned with varying magnetic flux-density, $\partial\boldsymbol{B}/\partial t$. No, they speak only of a single electric field, $\boldsymbol{E}$, defined uniquely by purely local measurements of force on a probe, and to which *both* equation 1 and 2 apply. There is but one electric field, $\boldsymbol{E}$, and Maxwell is its prophet!

Of course we may arbitrarily, or for our real or imaginary convenience choose to regard the one field, $\boldsymbol{E}_s$ as made up of an electrostatic part $\boldsymbol{E}_s$, and an inductive part, $\boldsymbol{E}_T$, so that

$$\boldsymbol{E} = \boldsymbol{E}_s + \boldsymbol{E}_T \tag{3}$$

and following our intuition (which may be wrong) that there is some real significance to this resolution of $\boldsymbol{E}$ we may try to find sufficient definitions of $\boldsymbol{E}_s$ and $\boldsymbol{E}_T$ from their supposedly inherently different properties!

Thus we may believe that $\boldsymbol{E}_s$ arises from charges only, and that $\boldsymbol{E}_T$ arises from changing magnetic fluxes only, so that following Cohn* we define $\boldsymbol{E}_s$ and $\boldsymbol{E}_T$ by the following two pairs of equations.

$$\text{div } \boldsymbol{E}_s = 4\pi\rho \tag{4}$$

$$\text{curl } \boldsymbol{E}_s = 0 \tag{5}$$

$$\text{div } \boldsymbol{E}_T = 0 \tag{6}$$

$$\text{curl } \boldsymbol{E}_T = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \tag{7}$$

However, if we confine ourselves and our knowledge of ρ and $\dfrac{\partial \boldsymbol{B}}{\partial t}$ to a limited region in space, such as outside the black box, and in its neighborhood, these equations 4 to 7 are not sufficient to determine $\boldsymbol{E}_s$ and $\boldsymbol{E}_T$. In that region we may make the field all "electrostatic," $\boldsymbol{E}_s$ or all "electromagnetically induced," $\boldsymbol{E}_T$, or quite arbitrary combinations of $\boldsymbol{E}_s$ and $\boldsymbol{E}_T$, and still satisfy equations 4 to 7.

Hence, this definition of $\boldsymbol{E}_s$ and $\boldsymbol{E}_T$ does not permit us to solve the problem of the essay and determine whether the field outside the black box is electrostatic, or electromagnetically induced.

If ρ or $\dfrac{\partial \boldsymbol{B}}{\partial t}$ respectively are known everywhere, so that equations 4 and 5 or 6 and 7 can be applied in our computation throughout all space, and if ρ or $\dfrac{\partial \boldsymbol{B}}{\partial t}$, respectively, are zero beyond some great distance, then $\boldsymbol{E}_s$ or $\boldsymbol{E}_T$ can be determined uniquely, from the defining equations. For example if ρ is known everywhere, then equations 4 and 5 give uniquely

$$\boldsymbol{E}_s = -\text{grad } \psi \tag{8}$$

where

* In my essays, generally I have given no literature references, as compiling such would be burdensome, and since I make no claim to scientific novelties in these essays. However, in this essay I do refer to Cohn (*EE, May '49, pp 441–7*), firstly, because of the very high excellence of the paper, secondly, because its pertinence to the subject of this essay, and thirdly, because it has appeared as recently in *Electrical Engineering*.

who discovered Thévenin approximately a quarter of a century ago, would have found out from a study of the history of circuit theorems who originated what they then "named" Thévenin's theorem.

Thévenin published his very short paper, in 1883[1]; its first paragraph (of altogether five) was reprinted on page 844 of the October issue of *Electrical Engineering.* In his paper, he merely states the theorem of the equivalent generator without any proof whatsoever, and one will understand Vashy's impression, who was not very enthusiastic about this "discovery." Helmholtz published his paper in 1853, 30 years before Thévenin[2]; it is a comprehensive analytical investigation which contains the rigorous proof. Perhaps it should be appropriate to go back to the name "principle of the equivalent generator" which has been in use long before some poor students of the history of science discovered Thévenin.

