Electrical Essays

Force on a Dielectric, or Physicists Are Just as Confusing

I had to go back to the Research Laboratory for some tools which I had left there the time I did that repair job on the Van de Graaff Atom Smasher. I found two physicists there, running the machine. "Here's a chance to get some expert opinion on my Electrostatic Space Ship (*EE, Feb '50, p 164*)," said I to myself, and I showed the drawing, Figure 1, to them. They both laughed heartily, and after they recovered told me I had failed to consider the force on the dielectric titanate.

This was a new idea to me, so I asked how much would that be, and would that spoil my invention? How could I figure it?

"Well," said one of them, "like you thought, there'll be no real or free charge on the titanate surface because $D_1=D_2$, but there will be an apparent surface charge $\sigma=\frac{1}{4\pi}(E_2-E_1)$ and there will be a force on account of that." "You mean an *apparent* force?" said I. "No," said he, "I mean a real force." "A *real* force on an *apparent* charge," I echoed, quite dazed.

"Bill," said the other, "Alter Ego is right. You can't

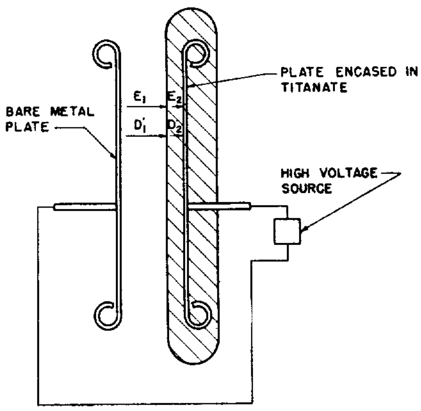

Figure 1. Electrostatic propellant for space ship

have a real force on an apparent charge. What you really have is a polarization, P, in the titanate due to the molecules being made into dipoles by the electric field. Within the titanate the field is uniform so the force on the dipoles is zero. But at the surface the field is strongly nonuniform, in the X direction, so there is then a real force on the real dipoles there."

"Don't be metaphysical, Charlie," said Bill, "Here, I'll figure the thing out. We have

$$D=E+4\pi P \qquad (1)$$

The real, volume charge density is

$$\rho=\frac{1}{4\pi}\frac{\delta D}{\delta x} \qquad (2)$$

The apparent or polarization charge density is as in the following:

$$\rho'=-\frac{\delta P}{\delta x}=-\frac{1}{4\pi}\frac{\delta}{\delta x}(D-E) \qquad (3)$$

The force per unit volume is then

$$f_B=[\rho+\rho']E=\left[\frac{1}{4\pi}\frac{\delta D}{\delta x}-\frac{1}{4\pi}\frac{\delta}{\delta x}(D-E)\right]E=\frac{1}{4\pi}\frac{\delta E}{\delta x}E=\frac{1}{8\pi}\frac{\delta}{\delta x}E^2 \qquad (4b)$$

"Integrating from just outside the titanate to just inside, we get the following for the force per unit area at that surface,

$$F_B'=\int_1^2 f_B dx=\frac{1}{8\pi}(E_2^2-E_1^2) \qquad (5b)$$

"Integrating from just outside the metal to just inside, we get for that surface

$$F_B''=\int_2^0 f_B dx=\frac{1}{8\pi}(0-E_2^2) \qquad (6b)$$

"So the whole force per unit area of structure is then found to be

$$F=F_B'+F_B''=-\frac{1}{8\pi}E_1^2 \qquad (7b)$$

which is just the negative of the force on Alter Ego's other metal plate, and so we can see that his invention won't work."

Charlie seemed impressed, but said, "Let's see how I do it with real polarization, P, but no force on the apparent charge, ρ'.

