mutual impedances, the impedances can be resolved into symmetrical components although the impedances are complex numbers, and not rotating vectors as are 3-phase voltages and currents. Their sequence components will not be the same for different reference phases of current, and there will be mutual coupling between sequences. This means, of course, a larger number of design constants that appear in equations relating voltages and currents. For example, the circuit of Figure 3 relates the symmetrical components of voltage drops and currents by the following equations:

(*a*). For phase *a:*

$E_0=1.154\, I_0 \angle 300^\circ+0.577\, I_1 \angle 120^\circ+0.577\, I_2 \angle 120^\circ$

$E_1=0.577\, I_0 \angle 120^\circ+0.765\, I_1 \angle 41^\circ+0.577\, I_2 \angle 120^\circ$

$E_2=0.577\, I_0 \angle 120^\circ+0.577\, I_1 \angle 120^\circ+0.765\, I_2 \angle 41^\circ$

(*b*). For phase *b:*

$E_0=1.154\, I_0 \angle 300^\circ+0.577\, I_1 \angle 240^\circ+0.577\, I_2$

$E_1=0.577\, I_0 +0.765\, I_1 \angle 41^\circ+0.577\, I_2 \angle 240^\circ$

$E_2=0.577\, I_0 \angle 240^\circ+0.577\, I_1 +0.765\, I_2 \angle 41^\circ$

(*c*). For phase *c:*

$E_0=1.154\, I_0 \angle 300^\circ+0.577\, I_1 +0.577\, I_2 \angle 240^\circ$

$E_1=0.577\, I_0 \angle 240^\circ+0.765\, I_1 \angle 41^\circ +0.577\, I_2$

$E_2=0.577\, I_0 +0.577\, I_1 \angle 240^\circ+0.765\, I_2 \angle 41^\circ$

It will be found that this circuit has the property of drawing the same current through the switches and ammeters when any two or all four switches are closed.

The question is: Can a linear static circuit be designed that will draw the same current through the ammeters when any two, any three, or all four switches are closed?

A. A. KRONEBERG (F '48)
(Southern California Edison Company, Los Angeles, Calif.)

Answers to Previous Essays

Delta Network. The following is the author's answer to his previously published essay of the foregoing title (*EE, June '50, p 550*).

All three statements are true.

1. The sum of the three delta impedances is zero, and the three star-connected impedances must be real and infinite. Z_a is negative, while Z_b and Z_c are positive. Such a circuit is an obvious impossibility.

2. When the circuit is connected to a source of balanced 3-phase voltage, the following values of voltage and current are readily computed:

$E_a=1 \angle 90^\circ;\ E_b=1 \angle 330^\circ;\ E_c=1 \angle 210^\circ$

$E_{ab}=E_a-E_b$:

$E_{ab}=\sqrt{3} \angle 120^\circ;\ E_{bc}=\sqrt{3} \angle 0^\circ;\ E_{ca}=\sqrt{3} \angle 240^\circ$

$I_{ab}=E_{ab}/Z_{ab}$:

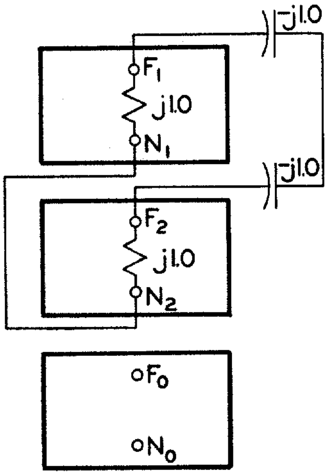

Figure 1

$I_{ab}=\sqrt{1/3} \angle 30^\circ;\ I_{bc}=\sqrt{1/12} \angle 90^\circ;\ I_{ca}=\sqrt{1/3} \angle 150^\circ$

$I_a=I_{ab}-I_{ca}$:

$I_a=1;\ I_b=-1/2;\ I_c=-1/2$

When one of the line connections is broken, the circuit is found by inspection to be a tuned parallel *LC* circuit with respect to the other two lines which presents an infinite impedance to single-phase voltages impressed on any one pair of its terminals.

