that the increased cost of totally enclosed motors is justifiable up to approximately 250 horsepower on the basis of maintenance savings alone. An outage of a motor up to this horsepower rating seldom involves the loss of a main unit, and therefore no considerable additional credit can be assigned on the basis of increased reliability.

For motors above 250 horsepower the difference in price between open-frame dripproof and totally enclosed motors increases at a greater rate than the evaluated difference in maintenance costs. However, an outage of a motor rated 250 horsepower and above generally involves a main unit, and the higher cost of the totally enclosed motor is justifiable on the basis of increased reliability in addition to the savings in maintenance costs. The loss of capacity factor and the incremental heat rate cost may outweigh all other factors for the larger auxiliary drive motors as for circulating water pumps, forced- and induced-draft fans, and boiler feed pumps.

Whether or not the purchase of totally enclosed motors can be justified depends on the type of design of the plant and on the degree of reliability which is expected. The adoption of the unit type of plant design, the extent to which duplication of auxiliaries is provided, and the amount of reserve capacity available to the system obviously will influence the decision.

On the Consolidated Edison system it generally has been found that for essential auxiliaries in power plants totally enclosed motors can be economically justified. On this basis a total of 580 totally enclosed motors with an aggregate rating of 52,000 horsepower has been bought since 1943.

It appears reasonable to expect that a thorough investigation of all of the factors entering into the design of auxiliary power drives and their comparative evaluation will disclose that totally enclosed motors may be economically justifiable to a greater extent than in the past.

REFERENCES

1. Rotating Electrical Machinery. *Standard C-50,* American Standards Association, New York, N. Y., 1943.

2. Standards for Motors and Generators. *Publication MG-1-4.15,* National Electrical Manufacturers Association, New York, N. Y., 1949.

Electrical Essays

Motionally Induced Electric Fields—Part IV

Motional Electromotive Force in Iron, Commutatorless D-C Motor, and Space Ship

Jack, the physicist, is continuing his lectures to Alter Ego and his friends on the basic principles underlying dynamo-electric machines.

Jack: "We now have the universal equation for the motional electric field, $\mathbf{E}_{mot}$, induced in any honest-to-goodness body moving with a velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$, namely

$$\mathbf{E}_{mot} = \frac{1}{c}[\mathbf{v} \times \mathbf{B}] \quad (1)$$

This universal equation is consistent with the principle of energy, for if $\mathbf{i}$ is the current density in the body, we will have

$$P_e = \mathbf{E}_{mot} \cdot \mathbf{i} = [\mathbf{v} \times \mathbf{B}] \cdot \mathbf{i} = \mathbf{v} \cdot [\mathbf{B} \times \mathbf{i}] \quad (2)$$

$P_e = c\mathbf{E}_{mot} \cdot \mathbf{i}$ is the electric power supplied per unit volume by the moving body external circuit.

"We find the mechanical power by means of the universal equation

$$\mathbf{F}_m = [\mathbf{i} \times \mathbf{B}] = -[\mathbf{B} \times \mathbf{i}] \quad (3)$$

which gives the mechanical force, per unit volume $\mathbf{F}_m$, which the magnetic field $\mathbf{B}$ exerts upon any body carrying a current density $\mathbf{i}$. The mechanical power, per unit volume, which the moving body delivers to other bodies is then

$$P_m = \mathbf{v} \cdot \mathbf{F}_m = -\mathbf{v} \cdot [\mathbf{B} \times \mathbf{i}] \quad (4)$$

"Combining equations 2 and 4 we have

$$P_e + P_m = 0 \quad (5)$$

That is, if the moving body is delivering electric power, as in a generator, then equal mechanical power must be supplied to it, and if the body is receiving electric power by $\mathbf{i}$ flowing against $\mathbf{E}_{mot}$, as in a motor, then it will deliver an equal mechanical power. Universal equations 1 and 3 are entirely in order as far as the principle of energy is concerned.

"But now, Alter Ego has something to say."

Alter Ego: "I am very glad to learn of the universal validity of equations 1 and 3 because based on them I have made two very important inventions; namely, a commutatorless d-c motor or generator, and a d-c driven space ship. I won't go into the details of these inventions, but will limit myself to describing their basic principles.

