and special louver styles of both the exterior horizontal type and the interior vertical type may be employed for rooms where the occupant's direction of view is established.

Heating and Ventilation. A heating and ventilation system employing a hot-water base-load type controlled by exterior thermostats and supplemented by a circulating air system with final temperature control by interior thermostats is capable of providing excellent temperature control as well as comfortable air circulation. The system is also capable of air conditioning through the air component system.

The radiators of the water-component system should be installed fairly low on the outside walls to provide radiation slightly less than the heat loss of the building. The radiators consist of fins attached to a pair of circulating water pipes. These radiators may be located conveniently just below the standard 30-inch table height so that the tops of such tables may extend over the radiators slightly. The power distribution duct system for small laboratories may be located on the wall above the radiators without being exposed to excessive temperature because the water temperature is not extremely high even for unusually cold winter days.

Furred ceilings in the corridors may be used to good advantage for the air ventilating system as well as other building services such as power distribution bus ducts and so forth. The forced air may be carried in air ducts with vents into the rooms on a modular dimension equal to twice the window modular dimension. Return air is carried in the open furred space with vents from the rooms in the alternate modular spaces as well as with vents in the ceiling of the corridor to pick up cool air from that location.

CONCLUSIONS

When building planning is begun well in advance of the architect's drawings, fine co-operation between the architect and the user can be achieved. The results are buildings that are highly functional, reasonable in cost, and beautiful to the eye.

Electrical Essays

Motionally Induced Electromotive Force—Part II

The Hall Effect

Jack, the physicist, is continuing his lecture to Alter Ego and his friends explaining the principles of electric motors and generators (see Part I, *EE, Nov '50, pp 1025–26*).

Jack: "As I was saying before Alter Ego interrupted me, the electrons in the cross bar of Figure 1 have to move with its velocity $\mathbf{v}$ in the direction perpendicular to the magnetic field, and therefore will have acting on them on this account a force

$$\mathbf{F}=e\frac{1}{c}[\mathbf{v}\mathrm{x}\mathbf{B}] \tag{1}$$

"Since this force is proportional to the charge e, it produces the same effect as if there were an electric field acting on it:

$$\mathbf{E}_{\mathrm{mot}}=\frac{1}{c}[\mathbf{v}\mathrm{x}\mathbf{B}] \tag{2}$$

"We'll call $\mathbf{E}_{\mathrm{mot}}$ of equation 2 the motional field, or motionally induced electric field. If we integrate it from one point to another, we'll call the result the motionally induced electromotive force, although if $\mathbf{v}$ is not constant throughout the material, this integral may depend on the path of integration.

"In a moving metal, then, since not only the regular electric field $\mathbf{E}$ but also $\mathbf{E}_{\mathrm{mot}}$ will be acting on the electrons, the current density will not be given by Ohm's law

$$\mathbf{i}=\sigma\mathbf{E} \tag{3}$$

where σ is the conductivity of the metal, but we will have instead

$$\mathbf{i}=\sigma\left(\mathbf{E}+\frac{1}{c}[\mathbf{v}\mathrm{x}\mathbf{B}]\right)=\sigma(\mathbf{E}+\mathbf{E}_{\mathrm{mot}}) \tag{4}$$

"If we make the current zero, as by opening the circuit of Figure 1, then we must have $\mathbf{E}+\mathbf{E}_{\mathrm{mot}}=0$, and $\mathbf{E}=-\mathbf{E}_{\mathrm{mot}}=-\frac{1}{c}[\mathbf{v}\mathrm{x}\mathbf{B}]$. To get the open circuit voltage then, since $\mathbf{E}=-\text{gradient } V$ where V is the electrostatic potential, we integrate through the wires of Figure 1 from the one open end to the other, and get

$$V_0=\int-\mathbf{E}\cdot d\mathbf{s}=\frac{1}{c}vBl \tag{5}$$

where l is the length of the slide wire between the two sliding contact points.