Instead of your equation $4b$ my equation for the force per unit volume will be

$$f_C=\rho E+P\frac{\delta E}{\delta x}=\frac{1}{4\pi}\frac{\delta D}{\delta x}E+\frac{1}{4\pi}(D-E)\frac{\delta E}{\delta x}=\frac{1}{8\pi}\frac{\delta}{\delta x}(2DE-E^2) \qquad (4c)$$

"My force per unit area at the first surface will be, therefore,

$$F_C'=\int_1^2 f_C dx=\frac{1}{8\pi}(2D_2E_2-E_2^2)-\frac{1}{8\pi}(2D_1E_1-E_1^2)$$

$$=\frac{1}{8\pi}(2D_2E_2-E_2^2-E_1^2) \qquad (5c)$$

"At the second surface I'll have

$$F_C''=\int_2^0 f_C dx=\frac{1}{8\pi}(0)-\frac{1}{8\pi}(2D_2E_2-E_2^2) \qquad (6c)$$

"The total force per unit area will be

$$F=F_C'+F_C''=-\frac{1}{8\pi}E_1^2 \qquad (7c)$$

"Your invention simply won't work, Alter Ego," Charlie and Bill both said, gleefully, with their arms around each other.

"Thank you, gentlemen," I said, glumly. "But I notice that Bill says that the force on the surface of the titanate tending to stretch the titanate is

$$F_B' = \frac{1}{8\pi}(E_2^2 - E_1^2) = -\frac{1}{8\pi}E_1^2\left(1 - \frac{1}{k}\right)^2 \qquad (\mathbf{5b^1})$$

where k is the dielectric constant, while Charlie here says that it is

$$F_C' = \frac{1}{8\pi}(2D_2E_2 - E_2^2 - E_1^2) = -\frac{1}{8\pi}E_1^2\left(1 - \frac{1}{k^2}\right) \qquad (\mathbf{5c^1})$$

Now how much will the titanate actually be stretched? Until you gentlemen agree on that, I'll have to reserve my acceptance of your conclusion that my invention won't work." I stalked out, but I could hear them arguing violently with each other as to how much the titanate will be stretched.

Physicists are just as confusing as engineers.

How much will the tension on the titanate arising from the electric field be? Is Bill right, or is Charlie right?

J. Slepian, Alter Ego

It is interesting to examine a few textbooks or treatises on electromagnetism to see whether they agree with Bill or Charlie as to the force per unit volume on a dielectric in an electrostatic field.

Page and Adams, "Principles of Electricity," tenth printing, pages 48–9, give for an uncharged dielectric, $f = P\frac{\delta E}{\delta x}$ and agree with Charlie.

Abraham and Foeppl, "Theorie der Elektrizitaet," fourth edition, volume 1, page 48, give a formula, equation 16–1, which reduces for this one dimensional case to $f = \frac{1}{8\pi}\frac{\delta}{\delta x}(DE)$ and they agree with neither Bill nor Charlie. Abraham and Becker, "Electricity and Magnetism," page 95, for the special case of a material with an isotropic dielectric constant, k, which is independent of field strength and is a function of material density only, seem to find it necessary to add a term to the foregoing thus obtaining the formula $f = \frac{1}{8\pi}\frac{\delta}{\delta x}\left(DE + E^2\frac{\delta k}{\delta \sigma}\sigma\right)$ where σ is the density.

Stratton, "Electromagnetic Theory," and Jeans, "Mathematical Theory of Electricity and Magnetism," agree with Abraham and Becker for this special case.

Doesn't anybody agree with Bill? Yes, Richardson, "The Electron Theory of Matter," in equation 2, page 206, gives a formula which we find agrees with Bill's formula, $f = \frac{1}{8\pi}\frac{\delta}{\delta x}E^2$.

All these various formulations, however, will agree that Alter Ego's invention will not work. This is because for all of them the volume force is given as the gradient of a function, and for all of them, this function reduces to $\frac{1}{8\pi}E^2$ when $D = E$. However, they disagree as to what the "tension" in the titanate will be.

So again, how much "tension" will the electric field exert upon the titanate?

