Polarization or Charge in a Dielectric?

A certain kind of wax is melted in an insulating tray, on the bottom of which is placed a flat electrode. Another flat electrode is laid on the upper surface of the melt, and a potential difference is applied to the electrodes. The wax is then allowed to cool until it is solid, the potential difference being maintained all the while.

On removing the solid wax it is found to be permanently electrified. A steady electric field is found to exist around it, as determined by the force observed on a charged small body or probe placed in the empty space around the wax.

It may be that when molten the wax had a slight electrical conductivity, and not necessarily a uniform one, so that opposite charges were conducted towards the two electrodes respectively. On cooling, the conductivity became zero, and the now perfectly insulating wax had frozen into it true or real volume charge densities, and it is these real charges which produce the external field.

Or it may be that the wax had a polarizability when molten, perhaps due to molecular dipoles which were free to rotate and line up in the field in the melt. On cooling, these dipoles lost their freedom to turn, and thus were frozen into their lined-up condition, leaving the wax permanently polarized, acting like the electrical equivalent of a permanent magnet. The charge then would be apparent and not real, and given by

$$\rho' = -\text{div } \mathbf{P}$$

where **P** is the polarization density, or the sum of the molecular dipole moments per unit volume, as Bill explained in a previous essay (*EE, May '50, p 456*).

How may one determine which of these views is correct?

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A Paradox?

Consider the case of a 100-kva 3-phase core-type 12-kv/-208–120-volt delta-Y platform-mounted transformer supplying a semirural area, and protected by fuses on the high- and low-voltage sides as in Figure 1.

Following confused complaints received from consumers, a trouble-shooter—whose previous experience, unfortunately for him, has been limited to single-phase systems—goes to the transformer to set things right. He fails to note that one high-voltage fuse has blown, and proceeds to check the outgoing low-voltage fuse with a lamp. He finds, by checking to neutral, that phase *A* seems all right, that phase *B* appears to be out, and that phase *C* is alive but that the voltage there seems low. Leaving his lamp connected between *C* and neutral, he removes the fuse from phase *B*, intending to replace it. The lamp immediately goes out, but relights as soon as the supposedly faulty phase-*B* fuse is replaced. Thinking that leads may be crossed, he removes phase-*C* fuse only to find that the lamp still goes out. Should the spirit of inquiry outweigh his sense of duty, he may connect his lamp between an active and neutral on the "live" side of the fuses. Those who have done this know the *dreadful* things that happen when the low-voltage fuses are removed and replaced!

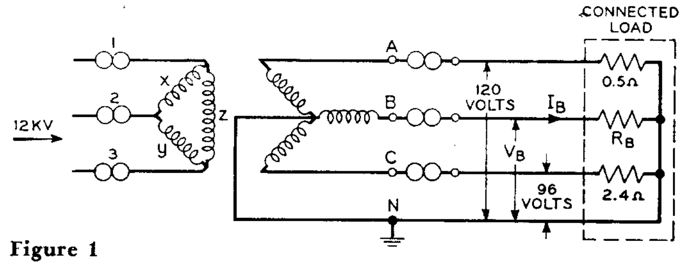

Figure 1

While the unhappy man is working feverishly, unfettered and unaided by theory or practice, let us examine the problem. Before he arrived, the connected load was purely resistive as shown in Figure 1. Assuming an ideal transformer:

1. Which high-voltage fuse was blown?
2. What are the values of V_B, I_B, and R_B?

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Answer to Previous Essay

Force on a Cut Iron Cylinder. The following is the author's answer to his previously published essay (*EE, Jul '50, p 635*).

J. H. Gittings and A. E. Stitely have made a direct test in East Pittsburgh of the configuration shown in Figure 1*C*, of the essay and reported the results to me. They made cuts in a 5/16-inch-diameter iron rod, and mounted the parts in a slightly larger diameter brass hollow cylinder, so that the iron parts, though compelled to stay on the same vertical axis, could slide up and down vertically freely. The middle two parts rested on each other and on the lower part of the iron cylinder, the upper part set so that the horizontal cut under it made a gap of 0.015 inch.

On turning on a vertical magnetic field of 1,000 gauss or so, the top middle part jumped up, closing the horizontal air gap, and opening up the diagonal 45-degree gap.

Bill tells me that this result confirms his formula for the force on the apparent charge (*EE, June '50, pp 551–2*). Charlie, however, claims that this experiment is a great victory for his different formula based on the action of magnetic dipoles in a nonuniform magnetic field. Jack said that this experiment shows very clearly the correctness of his still different formula based on the action of magnetic fields on amperian currents.

If the reader will look up my discussion of Maxwell's stress integral (*EE, Nov '49, pp 985–7*), he will discover how to make up a few formulas of his own, which will be confirmed by the experiment of Gittings and Stitely.