"Go back to the slide-wire experiment, Figure 1, and consider what will happen if the slide bar is a cylinder of very high permeability iron. The lines of force of the otherwise uniform magnetic field $\mathbf{B}$ will converge as is shown in Figure 2, and $\mathbf{B}$, within the circular section iron bar, will have almost twice the strength which it would have in a similarly placed copper bar. Therefore, according to your universal equation 1 which you say is absolutely right, the motional field $\mathbf{E}_{mot}$ in the iron bar will be twice what you would get for a copper bar.

"Now for my invention. Make a coil like I show in Figure 3, with two parallel sides, but make one of these

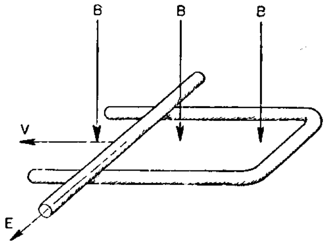

Figure 1 (above). The slide-wire experiment

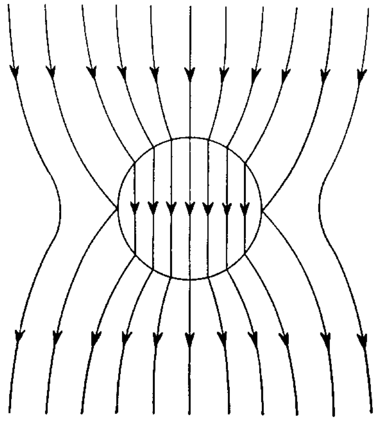

Figure 2 (right). Converging field in iron cylinder

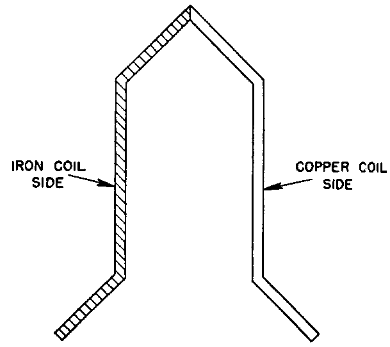

Figure 3. Iron-copper coil

parallel sides of iron and the other of copper. Now translate the coil in the magnetic field. There will be motional fields induced in the coil sides. If the coil had been all copper, then the motional field $\mathbf{E}_{mot}$ in the one coil side would oppose and just cancel the motional field $\mathbf{E}_{mot}$ in the other coil side, so far as the production of a total electromotive force around the coil is concerned. However, with one coil side iron, the motional field $\mathbf{E}_{mot}$ in it will be twice the motional field in the other side, copper coil, and these motional fields will not cancel round the coil, but the coil will give a net, not zero, electromotive force as it is moved in the uniform magnetic field. It is easy to see how we could mount a lot of these coils in series on a rotating member with slip rings, in a uniform magnetic field, and get a beautiful constant d-c voltage. There's your commutatorless d-c generator.

"Now, consider what happens if I send current through my (patent-pending) iron-copper coil while it is in a uniform magnetic field. If the coil had been all copper, then, according to your universal and absolutely right equation 3, the forces exerted by the magnetic field on the coil sides would be equal and opposite and would exactly cancel. However, in my iron-copper coil, the field **B** in the iron side is twice what it is in the copper side, and therefore, according to your universal equation 3, the force on the iron side will be twice the oppositely directed force on the copper side, and I will get a net force tending to move the whole coil. If I arrange a lot of these coils around on a rotor as in my generator, I'll get a commutatorless d-c motor. However, if I lay the coils out in a plane, I'll get a space ship, since the reaction on the field-producing member will be the same as if the coils were all copper, namely, zero. Aren't those wonderful inventions, Dr. Jack?"

Jack: "I don't like to throw cold water on your enthusiasm, Alter Ego, but I doubt that your inventions will work. You will remember that in my last lecture (*EE, Jan '51, pp 67–68*), I showed that the sum of all the electric fields, the electrostatic, that induced by transformer action, and the motional, when integrated round any closed material circuit, was equal to $1/c$ times the rate of change of the integral of **B** over any surface enclosed by the circuit, or in other words, to $1/c$ times the rate of change of the total flux linked by the circuit. Now, as your coil moves in the uniform magnetic field, the amount of flux linked by it does not change. Therefore, the total current-producing electromotive force which is around your coil stays zero.

"Of course, you should have obtained the same result using the motional equation 1, but you did not. Some people think that the **v** in equation 1 stands for the motion of the moving body, not relative to space or some suitable material frame of reference but relative to the magnetic field itself, and that where the field lines bunch up as in your iron bar the bar carries them along somewhat, so that the lines of force slide or cut through the bar with only half that velocity with which they are cut by the copper bar so that the motional field is the same in the iron as the copper. Maybe that is the way out...."