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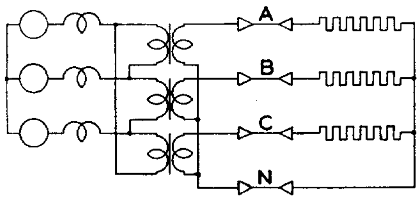
Unbalanced Load

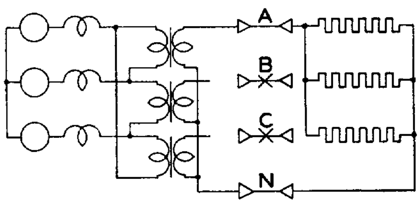
A balanced 3-phase star-connected resistance load is supplied from an efficient distribution system through a delta-star-connected transformer bank. The resistance in the supply system may be neglected. The resistance load is connected to the transformer bank by four short

Figure 1



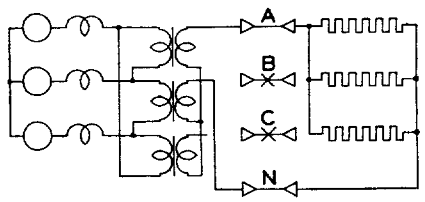
underground cables as indicated in Figure 1. The voltage drops 5.13 per cent when the load is connected to the transformer bank. A drill is accidentally driven through phase cables B and C and serious damage to these cables results from the short circuit. In attempting to maintain operations while replacement cables are being rushed to the job, the plant electrician connects the three resistors to the remaining sound phase cable as shown in Figure 2

Figure 2



and prepares to pour water on the transformer cases. However, he notes that the resistors develop only two-thirds the heat obtained in 3-phase operation. In attempting to increase the heating of the resistors, the electrician changes the connection of the neutral cable from the neutral connection of the transformer to phase B terminal as shown in Figure 3. He is disappointed to find, however,

Figure 3



that the heaters are still developing only two-thirds the heat obtained in 3-phase operation.

The electrician wants to know the answers to the following questions: What are the system characteristics? What else can he do to obtain at least 85 per cent normal heat in his equipment?

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where the subscript 1 refers to the fields normal to the ends outside the end surface, and the subscript 2 refers to the normal fields inside the end surface.

I had an idea. I would show Bill and Charlie that I had no hard feelings, and also show them how to find out which is the right tension, T_B or T_C. I picked up an iron cylinder and a hacksaw, and at lunchtime I rushed over to the Research Laboratory cafeteria, and pushed over to the table beside Bill and Charlie.

Well, it was like I said. Bill claimed T_B, and Charlie T_C. "Now," I said, "I'll settle it. There is a magnet up here, I am told, a cyclotron or something, so big that I can stand between the poles. I'll get in there and place this iron cylinder in the middle parallel to the field. The net force on it will be zero, but there will be a pull on each end trying to pull the ends apart. Then with my hacksaw I'll cut the cylinder across the middle, and measure how much force it takes to keep the two parts from being pulled away from each other by the pull on the ends."

"That's no good!" said Bill, "as soon as you make that saw cut new apparent charges, ρ_m', will appear there, and the force on that new apparent charge will keep the two parts from pulling apart."

"You're right, it's no good," said Charlie, "but that's because at the new surfaces you produce by the cut, there will be a discontinuity in **H**, and the magnetic dipoles there will now have a pull on them which will keep the two parts from separating."

The fellow next to us began to have a violent coughing spell, choking over his soup. His name was Jack, and I could tell by his looks he was a physicist. "You are all wrong, all of you!" he spluttered. "There is no such thing as magnetic charge, so you can't have unipoles, and if you can't have unipoles, you can't have dipoles. Everybody knows that magnetism in a body is due to little circulating electric currents, the Amperian currents, I. Their density is given by $\mathbf{I}=\text{curl }\mathbf{M}=1/4\pi \text{ curl }\mathbf{B}$, since $\text{curl }\mathbf{H}=0$. Inside the cylinder these Amperian currents will overlap and cancel out giving $\mathbf{I}=0$ as you can see since $\text{curl }\mathbf{B}=0$ there. But at the cylindrical surface the parallel component of B has a discontinuity. The Amperian currents don't cancel out there but form a surface current sheet of magnitude $I_s=-\frac{1}{4\pi}(B_1-B_2)$, as shown in Figure 1$A$. The mean field this current sheet is in is $\frac{1}{2}(B_1+B_2)$. Therefore there will be a radial force or tension on the side walls given by

$$T_J=-\frac{1}{4\pi}(B_1-B_2)\frac{1}{2}(B_1+B_2)=-\frac{1}{8\pi}(B_1^2-B_2^2) \qquad (3J)$$

per square centimeter, tending to swell out the iron cylinder radially."