REFERENCES

1. Sur un Nuveau Théorème d'Électricité Dynamique, **L. Thévenin.** *Comptes Rendus Hebdomadaires Des Séances De L'Académie Des Sciences,* tome 97, 1883, page 159.

2. Ueber Einige Gesetze der Vertheilung Elektrischer Stroeme in Koerperlichen Leitern mit Anwendung auf die Thierisch—Elektrischen Versuche, **H. Helmholtz.** *Annalen der Physik und Chemie,* band 89, 1853, S. 213 ff.

ERIC T. B. GROSS (F '48)

(Graduate School, Illinois Institute of Technology, Chicago, Ill.)

Circuit Calculations

To the Editor:

Electric circuit problems are seldom described by unique solutions. This is particularly true for networks comprising a plurality of power sources and power receiving stations.

These introductory words and the rest of this letter are essentially a part of the answer recently given to a group of students who brought up the question, "How can the flow of current and/or energy be traced from source to sink through electric networks?"

The objective of this letter is not at all an exposition or even an adequate exploration of this difficult problem. The writer does, however, think readers of *Electrical Engineering* may be interested in the introductory part of the discussion which he presented in a form he has not seen presented by any other person.

Reference is made to Figures 1 and 2 which illustrate a resistance R fed by two batteries supplying electromotive forces E_1 and E_2. In Figure 1 the two supplies are additive and in Figure 2 the voltage E_2 opposes that of E_1.

Since our electromotive forces sources are direct current selected for simplicity in illustration, we know for Figure 1 that if $I_1=3$ amperes and $I_2=4$ amperes, the current in $R=7$ amperes and the I^2R power input to resistance $R=49$ R watts. If this solution were unique, we would know that the two currents I_1 and I_2 coming together make a current I_3 with a value of 7 amperes.

It can be demonstrated on the other hand that we are equally justified in considering I_1 and I_2 as distinctly separate currents, each independently of the other, wending its way through resistance R. In this case the power in R in watts equals $(I_1+I_2)^2R$ which equals $(I_1^2+2I_1I_2+I_2^2)R$. Using the values noted, namely 3 amperes and 4 amperes,

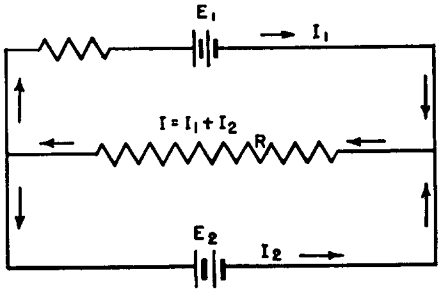

Figure 1. Currents I_1 and I_2 in same direction through resistance R

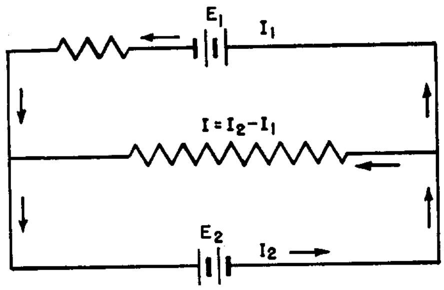

Figure 2. Current I_1 in direction opposite to current I_2 through resistance R

we have $(3+4)^2R$ or $(9+24+16)R=49$ R.

The same value as obtained by use of the more commonly used but less general 7^2R computation. In Figure 2 with the currents subtractive with respect to each other, we have power equals $(4\text{-}3)^2=(16-24+9)R=$ 1 R; thus by the algebraic solution checking the more commonly used arithmetic solution of the problem. Either analysis solves the problem correctly, and we may argue with equal weight that we have a single current of seven amperes the arithmetic sum of two currents or we may say that we have two distinct currents each circulating independently through R.