I interrupt Jack at this point, because I am already behind in my reply to the preceding essay and I must catch up.

Will Jack be able to explain away the contradiction, along the line he has indicated? Watch for the next exciting episode in the next installment.

J. SLEPIAN (F '27)

(Westinghouse Research Laboratories, East Pittsburgh, Pa.)

Three Impedances

Three impedances are so proportioned that when connected in star and energized from a 60-cycle system

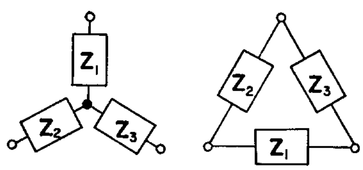

Figure 1. Star and delta connections of three impedances

they form a network that is equivalent to a network formed by the same three impedances connected in delta. Is this possible?

A. A. KRONEBERG (F '48)

(Southern California Edison Company, Los Angeles, Calif.)

Answers to Previous Essays

Correction to Motionally Induced Electromotive Force. The following is the author's correction to the Part II essay and the answer to Part I (*EE, Dec '50, pp 1086–9*).

The units used in these essays on "Motionally Induced

Answers to Previous Essays

Motionally Induced Electric Fields—Part IV. The following is the author's answer to his previously published essay (*EE, Feb '51, pp 159–60*).

As the author has observed so many times in the past in these various essays, relative to any particular suitable frame of reference, there is but one electric field **E**, and also but one **D**, one **H**, and one **B**, and these are the quantities which enter into Maxwell's field equations. These vectors may be defined in empty space through local observation of the forces on charged probes, placed in the empty space, by equation 3,[1] and may be defined within bodies stationary or moving through observation of the forces on charged probes within crevices in the bodies by equations 2 to 4.[2] The charge and current densities, ρ and **i**, are then defined from the quantities **D** and **H** by equations 3 and 10.[3]

Maxwell's field equations are not enough to fix or determine in a mathematical sense the various field quantities which we have defined operationally through observations on charged material probes in empty space. To get a mathematically complete system we adjoin the constitutive equations of the various materials in question, which are determined by experiment or otherwise.

If we know the constitutive equations for the matter of a body at rest, then the principle of relativity enables us to determine the constitutive equations for the body in a state of uniform motion. We merely make the transformation of the field quantities called for by relativity.[4] In this way, we obtained the constitutive equations for the glass of a glass slide bar.

$$\mathbf{D}=\epsilon\mathbf{E}+(\epsilon-1)\frac{1}{c}[\mathbf{v}\times\mathbf{B}] \tag{1}$$

$$\mathbf{B}=\mathbf{H}-\frac{\epsilon-1}{\epsilon}\frac{1}{c}[\mathbf{v}\times\mathbf{D}] \tag{2}$$

$$\mathbf{i}=\frac{1}{c}\mathbf{v}\rho \tag{3}$$

To deal with the iron slide bar, we similarly determine the constitutive equations for moving iron from the known equations for iron at rest, that is

$$\mathbf{i}'=\sigma\mathbf{E}' \tag{4}$$

$$\rho'=0 \tag{5}$$

$$\mathbf{D}'=0 \tag{6}$$

$$\mathbf{B}'=f(\mathbf{H}') \tag{7}$$

Making the substitutions of equations 1,[4] we get these equations:

$$\mathbf{i}-\frac{1}{c}\mathbf{v}\rho=\sigma\left(\mathbf{E}+\frac{1}{c}[\mathbf{v}\times\mathbf{B}]\right) \tag{8}$$

$$\rho+\frac{1}{c}\mathbf{v}\cdot\mathbf{i}=0 \tag{9}$$

$$\mathbf{D}+\frac{1}{c}[\mathbf{v}\times\mathbf{H}]=0 \tag{10}$$

$$\mathbf{B}-\frac{1}{c}[\mathbf{v}\times\mathbf{E}]=f\left(\mathbf{H}-\frac{1}{c}[\mathbf{v}\times\mathbf{D}]\right) \tag{11}$$

Neglecting higher powers of v/c than the first, we get that one of the constitutive equations of which we shall make use in this essay, namely

$$\mathbf{i}=\sigma\left(\mathbf{E}+\frac{1}{c}[\mathbf{v}\times\mathbf{B}]\right) \tag{12}$$

Maxwell's field equations plus the constitutive equations just mentioned are sufficient for solving the two slide-bar problems which Alter Ego has presented to us. For this purpose it is convenient to use an integral of that Maxwell equation concerned with curl **E**, which is slightly more general than that given by Stokes theorem. Stokes theorem itself tells us that for any closed curve, lying wholly or partly in the matter of stationary or moving bodies or in empty space

$$\int\mathbf{E}\cdot d\mathbf{s}=-\int\int\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S} \tag{13}$$

In equation 13 the integral on the left side is taken round the closed curve. The integral on the right side is taken over any 2-sided surface bounded by the closed curve, and lying where it may, wholly or partly within stationary or moving matter.