"Well," said I, "here's a force trying to pull the cylinder apart sideways. Then I'll cut the cylinder in two parts lengthwise." "No good," said Jack, "as soon as you make the cut there will be the same Amperian surface current density, giving the same normal tension force T_J, equation 3J, on the new surfaces. (See Figure 1B.) The two parts will still be in equilibrium and will not pull apart."

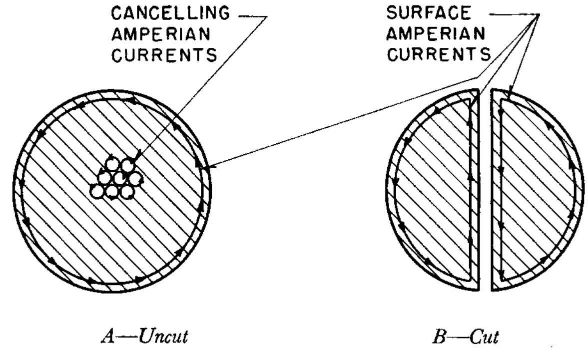

Figure 1. Amperian currents in iron cylinder

Well, lunch hour was over, and I rushed back to my job, leaving the three of them arguing at a great rate. Are there as many opinions as there are physicists? Is there an axial tension, T_B, or T_C, or is there a radial tension, T_J? Is a magnetized iron cylinder stretched or swelled? And what kind of forces are these that I can't measure by an experiment?

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Answers to Previous Essays

Force on a Dielectric. The following is the author's answer to a previously published essay of the foregoing title (*EE, May '50, pp 456–7*).

By this time, perhaps, the reader is prepared to believe that no uniquely valid meaning can be given to the notion of the force per unit volume within material bodies, which arises from the action of the local electromagnetic field there. No operational definition of such volume force can be given.

The expectation that such a uniquely definable and observable volume force exists arose during the period when scientists believed that the electrical properties of bodies could be accounted for by assuming that they were made up of charged microscopic particles, which obeyed Newtonian mechanics, and which had acting upon them the Lorentz forces $\mathbf{F}=q(\mathbf{E}+[v\times B])$, and also other nonelectric or mechanical forces which equilibrated the Lorentz forces. However, this expectation of the scientists has failed. In our present-day theories of the structure of matter, the microscopic particles, our electrons and nuclei, do not obey Newtonian mechanics, but quite another, that is, quantum mechanics. Also, the nonelectric, or mechanical, forces on the particles have completely disappeared, at least when we do not deal with energetic nuclear interactions. Hence, there is no theoretical basis for speaking of a volume force in matter arising from the macroscopic Maxwellian electromagnetic field.

However, for macroscopic bodies, we do have Newtonian mechanics, and we are able to determine the mechanical forces exerted upon such bodies by noting the strains in abutting bodies, such as twisted strings or compressed push rods, or the remoter presence of gravitational matter. In fact, the definitions of the electric and magnetic field and electric charge depend upon our capacity to observe and recognize these "purely" mechanical forces.

It has meaning, therefore, to speak of the total force arising from the electromagnetic field acting on a macroscopic or ordinary body which is separated from all other bodies except those which we recognize as purely mechanical in their action, such as pull strings or push rods. The total electric force is then determined by the amount by which the mechanical forces fail to balance according to Newtonian mechanics.

Similarly, we may speak meaningfully of the total electric torque on a macroscopic body, which is separated from all other bodies influencing the electromagnetic field.

Whatever expressions or formulas we may use for the electric force density or torque density in matter, we should expect or require them to integrate to the same total force or torque, namely, the one we observe by mechanical means, for any body in empty space. We see that the various electric force densities used by Bill, Charlie, and the other even more eminent physicists, all integrate to the same total value when applied to the plate structure of Alter Ego's invention. Apparently they are all equally valid in agreeing with what can be observed for total bodies.

