and stopping. A typical application is to sugar centrifugals where it is necessary to accelerate to top speed quickly and to stop quickly in order to meet production schedules. During the operating cycle considerable losses occur, particularly in the motor rotor. These losses must be calculated before the proper motor selection can be made. Two-speed squirrel-cage motors are used and the control is arranged to accelerate the motor to approximately one-half speed by applying full voltage to the low-speed winding, then to accelerate to full speed by applying full voltage to the high-speed winding. The centrifugal is decelerated by applying full voltage to the low-speed winding until approximately one-half speed is reached at which time the motor is disconnected and mechanical braking applied.

Consider a 6/12-pole 60-cycle squirrel-cage motor direct-connected to a centrifugal which requires the schedule of 40 seconds for acceleration, 20 seconds running time, and 40 seconds for deceleration. The WK^2 at the motor shaft including both motor and basket is 9,500 pound-feet squared. Assuming a negligible friction load, calculate the rotor losses during one cycle of operation including starting, running, and stopping. If the ratio of secondary resistance to primary resistance is four to one, estimate the total motor losses neglecting iron loss.

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Electrical Essay

Self-Running Electrostatic Motor

My inventive ability has got me in a fix. I was doing a repair job on the Van de Graaff atom smasher up at the Research Laboratory and got to thinking about the forces on electric charges like the books talk about. I cooked up the following.

Two endless insulating belts run round pulleys like in Figure 1. These belts are each kept charged equally and oppositely by high-voltage corona points as in Figure 1, and the belts are moved around until they are charged all over, uniformly. Of course for any position of the belts, the electrostatic forces balance. The attractions of the opposite charges on the upper parts of the belts are just balanced by the attractions of the opposite charges on the lower parts of the belt.

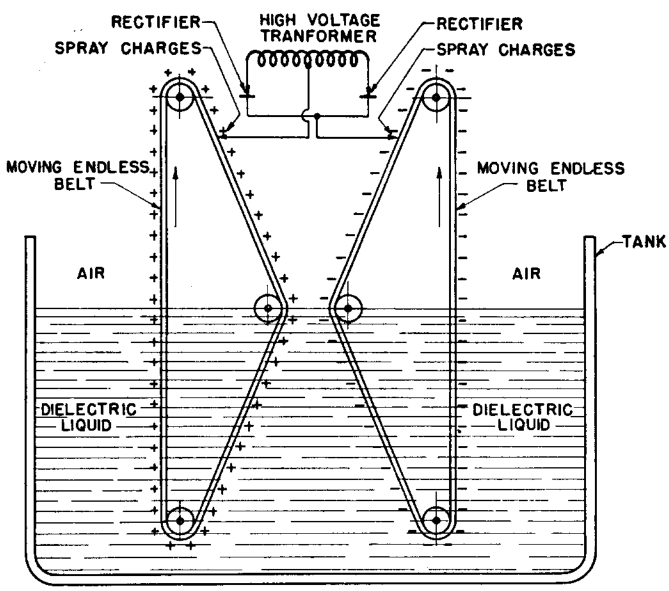

Figure 1. Self-running electrostatic motor

Now I change all this by immersing the lower halves of the belts in inerteen oil. Besides being fireproof, this inerteen oil has a dielectric constant of five, so according to what the books say, the attractions of the charges on the lower parts of the belt will be reduced by five so that they no longer balance the attractions of the charges on the upper parts of the belts. The belts will start rotating therefore, the inner nearer parts moving downward, and the outer parts moving upwards.

Now I am proud of being an inventor of space ships, but this last invention looks like perpetual motion, and I don't want to have none of that! How can I get out of this?

J. Slepian, Alter Ego

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Answers to Previous Essays

Electrostatic Space Ship. The following is the author's solution to a previous essay (*EE, Feb '50, p 164*).

Yes, Alter Ego, you are good, not as an inventor, perhaps, but for raising very interesting basic theoretical questions. You raise too many of these to discuss properly in this one essay, but perhaps we can get to them in the next few essays, dealing with electrostatics and magnetostatics which I expect you will write.

Your equation 1, Coulomb's law, $F=q_1q_2/r^2$, for charged small bodies in empty space, and your equation 2, Lorentz's force equation for a small charged body at rest in empty space, $\mathbf{F}=q\mathbf{E}$, serve to define the charge q on a small body

Will some reader contribute his half of the invention by getting rid of that other set of brushes?

P.S.: Just after writing the foregoing, I suddenly discovered how to make a commutatorless d-c machine, and completely. It is along a little different line, and I thought I had better get this postscript in before my readers spend too much time on the half-solution.

I start with an a-c machine, where a d-c winding on the rotor gives a succession of north and south poles. As the lines of force from a north pole cut the sides of a coil in the stator, it generates a certain voltage. However, the lines from the following south pole cut the coil sides and generate voltage in the opposite direction. Thus we get alternating current in the coil.