As one considers the philosophy of electric current flow it seems to the writer that the $(x+y)^2R$ algebraic method is the more logical and more general of the two even though its use is negligible as compared to the use of arithmetic method.

Since even this simple problem cannot be said to have a unique solution it seems unreasonable to think of any network as yielding only to a unique description of electric current and energy flow.

ROYAL W. SORENSEN (F '19)

(President AIEE 1940–41, Professor of Electrical Engineering, California Institute of Technology, Pasadena, Calif.)

Electrostatic or Electromagnetically Induced Electric Field?

To the Editor:

"There seems then to be no unique and physically significant definition of the electrostatic part of an electric field observed in a limited region of space" . . . "There is but one electric field, **E**, and Maxwell, our prophet, has proclaimed its properties" . . . . "A resolution of a local field into an electrostatic part and an inductive part . . . must be irrelevant, and without intrinsic physical meaning."

Thus speaks Dr. J. Slepian, High Priest of Maxwell's theory (*EE, Nov '49, pp 985–7).*

There is but one physical meaning to the term "electric field intensity," **E**, at a point, namely that it gives, in magnitude and direction, the force experienced by a stationary unit electric charge situated at the point. However, experience shows that other measurable quantities can be related to the value of **E**, and I do not think that anyone has ever actually measured **E** by directly measuring the force on one unit point charge. As an important example of a related quantity, if in a homogeneous conductor of resistivity ρ a current-density **i** exists at any point, then experience with circuits leads us to accept the law:

$$\mathbf{E}=\mathbf{i}\rho \qquad (1)$$

where **E** is the electric field intensity at the point.

Now the particular property of an electrostatic field, which distinguishes it from an electromagnetically induced electric field, is that it is derivable from a potential, and therefore its line-integral around a closed path is zero:

$$\oint \mathbf{E}_s\cdot d\mathbf{l}=0 \qquad (2)$$

Consequently, between any two points there is a perfectly definite difference of potential, independent of the integration path between the points:

$$V_{ab}=\int_a^b \mathbf{E}_s\cdot d\mathbf{l} \qquad (3)$$

and it is the function of many electromagnetic devices, such as the transformers manufactured by the company which employs Dr. Slepian, to produce potential differences between their terminals of accurately known magnitudes.

Let us therefore apply Dr. Slepian's doctrine to one of his company's transformers, with its secondary winding on open circuit. Since no secondary current is flowing, if **E** is the resultant, total, one and only Maxwellian electric field intensity at a point inside the copper of the secondary winding, we know, from equation 1, that:

$$\mathbf{E}=0 \qquad (4)$$

Dr. Slepian teaches that it is immaterial as to whether we regard **E** as electrostatic or as electromagnetically induced, and further that it is meaningless to try to split it into two components. But we may be forgiven for wanting to know how many volts the transformer will provide, or in other words its terminal potential difference. Therefore, since potential difference is the line-integral of an electrostatic field, following Dr. Slepian we shall assume that **E** in equation 4 is electrostatic, and surely we can make no mistake if we take the line-integral from one secondary terminal to the other through the path of the winding which connects them. Hence we immediately obtain:

$$\text{Secondary terminal potential difference on open circuit}=\int_{T_1}^{T_2}\mathbf{E}\cdot d\mathbf{l}=0 \qquad (5)$$

Thinking this result a little puzzling (for, after all, the transformers of Dr. Slepian's company are reputed to work), suppose we take the line-integral of the electrostatic field along a path between the terminals in air. But on second thought we give up this idea for we know that potential difference depends only on the terminal points, and not on the particular integration path, so we should still get zero potential difference.