Maxwell has given a general method for determining the total electric force on a body by integrating a certain tensor function of the electric and magnetic fields over a surface lying in empty space and enclosing that body. In a later essay the author will show how from Maxwell's tensor, infinitely many valid electric force density formulas may be deduced.

Returning to the essay, our conclusion is that so far as observable phenomena go, both Bill and Charlie are right.

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Unbalanced Load. The following is the author's solution to a previously published essay of the foregoing title (*EE, May '50, p 457*).

In selecting a base for the analysis of the problem, the author notes the voltage drop taking place when the load is connected to the system. From this he deducts

$$\frac{IR}{E}=\frac{R}{Z}=\cos\theta=0.9487=\frac{3}{\sqrt{10}}=3\sin\theta \qquad (1)$$

The author selects the following base:

$E=10$
$I=\sqrt{10}$
$Z=\sqrt{10}$

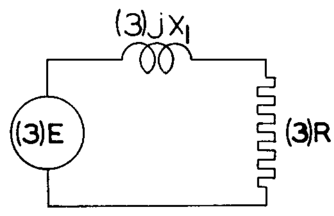
and derives the following:

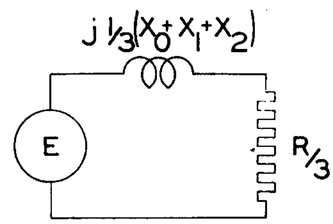
$R=3$
$X_1=1$
$P=3I^2R=90$

An equivalent single-phase circuit is shown in Figure 1.

The electrician reported two-thirds normal power developed in the parallel connection of three resistors and the author notes this as follows:

$P'=60$
$R'=1$
$I'=\sqrt{60}$





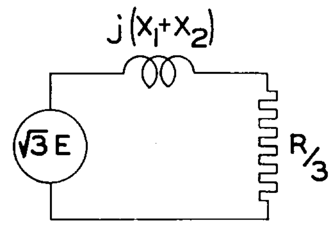
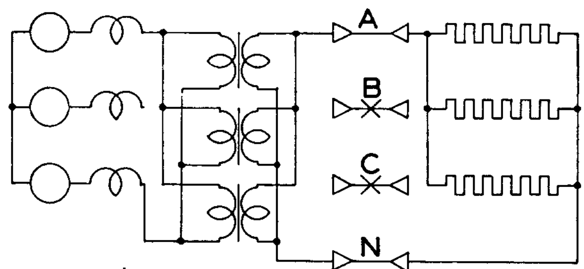


Figure 1 (above, left)

Figure 2 (above, right)

Figure 3 (left)



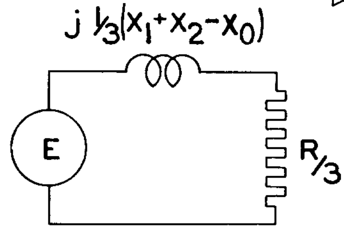


Figure 4

The method of symmetrical components gives the following two formulas for calculating currents in single-phase load connected from line to neutral and from line to line on a 3-phase system:

$$I'=\frac{3E}{\sqrt{(3R')^2+(X_0+X_1+X_2)^2}} \qquad (2)$$

$$I'=\frac{\sqrt{3}E}{\sqrt{(R')^2+(X_1+X_2)^2}} \qquad (3)$$

The equivalent circuits of Figures 2 and 3 corresponding to connections shown in figures of the same number in the essay illustrate the foregoing formulas.

The only unknown in equation 3 is the negative sequence reactance X_2, which can, therefore, be evaluated. Following this, the only remaining unknown in equation 2 is the zero sequence reactance X_0. The two reactances then are

$X_2=1.00$
$X_0=0.45$

Since transformer connections are delta-star, the zero sequence reactance is that of the transformer and the reactance of the system is obtained by subtraction from the positive sequence reactance.

In reply to the electrician's second question, the author suggests the connections of Figure 4, which will produce 87.7 per cent normal heat in the resistors.

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