"If I close the circuit, the total current I will be given by

$$RI=V-V_0=V-\frac{1}{c}vBl \tag{6}$$

where R is the resistance of the circuit.

"Multiplying equation 6 by I, we get

$$VI=RI^2+V_0I=RI^2+\frac{1}{c}vIBl \tag{7}$$

where VI is the electrical power input, RI^2 is the Joulean heat developed, and $V_0I=\frac{1}{c}vIBl$ is the mechanical power, since $\frac{1}{c}IB$ is the force which acts per unit length on a conductor carrying a current I, and which lies in a perpendicular magnetic field $\mathbf{B}$. But I see that Alter Ego has been trying to ask a question. What is it, Alter Ego?"

Alter Ego: "This **v** in your equations is the velocity of the electrons in the magnetic field, which you produced by moving the slide bar, but there are other ways of giving electrons velocity. For example, in a piece of metal, at rest, if I send current in it, I'll have

$$\mathbf{i}=ne\mathbf{v} \text{ or } \mathbf{v}=\mathbf{i}/ne \tag{8}$$

where n is the density of electrons in the metal.

"Now, if I put this metal carrying current in a magnetic field, even though the metal is at rest, the electrons are moving, and I'll get a motional field

$$\mathbf{E}_{\text{mot}}=\frac{1}{c}[\mathbf{v}\times\mathbf{B}]=\frac{1}{cne}[\mathbf{i}\times\mathbf{B}] \tag{9}$$

Is that right?"

Jack: "Well, we do not usually call it a motional field, but what you have done is to independently discover the Hall Effect. Not only that but you have even given a formula for the Hall Effect coefficient, $1/cne$, which is just about right. That is wonderful, Alter Ego!"

Alter Ego: "I don't know how big n is, but at least I know it is positive, and c is positive, and e is negative, so I can be very sure that the Hall Effect coefficient is always negative, isn't that right, Dr. Jack?"

Jack: "Well no, some metals, zinc, cadmium, and lead, for example, have Hall Effect coefficients of the opposite sign."

Alter Ego: "Gosh! Then the electrons in these metals have a positive charge?"

Jack: "No. You see, the electrons in the metal are not quite as simple as I made out. They are really wave functions in the periodic lattice potential and because they are degenerate, they satisfy Fermi-Dirac statistics, and lie in the bottom portion of a band of allowed energy levels. Some of the electrons get pushed into the upper energy levels and leave holes in the assembly of electrons in the lower levels, and these holes act like electrons with positive charge."

Alter Ego: "Double gosh! Then you can have a force like $\frac{1}{c}[\mathbf{v}\times\mathbf{B}]$ acting on a hole?"

Jack, weakly: "Yes."

Alter Ego: "Well, anyway, since you told me the Lorentz force formula is all right, I can be sure that the Hall Effect is proportional to **B**, isn't that right?"

Jack: "Well, no. The Hall Effect in the magnetic metals is not proportional to either **B** or **H**. The Hall Effect is quite complicated in these metals."

Alter Ego: "Then, when calculating the motional electromotive force of a conductor moving in a magnetic field, like in the slide-wire experiment, you say that the electrons are small bodies with charge e, moving in the Maxwell field **B**, subject to the Lorentz force, $\mathbf{F}=e\frac{1}{c}[\mathbf{v}\times\mathbf{B}]$, thus giving the motionally induced electric field $\mathbf{E}_{\text{mot}}=\frac{1}{c}[\mathbf{v}\times\mathbf{B}]$, which is the same for all metals; but when I move the electrons in the stationary metal to calculate the Alter Ego Effect, also known as the Hall Effect, then you say the electrons become waves, or holes, their charge is no longer the negative number e, they no longer move in Maxwell's **B** or even **H**, so they certainly can't have acting on them the Lorentz force, $e\frac{1}{c}[\mathbf{v}\times\mathbf{B}]$. I am confused."

Jack: "I am a bit confused, too. I knew that the simplified electron theory I gave you was wrong, but at least you would understand it. Instead I should have given you the true theory, which you could not possibly understand."