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"bucking" voltage is then applied at the control grid until the resulting grid voltage during normal set operation is approximately minus nine volts. The plate current of the triode section is effectively cut off when operating in this condition. The plate of the triode section is connected to the *3NP4* grid through a 22,000-ohm resistor. The triode plate voltage and the *3NP4* grid voltage are derived through a high-impedance circuit (one megohm). A failure of either the horizontal or vertical sweeps will allow the triode grid to approach zero bias. The triode section will conduct and the *3NP4* grid voltage will be reduced to a point near zero. This effectively increases the *3NP4* bias beyond cutoff, thus reducing the possibility of sweep burns.

Function Switch. A 2-section range switch is used. It switches video, deflection coils, alternating current, picture-tube bias, and direct-view cathode-ray tube grid, cathode, and heater. In the case of video, it merely connects the video output of the set to the direct-view tube or the input of the Duo-Vue amplifier. The switch is physically positioned at the center and extreme rear of the unit, which places it immediately under the direct-view tube socket in most receiver designs. This position is selected to reduce lead capacity. In "direct-view" position the direct-view tube is connected normally. In "projection" position, the direct-view heater, cathode, and grid are opened, and bias voltage from the selenium rectifier is applied to the grid and cathode to cut off residual spot emission. When switching from "projection" to "direct-view," the *3NP4* tube is biased beyond cutoff by simply grounding its grid. The deflection energy is transferred from the direct-view to the Protelgram deflection coils by short-circuiting-type switches. This insures ample load on the receiver output transformers at all points of the switching sequence. The function switch assembly is designed to be the central connector for all units. Two cables are installed in the television receiver and fitted with connectors which, in turn, are mated to the switch assembly. The Duo-Vue chassis and projection deflection yoke are plug connected to the switch.

The unit contains front panel brightness and focus knobs and auxiliary brightness and focus controls. This method of design is selected to make the front-panel controls of the vernier type and simple to operate. Height and width controls for the projected picture are incorporated as service controls. These controls are of the attenuator type. This is made possible by the excellent deflection sensitivity of the Protelgram system. More than adequate sweep is obtained from any 10-, $12^1/_2$, or 16-inch picture-tube set. The height control is a simple resistive shunt across the vertical deflection coils. The width control consists of a permeability tuned coil in series with the horizontal deflection coils. The inductance is damped by a resistor to prevent ringing.

Sockets are provided on the Duo-Vue chassis for easy connection of the focus coil and high-voltage unit.

The Duo-Vue unit has been operated with television receivers of many makes and prices. It has successfully produced a fine quality 3- by 4-foot picture, and has apparently not reduced the picture quality of the television receiver. The highlight brightness of the 3- by 4-foot picture measures approximately three to four foot lamberts. The resolution of the picture has been measured at 425 lines and has always shown itself, in tests, to be as good or better than the direct-view picture of the receiver.

A Duo-Vue is simple to operate and after proper installation, adds little more adjusting than the original receiver required.

It is believed that this instrument will do much to interest the small picture television-set owner in real large-screen television since it can be purchased and installed at low cost.

Answers to Previous Essays

Polarization or Charge in a Dielectric? The following is the author's answer to his previously published essay (*EE, Aug '50, p 710*).

It is evident that to answer the question raised we must somehow probe into the dielectric itself. Examination of the external field alone cannot give the desired information, since for any distribution of polarization density **P** within a body which gives a particular external field **E** there is a possible distribution of charge density ρ namely $\rho = -\text{div}\ \mathbf{P}$ which will give exactly the same external field. Probing into the dielectric means drilling little holes or milling little cuts into it, and inserting instruments of some sort to determine the electrical conditions in these holes or cuts. According to Faraday's and Maxwell's views these "electrical conditions" are completely described by knowledge of the electric and magnetic fields existing within these holes and cuts. We are thus led to the Kelvin Cavity definition of the electromagnetic field within matter, which is essentially the only satisfactory operational definition, for macroscopic electromagnetic theory.

Returning to our wax, we determine that its total charge is zero by integrating the normal component of the electric field over a surface surrounding it and lying in empty space, where the electric field is defined through the force on a small charged probe. This integral, which equals $4\pi Q$ where Q is the (thus defined) total charge on the wax body, will be zero in this case. Now to determine whether any part of the wax has a charge we cut the wax up into small parts by fine cuts as in Figure 1, and assume that we do this so skillfully that we do not affect the electrical field outside. We assume then that we have also not affected the charges or polarizations of the individual parts of the wax. Now to find out whether any particular small

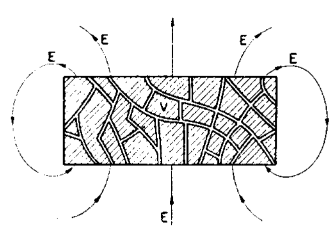

Figure 1. Wax cut into small volumes

part V has a total charge on it, we surround it by a surface S^*, lying wholly within the narrow cut around V, and integrate $\mathbf{E}_n^*$, the normal component of the electric field within the cut over the surface S^*, obtaining

$$\int\int \mathbf{E}_n^* dS^* = 4\pi q. \quad (1)$$

q, we define as the real or true charge on the volume V.