If, on the other hand, we admit the existence of two components of electric field at the point in the secondary conductor, $\mathbf{E}_s$ which is electrostatic and $\mathbf{E}_i$ which is induced, we put:

$$\mathbf{E}=\mathbf{i}\rho=\mathbf{E}_s+\mathbf{E}_i=0 \qquad (6)$$

or, on open circuit:

$$\mathbf{E}_s=-\mathbf{E}_i \qquad (7)$$

Now potential difference has meaning only when applied to $\mathbf{E}_s$, so we have, for the terminal potential difference:

$$V=\int_{T_1}^{T_2}\mathbf{E}_s\cdot d\mathbf{l}=-\int_{T_1}^{T_2}\mathbf{E}_i\cdot d\mathbf{l}=-E \qquad (8)$$

where E is the induced electromotive force, and the minus sign tells us that, if we move along the winding in the direction of fall of potential, we move in a direction opposite to that in which $\mathbf{E}_i$ (and E) act.

We can, of course, calculate V as a rise of potential between terminals, in which case we get, on open circuit:

$$V=E \qquad (9)$$

or, if the transformer is loaded and I is the secondary current, from equation 6:

$$-\mathbf{E}_s=\mathbf{E}_i-\mathbf{i}\rho \qquad (10)$$

which on integration along the winding gives:

$$V=E-IR \qquad (11)$$

where of course E is the total electromagnetically induced electromotive force, due to both mutual and leakage fluxes.

Since Dr. Slepian apparently denies the relevance of all this, it would be most interesting to know his explanation of the terms potential difference (V) and induced electromotive force (E) as applied to electric circuits, and his definition of these measures in terms of the two "eternal equations."

E. G. CULLWICK (M '33)
(Professor of Electrical Engineering, St. Andrews University, Dundee, Scotland)

To the Editor:

Professor Cullwick misquotes me badly when he says that I "teach that it is immaterial as to whether we regard E as electrostatic or as electromagnetically induced." What I did say in my electrical essay was that the resolution of $\mathbf{E}$, the electric field in a given region in space which can be observed uniquely and locally by local operational means into an electrostatic part, $\mathbf{E}_s$, and an electromagnetically induced part $\mathbf{E}_T$, $\mathbf{E}=\mathbf{E}_s+\mathbf{E}_T$, does not have any unique significance, and expresses no physically determinable reality. I did not say that we may regard any local field as all "electrostatic," meaning by "electrostatic" the quality of being the gradient of a scalar. Of course, fields exist which are not the gradients of scalars. But the resolution of such a field into the sum of a part which is the gradient of a scalar, and another part which is not can be done in infinitely many ways, and such resolution has no unique significance.

In his book, "Fundamentals of Electromagnetism," Macmillan, 1939, Professor Cullwick apparently defines the electrostatic component of the field at a point in space as that field which is calculated at that point from the Coulomb potentials of all the charges in the universe. The remaining part of the electric field at the point in question is then called magnetically induced.

Let us apply these definitions which I attribute to Professor Cullwick, I hope rightly, to the field outside a cavity oscillator, that is a completely enclosed metal container within which there is an oscillatory electromagnetic field. Let us assume that the net field, outside, which can be observed directly, without peering into the box, is zero; that is that any small charged body at rest there experiences no unbalanced mechanical force. Then, as I understand Professor Cullwick, he would say that this zero electric field outside the box, which obviously can have no observable electrical influence upon anything, is "really" made up of an "electrostatic" part, which Professor Cullwick can calculate from the charges inside the cavity oscillator, and a magically just equal and oppositely directed magnetically induced electric field which Professor Cullwick can cleverly also calculate from the varying magnetic fields inside the metal box.

Now, I am broad-minded and am perfectly willing to let Professor Cullwick, "for his real or imaginary convenience," assert that there are these equal but oppositely directed electric fields penetrating through the thick metal walls of the container and reaching outside, but I balk at regarding this resolution of the external zero field as having any unique or physically verifiable significance. I myself can resolve the actual zero field into equal and opposite components, in a thousand different ways, all of which are just as good as Professor Cullwick's as far as any physical test goes. However, I would regard that only as a manifestation of my own mathematical ability, and would not believe that these component fields had any verifiably unique reality. Similarly, the nearly zero electric field inside the metal conductor of the transformer coil to which Professor Cullwick refers, can be resolved by the designer into an induced field and an almost equal and opposite electrostatic field. I insist however, that this resolution is only for the designer's convenience, and reflects no demonstrable reality.