(*To be continued.*)

There are several questions to be asked: 1. Should Jack have appealed to electron theory at all? 2. Did Jack present the simple Lorentz picture correctly? 3. Is there not some more general macroscopic principle or theory that Jack might have appealed to rather than the electron theory of the microstructure of matter?

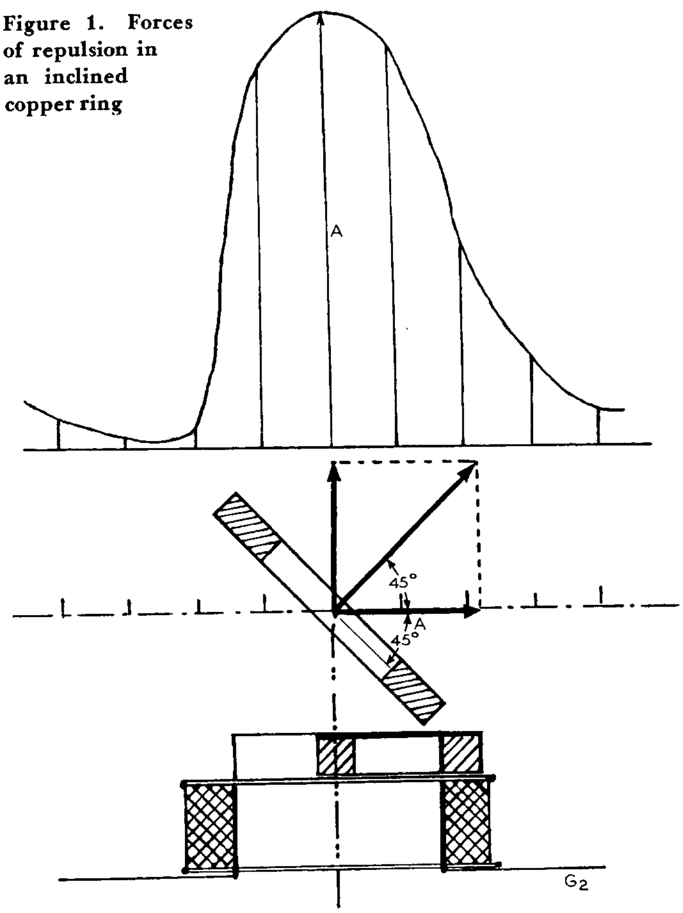
J. SLEPIAN (F '27)
(Westinghouse Research Laboratories, East Pittsburgh, Pa.)

A Repulsion Motor Problem

The operation of a repulsion motor depends on the fact that a copper ring moves out of an alternating magnetic field or turns until the plane of the ring is parallel to the lines of force (Thomson's experiment).

In practice, the switching devices of these motors (commutator, brushes, wound armature and its attendance)

Figure 1. Forces of repulsion in an inclined copper ring



Answers to Previous Essays

Motionally Induced Electromotive Force—Part II. The following is the author's answer to his previously published essay (*EE, Dec '50, pp 1086–87*).

> "O, what a tangled web we weave,
> When first we practice to deceive."

Question 1. Poor Jack! He should not have appealed to electron theory at all. He knew that the simple Lorentz electron theory picture of matter is not true, but it had the great merit of being understandable to his students, Alter Ego, and his friends. How many teachers face this dilemma! To give a false theory, which the students will say they understand, or a more sophisticated and truer statement which will be incomprehensible to the immature student?

Questions 2 and 3. No, Jack did not present the simple Lorentz picture correctly, and yes, there is a more general macroscopic principle that Jack might have appealed to, and then he could have left the electrons out. In fact, Jack tacitly used this general principle in his faulty treatment of the Lorentz theory, although of course that is not what made his treatment faulty.