3. The capacitor $-j6$ can be replaced by a capacitor $-j2$ and a reactor $j3$ connected in parallel with it. The network then can be represented by three reactors $j1$ connected in star, and a capacitor $-j2$ connected to terminals *b* and *c*. The star-connected reactors will appear in the sequence networks of the system as shunts $j1$ across the positive and negative sequence networks. The capacitor $-j2$ is connected from line to line and will appear as two capacitors $-j1$ connected in series between the F_1 and F_2 terminals of the networks. Terminals N_1 and N_2 are connected to each other. This connection is illustrated in Figure 1.

A. A. KRONEBERG (F '48)
(Southern California Edison Company, Los Angeles, Calif.)

Magnetized Iron, Stretched or Swelled? The following is the author's answer to a previously published essay of the foregoing title (*EE, June '50, pp 550–1*).

You are getting a bit tiresome, Alter Ego, with your concern about the ponderomotive forces per unit volume inside matter due to electric or magnetic fields. By this time your readers are quite well aware that there is no uniquely valid or observable electrical or magnetic ponderomotive force within matter. They know that physicists will give different values for their ponderomotive force per unit volume with the related surface force, but that they won't let you observe their individual ponderomotive force densities by any experiment, because for a complete body surrounded by empty space, they will all integrate their forces up to give the same actually observable total force.

However, your idea about making cuts with a hacksaw is a good one because it gives me a chance to talk about Maxwell's stress tensor, and also show you how to invent as many new and all equally valid force densities as there are physicists, with enough left over to take care of the generations of physicists to come.

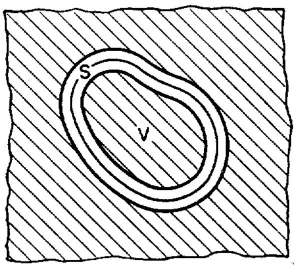

Figure 1. Inside matter

I shall limit my remarks to the case of the action of steady electric fields acting upon perfectly insulating bodies or dielectrics, at rest. The ideas can, however, be carried out in the more general case. I shall also, for economy of words, neglect gravitational action.

Consider a body at rest in empty space. For such a body Maxwell has given a method for determining from a knowledge of the electric field outside the body the total electric force, and that is the honest-to-goodness mechanical force which must be applied to keep it at rest.

We consider a surface enclosing the body. Then the total electric force $\mathbf{F}$, acting on the body, and which balances the mechanical force, is given by the following integral over the surface,

$$\mathbf{F} = \frac{1}{8\pi}\int\int (2\,\mathbf{E}\,\mathbf{E}\cdot d\mathbf{S} - E^2 d\mathbf{S}) \quad (4)$$

The x, y, and z components of F are

$$F_x = \frac{1}{8\pi}\int\int (E_x^2 - E_y^2 - E_z^2)dS_x + 2E_xE_ydS_y + 2E_xE_zdS_z \quad (5)$$

$$F_y = \frac{1}{8\pi}\int\int 2E_xE_ydS_x + (E_y^2 - E_x^2 - E_z^2)dS_y + 2E_yE_zdS_z \quad (6)$$

$$F_z = \frac{1}{8\pi}\int\int +2E_xE_zdS_x + 2E_yE_zdS_y + (E_z^2 - E_x^2 - E_y^2)dS_z \quad (7)$$

The coefficients of dS_x, dS_y, and dS_z in equations 5 are called the components of Maxwell's stress tensor.