Replying to the specific questions in the last paragraph of Professor Cullwick's letter, I define the potential difference from one point to another in an electric field as the integral of the electric field, $\int \mathbf{E}_s ds$, along some path joining the two points, provided the field has a potential, which will not generally be the case, that is provided $\int \mathbf{E}_s ds$ is independent of which particular path is chosen in going from one point to the other. I define induced electromotive force only for closed paths and as $\int \mathbf{E}_s ds$ around such a closed path.

J. SLEPIAN (F '27)
(Westinghouse Research Laboratories, East Pittsburgh, Pa.)

Of What Value Is History?

To the Editor:

I enjoyed the recent article, "Of What Value Is History?" (*EE, Nov '49, pp 945–8*) very much. There is indeed much to be learned from the history of electrical engineering, and also from the psychological attitude of professional men toward any radically new trend of thought. It brings to mind an exceedingly mysterious case, as recent as 1944, in which a man and his work were completely ignored by all the technical journals—in spite of widespread articles in the semitechnical magazines. I refer to the researches of Dr. Felix Ehrenhaft. He claimed that the traditional belief that magnetic poles occur only in equal and opposite pairs is contrary to fact, and gave clear and explicit experimental procedures whereby this could be shown. One of the best articles on him appeared in *Popular Science*, June 1944.

It is not my intention to express any opinion whatsoever on the merits of Ehrenhaft's research. I only want to point out that he made what is, to electrical engineers, a fantastic claim; that he backed up his claims by completely adequate diagrams and directions whereby anyone could attempt to duplicate his work; and that as far as I have been able to determine not a single electrical engineer even took the trouble to publish a criticism of his work. For no understandable reason, all mention of Dr. Ehrenhaft ceased abruptly. The last article I could find appeared in *Science Digest*, issue of October 1944.

Does anyone know the real truth behind the "Ehrenhaft mystery?"

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(319 Lincoln Avenue, Bridgeport, Conn.)

NEW BOOKS • • • • •

The following new books are among those recently received at the Engineering Societies Library. Unless otherwise specified, books listed have been presented by the publishers. The Institute assumes no responsibility for statements made in the following summaries, information for which is taken from the prefaces of the books in question.

INGENIEURE, BAUMEISTER EINER BESSEREN WELT. By F. Münzinger. Third edition. Springer-Verlag, Berlin/Göttingen, 1947. 263 pages, charts, tables, 8½ by 5¾ inches, stiff cardboard, apply. The important topics dealt with are the relation between technology and the sociological aspects of civilization, and the contributions and obligations of the engineer to society. The author also discusses, among other things, the influence of engineering training on men, and follows the history of certain inventions with particular attention to their consequences. The book closes with a speculation on the hypothetical reaction of a resurrected member of a past generation to the current disturbed state of affairs.

INTRODUCTION TO THE ELECTRON THEORY OF METALS. (Institute of Metals Monograph and Report Series number 4). By G. V. Raynor. Institute of Metals, 4 Grosvenor Gardens, London, S.W.1, England, 1947. 98 pages, diagrams, charts, tables, 8¾ by 5½ inches, cloth, 7s.6d. Written for the metallurgists who have not had recent training in physics, this text gives a picture of the development of the electron theory of metals. It outlines some of the ideas which lie behind the theory, and indicates its applications to physical metallurgy and the properties of metals.

KONTAKTUMFORMER MIT SCHALTDROSSELN. By A. Goldstein. Verlag AG. Gebr. Leeman and Company, Zürich, Switzerland, 1948. 179 pages