Now what Jack is after, of course, is the constitutive relation for a metal in motion, assuming that for a metal at rest it is

$$\mathbf{i} = \sigma \mathbf{E} \tag{1}$$

Now, Jack did assume in his discussion that in some sense the moving bar had exactly the same properties as the stationary bar, and that relative to the electrons inside it, or relative to any other objects moving along with it, the bar would behave the same as the bar at rest, relative to the electrons within it, or relative to other objects at rest with it. But this is the principle of relativity, and Jack did not need to go farther to derive the constitutive equation for the moving bar. Knowing the constitutive equation for the bar at rest, say Ohm's law, or equation 1, then relativity alone is sufficient to determine the constitutive equation for the bar in motion, and further inquiry into the details of microscopic electron theory is unnecessary, and for the desired purpose, irrelevant.

If we include among the objects moving along with the moving bar an observer, B, then the principle of relativity just described would assert that to the observer B the bar would appear, through such experiments as B could make, to have the same constitutive equation as that for the bar at rest, as determined by an observer A, also at rest with the bar.

Therefore, if B sees the field $\mathbf{E}'$, and the current $\mathbf{i}'$, B will find the constitutive equation

$$\mathbf{i}' = \sigma \mathbf{E}' \tag{2}$$

But the fields $\mathbf{E}'$, and so forth, and the charge and current densities ρ', $\mathbf{i}'$, which B sees will not be the same as the fields $\mathbf{E}$, and so forth, and the charge and current densities, ρ, $\mathbf{i}$, which A sees, since B and A will differ as to what is the velocity of the charged probes which they use for observing the fields. If A and B use relativistic particle mechanics, the field $\mathbf{E}'$ will be related to the fields $\mathbf{E}$ and $\mathbf{B}$ by the equations

$$\mathbf{E}_x' = \mathbf{E}_x$$

$$\mathbf{E}_y' = \left(\mathbf{E}_y - \frac{1}{c} v \mathbf{B}_z\right)\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

$$\mathbf{E}_z' = \left(\mathbf{E} + \frac{1}{c} v \mathbf{B}_y\right)\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \tag{3}$$

where the velocity $\mathbf{v}$ is entirely in the x direction, and is of magnitude v.[1] If we neglect v^2/c^2 compared to 1, then equation 3 reduces to

$$\mathbf{E}' = \mathbf{E} + \frac{1}{c}[\mathbf{v} \times \mathbf{B}] \tag{4}$$

which is also what we would get if we used Newtonian particle mechanics.

Similarly, if we used relativistic particle mechanics and neglect v^2/c^2 compared to 1, we get

$$\mathbf{B}' = \mathbf{B} - \frac{1}{c}[\mathbf{v} \times \mathbf{E}] \tag{5}$$

If we had used Newtonian particle mechanics, with the definition we have given for $\mathbf{B}$, we would have had

$$\mathbf{B}' = \mathbf{B} \tag{5A}$$

However, equation 5 with the other related primed vectors satisfies Maxwell's equations except for terms involving v^2/c^2, whereas equation 5A would fail to satisfy Maxwell's equations by terms involving v/c.

Similarly for the other field quantities, neglecting v^2/c^2 compared to 1, we have

$$\mathbf{H}' = \mathbf{H} - \frac{1}{c}[\mathbf{v} \times \mathbf{D}] \tag{6}$$

$$\mathbf{D}' = \mathbf{D} + \frac{1}{c}[\mathbf{v} \times \mathbf{H}] \tag{7}$$

$$\mathbf{i}' = \mathbf{i} - \frac{1}{c}\rho \mathbf{v} \tag{8}$$

$$\rho' = \rho + \frac{1}{c}\mathbf{v} \cdot \mathbf{i} \tag{9}$$

Equations 8 and 9 may seem strange, suggesting that relatively moving observers will disagree as to the charge and current densities in a given body, but they are true, (except for higher powers of v/c) and to be expected, as will be developed in later essays.