We may apply Gauss's theorem to any one of the foregoing surface integrals. For example we might take that for F_x but start with the more convenient form derived from equation 4, with $\mathbf{i}$ the unit vector in the x direction

$$F_x = \frac{1}{8\pi}\int\int 2E_x\mathbf{E}.\,d\mathbf{S} - E^2\mathbf{i}.\,d\mathbf{S}$$
$$= \frac{1}{8\pi}\int\int\int \text{div}\,(2E_x\mathbf{E} - E^2\mathbf{i})dT \quad (8)$$

We may then interpret the integrand in the volume integral as the x component of a force per unit volume, since it integrates up to the correct total F_x. In this case the integrand

$$\frac{1}{8\pi}\,\text{div}\,(2E_x\mathbf{E} - E^2\mathbf{i}) = \frac{1}{4\pi}E_x\,\text{div}\,\mathbf{E} = [f_B]_x \quad (9)$$

That is, we get the electric volume force of Bill, (*EE, May '50, pp 456–7*). It gives zero volume force in empty space as it should, and Bill's surface forces on surfaces where $\mathbf{E}_n$ is discontinuous.

But now in applying Gauss's theorem, we do not need to use precisely the tensor suggested in the equations 5, 6, and 7. We may use any nine functions we may please which can serve as components of a tensor, provided these nine functions in empty space, that is where $\mathbf{D}=\mathbf{E}$, reduce to the tensor components of Maxwell for empty space, that is, the nine functions given in equations 5, 6, and 7. For example we may use

$$F = \frac{1}{8\pi}\int\int 2\mathbf{E}\mathbf{D}\cdot d\mathbf{S} - E^2 d\mathbf{S} \quad (4')$$

which gives

$$F_x = \frac{1}{8\pi}\int\int (2E_xD_x - E_x^2 - E_y^2 - E_z^2)dS_x + 2E_xD_ydS_y + 2E_xD_zdS_z \quad (5')$$

and so forth.

It is clear that equation 4′ reduces to equation 4 when applied over a surface lying in empty space, so equations 4′ and 4 must agree as to what the force is on the body which the surface encloses. However, if we apply Gauss's theorem to equation 5′, and so forth, we get with a little mathematical manipulation for the volume force, $\mathbf{f} = \rho\mathbf{E} + \mathbf{P}\cdot\nabla\mathbf{E}$ which is Charlie's formula. Thus Charlie will always agree with Bill as to what the total force is on a body surrounded by empty space. Suppose now we take

$$\mathbf{F} = \frac{1}{8\pi}\int\int 2\mathbf{D}\mathbf{D}\cdot d\mathbf{S} - \mathbf{D}^2 d\mathbf{S} \quad (4'')$$

which also agrees with equation 4 for any surface in empty space.

Again applying Gauss's theorem, and with a little mathematical manipulation, we get for the volume force, $\mathbf{f} = \frac{1}{4\pi}\mathbf{D}\,\text{div}\,\mathbf{D} - \frac{1}{4\pi}[\mathbf{D}\times\text{curl}\,\mathbf{D}]$, which, when translated over to the magnetic case, gives the formula of Jack. Hence Jack also will agree with Bill and Charlie as to the total force on a body in empty space, even though he disagrees as to the force per unit volume within the body.

Equations 4′ and 4″ are examples of expressions which define tensors which reduce to Maxwell's tensor in empty space. It should be clear how to write as many others as one may wish.

Now how about the force on a volume which is completely inside matter? Well, what can we do other than what Alter Ego suggested? How else can we tell whether a given formula for the force is correct except to apply Alter Ego's hacksaw test and cut the volume away from the rest of the material so that the force can be observed. But after we do that, the volume V lies in empty space (Figure 1), a thin shell perhaps, but nevertheless an empty space in which we may place an enclosing surface S and apply Maxwell's stress tensor, equation 4. Then as before, instead of Maxwell's stress tensor we may use any of the infinitely many tensors which reduce to Maxwell's in empty space, and still correctly calculate the force of the cutaway volume.

We conclude then that there is no uniquely valid or correct formula for the electric force on a volume inside matter. Infinitely many different formulas may be devised which will all meet the test of any experiment successfully.

J. SLEPIAN (F '27)
(Westinghouse Research Laboratories, East Pittsburgh, Pa.)