We are thus led to study the electric field $\mathbf{E}^*$ in narrow cuts within the material. At any point in the material, $\mathbf{E}^*$ will depend on the state of electrification of the material, but it will also depend on the particular orientation chosen for the cut at that point. If this orientation is varied, $\mathbf{E}^*$ will vary, and also particularly $\mathbf{E}_n^*$ will vary. If however, the integral of equation 1 is to be proportional only to the volume of V, and not be dependent on its shape, then it may be shown that $\mathbf{E}_n^*$ is the normal component of a vector $\mathbf{D}$ which does not change as the orientation of $\mathbf{dS}^*$ is changed. This vector $\mathbf{D}$ we call the electric induction, which is thus defined as having the same normal component as $\mathbf{E}^*$, the field within a narrow cut.

D is defined by

$$\mathbf{D}_n = \mathbf{E}_n^* \quad (2)$$

If now ρ the charge density is q of equation 1 divided by V, we may equivalently say that ρ is defined by

$$4\pi\rho = \text{div } \mathbf{D} \quad (3)$$

Studying E^* further, we should expect that since it is in empty space, we should have

$$\oint \mathbf{E}^* dS = 0 \quad (4)$$

where the integral of equation 4 is taken around any closed curve lying in the cut. It follows then that $\mathbf{E}_s^*$ is the s component of a vector $\mathbf{E}$ which at any given point does not change as the orientation of the cut in the material is changed. This vector $\mathbf{E}$ we call the electric field or intensity, which is thus defined as having the same component parallel to the cut, of $\mathbf{E}^*$, the field within the narrow cut; that is, $\mathbf{E}$ is defined by

$$\mathbf{E}_s = \mathbf{E}_s^* \quad (5)$$

Finally, for the polarization, **P** is defined by

$$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P} \quad (6)$$

By making cuts in the wax, then, and observing the fields in the cuts, and using equations 1 through 6 we may answer the question raised in the essay.

This method of defining the electromagnetic field within matter by means of the fields observed within fine cuts can be extended without change directly to the general case for material media at rest provided there are no conduction currents. To meet the condition of conduction currents we need to postulate that we can introduce cuts of such short time duration of existence, that they do not affect the electromagnetic field; that is, that they do successfully define unique vectors E, D, H, B, and so forth. We would then have **D** defined by

$$\mathbf{D}_n = \mathbf{E}_n^* \quad (2)$$

E is defined by

$$\mathbf{E}_s = \mathbf{E}_s^* \quad (5)$$

P is defined by

$$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P} \quad (6)$$

ρ is defined by $4\pi\rho = \text{div } \mathbf{D}$ (3)

B is defined by

$$\mathbf{B}_n = \mathbf{H}_n^* \quad (7)$$

H is defined by

$$\mathbf{H}_s = \mathbf{H}_s^* \quad (8)$$

M is defined by

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M} \quad (9)$$

i is defined by

$$\frac{4\pi}{c}\mathbf{i} = \text{curl } \mathbf{H} - \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} \quad (10)$$

Maxwell's equations then further assert that

$$\text{div } \mathbf{B} = 0 \quad (11)$$

$$\text{curl } \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \quad (12)$$

and it follows that

$$\text{div } \mathbf{i} + \frac{\partial \rho}{\partial t} = 0 \quad (13)$$

For moving media, the definitions of E, D, and so forth need to be modified, but that is beyond the scope of the present essay.

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A Paradox? The following is the author's answer to his previously published essay (*EE, Aug '50, p 710*).

The transformer operates with one primary winding supplied from 12 kv single phase and the other two windings supplied in series from the same source.

Normal voltage transformation ratios must apply between any one primary winding and its corresponding secondary for the flux lag in any leg of the core is common to both windings. As V_{AN} is the only normal secondary voltage, then winding x must be the only primary supplied with 12 kv. Fuse 3 must therefore be the one blown.

As normal flux must be maintained for winding x, the algebraic sums of the fluxes through y and z must equal that through x. Thus $V_{BN}+V_{CN}=V_{AN}$, from which $V_{BN}=24$ volts.

Furthermore, as $\theta_y+\theta_3=\theta_x=$ a constant, $\Delta\theta_y=-\theta_0$. It follows that $I_B=I_C=40$ amperes, and that $R_B=V_{BN}/I_B=$ 0.6 ohm.

When our trouble-shooter removed low-voltage fuse B, I_B fell to zero. I_C in consequence fell to zero. At the same time V_{CN} fell to zero and V_{BN} rose to 120 volts. Had both B and C phase low-voltage fuses been removed at the same time, the corresponding terminal voltages at the transformer would have adjusted themselves to 60 volts each. In practice there is usually a slight discrepancy—for example, say 59 and 61 volts due to unequal flux paths through the core.

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