Now if observer B finds Ohm's Law or equation 2 holding for the metal body at rest relative to him, then observer A will find for that same body moving with velocity $\mathbf{v}$ relative to him

$$\mathbf{i} - \frac{1}{c}\rho \mathbf{v} = \sigma\left(\mathbf{E} + \frac{1}{c}[\mathbf{v} \times \mathbf{B}]\right) \tag{10}$$

obtained by substituting equations 4 and 8 in equation 2.

Observer B may also find that the charge density ρ' in a homogeneous metallic body at rest relative to him is always zero. If we set $\rho' = 0$ in equation 9, then, except for higher powers of v/c, equation 10 will reduce to

$$\mathbf{i} = \sigma\left(\mathbf{E} + \frac{1}{c}[\mathbf{v} \times \mathbf{B}]\right) \tag{11}$$

Thus, we get equation 11 which is basic for the usual treatment of voltages induced in moving wire coils as given in electrical engineering texts, but we see that it is limited to coils of material which at rest satisfy Ohm's Law, equation 2, have zero charge density, and move so slowly that v^2/c^2 is negligibly small compared to 1.

REFERENCE

1. Electromagnetic Theory (book), **Stratton.** McGraw-Hill Book Company, Inc., New York, N. Y., 1941, page 79.

J. SLEPIAN (F '27)
(Westinghouse Research Laboratories, East Pittsburgh, Pa.)

A Repulsion Motor Problem. The following is the author's answer to his previously published essay (*EE, Dec '50, pp 1087–88*).

Action of single-phase motors can be explained by the following:

The alternating field can be thought of as being composed of two inverse rotating magnetic fields with one-half the amplitude of the original field. In practice this would be a 3-phase motor with two inverse field windings. Then there will be two magnetic fields rotating in opposite directions. It is possible now to transpose these rotating fields to an alternating field with double amplitude. At standstill, both the inversely rotating fields generate an electromotive force in the armature by induction. The armature, in turn, will cause two inverse rotating fields again.

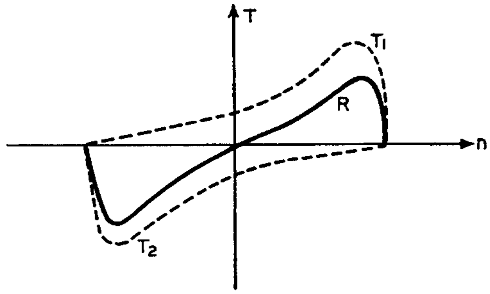


Figure 1. Two rotating magnetic fields in operation

Figure 1 shows graphically the two rotating magnetic fields in operation, where T = torque; R = resultant torque; and n = revolutions per minute. Torque is nullified by the inversion of the fields.

In starting the armature with an auxiliary motor, one of the rotating fields is favored. It grows along the T_1 curve of the diagram. The other field follows the T_2 curve.

Regarding the problem, one of the T curves is suppressed and the other is reinforced (dotted lines) by the means described here. Of course, it is rather difficult to check the conditions that really take place to prevent the armature from starting by itself.

R. J. GERHARZ
(Institute of Technology, Darmstadt, Germany)

Radio and Electric Trucks Form Team to Create New Handling Savings

Driver of the electric truck has just deposited a load at the outbound shipping dock. Without dismounting from the truck, and even before the forks leave the pallet, he is able to call the dispatcher for his next assignment

Use of 2-way radio as an aid to mechanized handling of materials is saving American industry thousands of dollars. The new technique accomplishes two purposes: it keeps industrial trucks busy carrying payloads instead of wasting time deadheading, and it moves them to busy spots where they are needed without a wasted minute. One firm putting the idea to work is the Johnson and Johnson company, whose new shipping center is at Metuchen, N. J. Internal handling, except for a dragline operating in the order picking area, is by a fleet of seven Skylift electric fork trucks, 2,000-pound capacity, made by the Automatic Transportation Company, Chicago, Ill. Two-way Motorola short-wave radio sets were installed on each of the Skylifts. They are placed to the right of the driver's seat, and are easily accessible. All sets, plus the master station from which all orders emanate, are on the same frequency. Thus, every driver hears all messages, and can better orient himself to the entire operation. A master short-wave station is located in central dispatching headquarters, which is both the voice and brain of the materials' handling system. Included are a space layout chart and stock record location cards. The dispatcher knows where all merchandise is, can keep up to the minute on the positions of his fleet, and is able to shift trucks and goods with exceptional speed.