Now! After the north pole has finished cutting the coil-side and before the south pole has begun, let us reverse the direct current in the rotor. Then the following south pole changes to a north pole, and the voltage it generates by cutting the coil will have the same direction as that produced by the preceding pole. Continue this way, reversing the rotor current just after each pole has done its work on the stator coil. Then the voltage in the stator coil will always be in the same direction, and if a filter is interposed we should have top-quality direct current.

J. Slepian, Alter Ego

When the author was working on the test floor of the Westinghouse Electric Corporation many years ago, a young engineer independently made the second of the two inventions described in the foregoing, and persuaded his superior to authorize the building of a small model. The engineering report describing it, however, was entitled, "A New Method for Generating Double-Frequency Current."

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Answers to Previous Essays

Self-Running Electrostatic Motor. The following is the author's solution to a previous essay (*EE, Mar '50, p 247*).

You'll avoid getting into such fixes in the future, Alter Ego, if you'll refrain from talking about the force exerted by the electric field on charges or charged bodies placed within matter, as if the notion of such force had any uniquely verifiable meaning.

Coulomb's law and the Lorentz force equation have meaning for small enough charged bodies in empty space. For such bodies, **F**, q, **E**, and **B** may be defined operationally and uniquely. However, the definitions used for empty space cannot be carried over directly to within matter. We shall see in later essays how we may define **E**, **D**, **B**, and **H**, and even ρ, the charge density within material, but **F**, supposedly the direct action of the electric field on the charges introduced into the material, is another matter.

So far, in this discussion of this and the preceding electrical essay, "Electrostatic Space Ship," the vectors **F**, **E**, **B**, the scalar q, have meaning only for small enough bodies in empty space, and sufficiently far away from other material bodies. It is only in this domain of observable phenomena that Coulomb's law and the Lorentz force equation have verifiable meaning. If we wish to extend electromagnetic theory to the interior of extensive material bodies, new operationally meaningful definitions of the field vectors, and charge and current density must be given since the definitions given for particles or sufficiently small bodies are clearly without verifiable meaning inside of extensive matter. It is possible to give such generally uniquely meaningful definitions of the electromagnetic field and charge and current densities, and the author hopes that in future essays such definitions will be presented. However, in the author's opinion, it is not possible to define in any significantly unique way the ponderomotive force density, and again the author hopes that in future essays this will be brought out.

Going from small-charged-body electromagnetism to that for extended material bodies is similar in some ways to going from Newtonian particle dynamics to rigid body or continuous medium mechanics. In this last case Newton's laws for particles are not sufficient. We must invoke new and independent principles, such as the principle of virtual work, or D'Alembert's principle.

Similarly, in the electrical case, to systematize or put in order the electromechanical effects for extended bodies we need some additional principle over and above Coulomb's law and the Lorentz force equation. Most investigators have chosen the principle of conservation of energy for this purpose. They assume the existence of an energy function which, for the particular electromechanical system, is a function of the electrical and mechanical parameters which specify the state of the system, and which the investigator may vary at will. The change in value of this energy function as the parameters are varied is set equal to the work done by the investigator in changing the parameters, and thus an expression for the force appropriate to any particular independently variable parameter may be obtained.

Certainly no objection can be raised to this procedure. If we apply it to Alter Ego's machine, it would appear that whatever the energy function may be, it must be independent of the position of the belts along their pulleys, since shifting the belts along themselves appears in no way to change the electrical or mechanical configuration. Hence it appears that the force tending to move the belts is zero. However, many investigators have gone further. They assume (unjustifiedly) that the total energy of the system can be resolved in some significantly unique way into the sum of a purely electrical part and a purely mechanical part. Then assuming that the system changes continuously from one state to another, with great mathematical skill, they transform the rate of change of the hypothetically unique electrical part of the energy into an integral over the moving parts of the system of some vector expression multiplied scalarly into the vector velocity of the material at each point.

There is no evidence that this transformation of the rate of change of the electrical part of the energy, dU_e/dt, into an integral of the form

$$\frac{dU_e}{dt} = \int\int\int \mathbf{f}_e \mathbf{v}\, d\tau \qquad (1)$$

is unique. In fact, we must expect the contrary generally to be the case. Nevertheless, the particular factor $\mathbf{f}_e$ which the particular investigator uses to multiply $\mathbf{v}$ in equation 1 is then called by him the electric ponderomotive force density. Presumably it will be equilibrated by a mechanical force density arising in a similar way from the presumably uniquely determinable mechanical part of the total energy. An example of $\mathbf{f}_e$ so calculated was given in last month's discussion (*EE, Mar '50, pp 247–9*) of the preceding essay (*EE, Feb '50, p 164*).