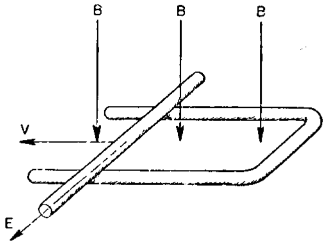


Figure 1 (above). The slide-wire experiment

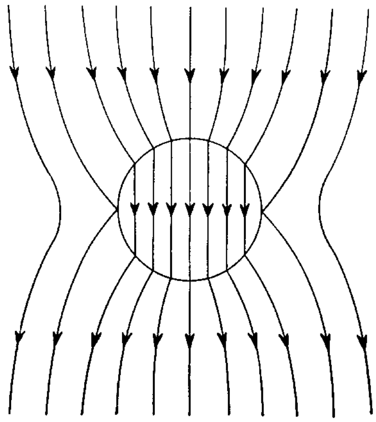


Figure 2 (right). Converging field in iron cylinder

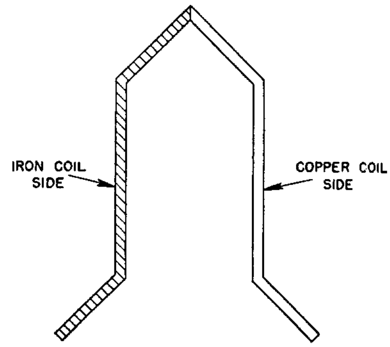


Figure 3. Iron-copper coil

parallel sides of iron and the other of copper. Now translate the coil in the magnetic field. There will be motional fields induced in the coil sides. If the coil had been all copper, then the motional field $\mathbf{E}_{mot}$ in the one coil side would oppose and just cancel the motional field $\mathbf{E}_{mot}$ in the other coil side, so far as the production of a total electromotive force around the coil is concerned. However, with one coil side iron, the motional field $\mathbf{E}_{mot}$ in it will be twice the motional field in the other side, copper coil, and these motional fields will not cancel round the coil, but the coil will give a net, not zero, electromotive force as it is moved in the uniform magnetic field. It is easy to see how we could mount a lot of these coils in series on a rotating member with slip rings, in a uniform magnetic field, and get a beautiful constant d-c voltage. There's your commutatorless d-c generator.

"Now, consider what happens if I send current through my (patent-pending) iron-copper coil while it is in a uniform magnetic field. If the coil had been all copper, then, according to your universal and absolutely right equation 3, the forces exerted by the magnetic field on the coil sides would be equal and opposite and would exactly cancel. However, in my iron-copper coil, the field **B** in the iron side is twice what it is in the copper side, and therefore, according to your universal equation 3, the force on the iron side will be twice the oppositely directed force on the copper side, and I will get a net force tending to move the whole coil. If I arrange a lot of these coils around on a rotor as in my generator, I'll get a commutatorless d-c motor. However, if I lay the coils out in a plane, I'll get a space ship, since the reaction on the field-producing member will be the same as if the coils were all copper, namely, zero. Aren't those wonderful inventions, Dr. Jack?"

Jack: "I don't like to throw cold water on your enthusiasm, Alter Ego, but I doubt that your inventions will work. You will remember that in my last lecture (*EE, Jan '51, pp 67–68*), I showed that the sum of all the electric fields, the electrostatic, that induced by transformer action, and the motional, when integrated round any closed material circuit, was equal to $1/c$ times the rate of change of the integral of **B** over any surface enclosed by the circuit, or in other words, to $1/c$ times the rate of change of the total flux linked by the circuit. Now, as your coil moves in the uniform magnetic field, the amount of flux linked by it does not change. Therefore, the total current-producing electromotive force which is around your coil stays zero.

"Of course, you should have obtained the same result using the motional equation 1, but you did not. Some people think that the **v** in equation 1 stands for the motion of the moving body, not relative to space or some suitable material frame of reference but relative to the magnetic field itself, and that where the field lines bunch up as in your iron bar the bar carries them along somewhat, so that the lines of force slide or cut through the bar with only half that velocity with which they are cut by the copper bar so that the motional field is the same in the iron as the copper. Maybe that is the way out...."

I interrupt Jack at this point, because I am already behind in my reply to the preceding essay and I must catch up.

Will Jack be able to explain away the contradiction, along the line he has indicated? Watch for the next exciting episode in the next installment.

J. SLEPIAN (F '27)

(Westinghouse Research Laboratories, East Pittsburgh, Pa.)

Three Impedances

Three impedances are so proportioned that when connected in star and energized from a 60-cycle system

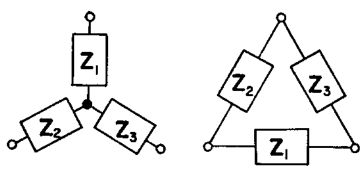


Figure 1. Star and delta connections of three impedances

they form a network that is equivalent to a network formed by the same three impedances connected in delta. Is this possible?

A. A. KRONEBERG (F '48)

(Southern California Edison Company, Los Angeles, Calif.)

Answers to Previous Essays

Correction to Motionally Induced Electromotive Force. The following is the author's correction to the Part II essay and the answer to Part I (*EE, Dec '50, pp 1086–9*).

The units used in these essays on "Motionally Induced

Electromotive Force" are Gaussian, that is, **E**, **D**, **P**, ρ, and V are in centimeter-gram-second electrostatic units, and **H**, **B**, **M**, and **i** are in centimeter-gram-second electromagnetic units. Other quantities such as **v**, c, **F**, and so forth, are in centimeter-gram-second units. With these units, **v** and c occur in all equations as the ratio $\mathbf{v}/c$. It seemed to the author that this relationship of **v** to c is worth emphasizing, and with these units the continual appearance of $\mathbf{v}/c$ gives this emphasis.

However, the author slipped from consistent adherence to these units in a few places in *Electrical Engineering*, December 1950. On page 1086, the lines following equation 7 should read, "where cVI is the electrical power input, cRI^2 is the Joulean heat developed, and $cV_oI=vIBl$ is the mechanical power, since IB is the force . . ."

On page 1089, column 1, line 17 from the bottom, the equation should read

$$4\pi\left(\mathbf{i}+\frac{1}{4\pi c}\frac{\partial \mathbf{D}}{\partial t}\right)=\text{curl } \mathbf{H}$$

instead of as given.

J. SLEPIAN (F'27)

(Westinghouse Research Laboratories, East Pittsburgh, Pa.)

Motionally Induced Electromotive Force—Part III. The following is the author's answer to his previously published essay (*EE, Jan '51, pp 67–68*).

No, there is no motional field $\mathbf{E}_{\text{mot}}$ in vacuum or, for that matter, anywhere else, in the author's opinion. Having chosen a particular frame of reference in which Newtonian particle mechanics holds, or better yet, relativistic particle mechanics holds, then we may define **E** and **H** from the forces which we observe on small enough charged material probes placed at that point in empty space where we wish to observe **E** and **H**. Then Maxwell's equations are found to hold for these vectors **E** and **H** thus defined in empty space. Within matter **E** and **H** are as yet undefined, since the operation of observing forces on a charged probe to be placed within matter has no meaning.

However, **E** and **H** are observable in empty crevices in matter, and if we assume that Maxwell's equations hold for **E** and **H** so observed in any and all the crevices which we can construct in the material body, then it follows that equations 1, 2, 3, and 4 given on page 1088 (*EE, Dec '50*) define vectors **E**, **D**, **H**, and **B**, which we call the electromagnetic field vectors for the material body.