Generally it consists of a term, $\rho\mathbf{E}$, plus more complicated expressions deriving from the so-called electrical part of the energy. Alter Ego knew only of the $\rho\mathbf{E}$ term, and believing that the only effect of the inerteen oil would be to reduce the intensity of the field $\mathbf{E}$ which would otherwise have been produced in air by the charges on the belts, came to the conclusion that there would be a net force tending to move the belts.

If Alter Ego had studied his books further, he would have discovered that they assert that there will also be "ponderomotive forces" on the oil, such as those given by the second and third terms of equation 4 in the discussion of the preceding essay. Taking into account these forces, Alter Ego would have discovered that he could not expect the oil to lie level as he shows in his Figure 1, but that it would heap up where the belts enter the oil surface. Also, Alter Ego would have been puzzled by the third term, $\left(\frac{1}{8\pi}\text{grad } E^2\frac{dk}{d\sigma}\sigma\right)$, of equation 14 in the discussion of the preceding essay, which contains the coefficient of variation of the dielectric constant k with density σ, in spite of the fact that in his problem, the oil is essentially incompressible. If he were a sufficiently good mathematician to take into account this force, he might be surprised to discover that no matter what value he took for the coefficient, $dk/d\sigma$, he would come out with the same configuration of the oil, since the term in question would give surface forces on the oil which would just balance the volume forces arising from this term. He might then wonder whether this so agreeably self-cancelling ponderomotive force had any unique validity or reality.

He might observe that, after all, these forces were all calculated by using the energy principle with an arbitrarily assumed division of the energy into an electrical and mechanical part, and that even with this arbitrary division, the forces were not determined uniquely by the mathematics. He might conclude then, that since these uncertainly determined ponderomotive forces were calculated from the principle of energy, he might just as well use this principle to solve his problem directly. For his electrostatic motor, as Alter Ego realized, with zero continuous current flowing, the principle of energy would predict that belts would not move. Any detailed calculation with "ponderomotive forces" also calculated from the energy principle could give no further information and must of course lead to the same conclusion.

Now for the patient reader who has come this far, the author has a surprise. The author believes that with any actual insulating oil, including Westinghouse inerteen, the electrostatic motor of Alter Ego *will run*, provided the friction at the pulleys and the stiffness of the belts is sufficiently reduced. No oil is prefectly insulating, and therefore a small continuous current will flow to the moving belt by way of the corona points, to replace the belt charge lost by electrical leakage or conductance of the oil. Hence there will be a continuous electric energy input into the system, and the principle of energy cannot be used to deny the possibility of a mechanical energy output from the moving belts.

In the language of "ponderomotive forces" the "attraction" of the oil for the charges entering will be less than the "attraction" for the charges leaving because the charges leaving will be reduced in amount by the electrical leakage of the oil. Any one who has watched oil being tested in the usual oil test cup knows that there is considerable continuing motion of the oil at quite a bit less than the breakdown voltage. The possible operativeness of Alter Ego's electrostatic motor is daily being demonstrated in testing laboratories.

There is another way of arriving at "ponderomotive forces" within material, but still not uniquely, namely through Maxwell's electromagnetic stress tensor. The author hopes to discuss this in some later essay.

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Polyphase Windings. The following is the author's answer to a previous essay (*EE, Feb '50, p 165*).

Figure 1 shows how the stator winding was rearranged. As three slots had to be left empty, I took six slots at 60 degrees around the circle, and left disconnected one copper bar in each, these being the top bar in slots *100*, *200*, *300*, and the bottom bar in slots *50*, *150*, *250*. The phase groups were then arranged as shown in Figure 1.

It will be seen that for the successive phase-pole groups I took five top and six bottom, and six top and five bottom conductors. The coil span which was 1 to 16 in the original winding was now 1 to 17 on 6, and 1 to 18 on 12 of the 18 poles of the reconnected stator. This caused no mechanical difficulty; it was only a matter of opening the ends of the bars a little more to reach over the slightly wider span. Though physically unsymmetrical, it resulted in an electrically symmetrical winding.

Each phase has two parallel paths. For phase A, for example, path A, in this arrangement, would go around the circle six times and path A' only five times, but if we come out at the back half way through the sixth round with top conductor in slot *139*, we have been $5^1/_2$ times around and have half of the phase conductors in series. We now take path A' five times around, coming out in front with the top conductor of slot *22* and take it across and pass it under the frame to join bottom conductor of slot *156* and complete our $5^1/_2$ times around the stator, with the half round left over from the sixth round of path A. Thus we have two equal paths in parallel as required. The extra resistance introduced by the interconnecting cable is negligible and balanced by an equal cable which brings the end of path A from top to bottom of the frame where all leads come out to the terminal box. These cables passing under the stator frame from front to back are bunched together for the three