The velocity **v** which enters into the definition of these field vectors is the velocity of the material of the body which bounds the crevice. If now we *define* the charge density ρ by the equation $4\pi\rho=\text{div } \mathbf{D}$, and the current density **i** by the equation $4\pi\left(\mathbf{i}+\frac{1}{4\pi c}\frac{\partial \mathbf{D}}{\partial t}\right)=\text{curl } \mathbf{H}$, then Maxwell's field equations will hold for all these quantities, thus defined ultimately by means of measurements of force on charged probes at rest or moving in empty space, and in some particular frame of reference.

Including ρ and **i**, we have then five vector field quantities and one scalar field quantity which enter in Maxwell's field equations. These last are two vector equations which refer to curl **E** and curl **H**, and two scalar equations which refer to div **D** and div **B**. Maxwell's field equations are then not mathematically determinate, since they have more dependent variables than the number of independent equations.

To make a mathematically determinate system, we postulate that sufficient additional relations are imposed upon the field quantities by the material itself. These constitutive relations are dependent upon the nature of the material, and may be expected to vary with the pressure, temperature, and so forth, of the matter, possibly also with its past history, and, important for our present purpose, also with its velocity. These constitutive relations may be found by experiment, or may be deduced from theories of the microscopic structure of matter, and of the behavior of microscopic particles, electrons, dipoles, nuclei, and so forth. They are the data of Maxwell theory and not any part of the conclusions from Maxwell theory.

Maxwell's field equations are equivalent to six independent scalar equations, and the field quantities are equivalent to 15 independent scalar variables. To make a mathematically determinate system, the constitutive relations must then be equivalent to nine independent scalar relations.*

Thus for empty space we have $\mathbf{D}=\mathbf{E}$, $\mathbf{B}=\mathbf{H}$, $\mathbf{i}=0$, and $\rho=0$, which are equivalent to nine independent scalar equations. Again, for a nonmagnetic metal at rest we may have $\mathbf{i}=\sigma\mathbf{E}$, $\mathbf{B}=\mathbf{H}$, $\mathbf{D}=0$, $\rho=0$, again equivalent to nine independent scalar equations.*

The problem of the electromagnetism of moving bodies is thus reduced to the problem of determining what are the constitutive equations for the matter of bodies in motion. Is there some general method whereby from a knowledge of the constitutive relations for the material making up a body at rest, we may deduce the constitutive equations for that body in uniform motion with velocity **v**?

Minkowski and Einstein described such a general method in 1905. Given a body at rest, for which constitutive equations connecting the field quantities **E′**, **D′**, **H′**, **B′**, **i′**, and ρ' are known. Then if in these equations we substitute

$$\mathbf{E}'=\mathbf{E}+\frac{1}{c}[\mathbf{v}\times\mathbf{B}]$$

$$\mathbf{D}'=\mathbf{D}+\frac{1}{c}[\mathbf{v}\times\mathbf{H}]$$

$$\mathbf{H}'=\mathbf{H}-\frac{1}{c}[\mathbf{v}\times\mathbf{D}]$$

$$\mathbf{B}'=\mathbf{B}-\frac{1}{c}[\mathbf{v}\times\mathbf{E}]$$

$$\mathbf{i}'=\mathbf{i}-\frac{1}{c}\rho\mathbf{v}$$

$$\rho'=\rho+\frac{1}{c}\mathbf{v}\cdot\mathbf{i} \qquad (1)$$

* Any vector equation may be written as the three equations between the component of the two sides of the equation. However, the three components of the curl of any vector **A** must not be regarded as independent since we have always the scalar equation div curl **A**=0. Therefore, the equation of Maxwell referring to curl **E** must be regarded as equivalent to only two independent scalar relations, and similarly for the equation concerning curl **H**. Also, the field quantities **i** and ρ must always satisfy the equation div $\mathbf{i}+\frac{\partial\rho}{\partial t}=0$. Therefore, **i** and ρ must be regarded as equivalent to only three independent scalar quantities.