Equation 13 refers to any particular closed curve with any bounded surface which we may arbitrarily select or construct at any particular instant of time. Let us now consider the temporal succession of such closed curves and bounded surfaces which we obtain by assigning an arbitrary continuous velocity **u** to the various points of the closed curve and bounded surface. **u** need have no relation to the velocity **v** of the matter through which the curve or surface may pass. **u** is a mathematical parameter arbitrarily assigned over the curve and surface of integration in matter or in empty space, wherever the curve or surface may lie.

At any particular instant of time, equation 13 applies for that particular instantaneous configuration of curve and surface of integration. Now add to both sides of equation 13 the quantity $\int\frac{1}{c}[\mathbf{u}\times\mathbf{B}]\cdot d\mathbf{s}$. We get

$$\begin{aligned}\int\left(\mathbf{E}+\frac{1}{c}[\mathbf{u}\times\mathbf{B}]\right)\cdot d\mathbf{s}&=\int\frac{1}{c}[\mathbf{u}\times\mathbf{B}]\cdot d\mathbf{s}-\int\int\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S}\\&=-\int\frac{1}{c}\mathbf{B}\cdot[\mathbf{u}\times d\mathbf{s}]-\int\int\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S}\\&=-\frac{1}{c}\frac{d}{dt}\int\int\mathbf{B}\cdot d\mathbf{S}\end{aligned} \tag{14}$$

That is, the integral of $\mathbf{E}+\frac{1}{c}[\mathbf{u}\times\mathbf{B}]$, where **E** and **B** are the one and only Maxwellian **E** and **B**, around any closed curve, the points of which move with arbitrarily assigned velocity **u**, is equal to $-1/c$ times the rate of change of the magnetic flux linked by the arbitrarily moving curve. We shall make frequent use of equation 14, so let us keep it in mind.

Let us now consider the two slide-bar problems. We shall confine our attention to the neighborhood of the plane of symmetry perpendicular to the slide bar at its middle, and we shall assume the bar long enough so that the effect of its ends will not disturb the 2-dimensional character of the field near the plane. That is, we shall assume that near the plane of symmetry, the magnetic vectors **H** and **B** are parallel to the plane, and that the electric vectors, **E** and **D**, and also **i**, are perpendicular to the plane, and that these vectors do not change in magnitude or direction as we move away in a perpendicular direction from the plane of symmetry.

Far enough away from the slide bar we assume that we have a uniform magnetic field, $\mathbf{H}=\mathbf{B}=\mathbf{B}_0$, and a uniformly zero electric field, $\mathbf{D}=\mathbf{E}=\mathbf{E}_0=0$. Within or near the slide bar, for either case, the magnetic and electric fields are altered and **B** and **E** may not be equal to $\mathbf{B}_0$ and $\mathbf{E}_0$ respectively. Consider the temporally changing rectangular path of integration constructed as follows, Figure 1.

Adjacent to or within the central part of the slide bar, draw a short path length, *AB*, parallel to the bar, and let *AB* move with the velocity **v**, keeping always in the same geometric relation to the slide bar. Draw the two path sides *AD* and *BC* parallel to each other and in the direction **v**. The final closing path side, *CD*, is parallel to *AB*, and is to be kept at rest. *AD* and *BC* are thus lengthening, and the area *ABCD* is increasing at the rate *vl* where *l* is the length of *AB* or *CD*.

Now apply the theorem of equation 14 to the closed path *ABCD*. At *CD*, we have $\mathbf{u}=0$, $\mathbf{E}=0$. *CD* therefore gives a zero contribution to the integral of equation 14. For sides *BC* and *AD*, **E** and $[\mathbf{u}\times\mathbf{B}]$ are perpendicular, and so these sides also give zero contribution. For side *AB*, which alone contributes to the integral, we have $\mathbf{u}=\mathbf{v}$, so that

$$\int_{ABCD} \mathbf{E}+\frac{1}{c}[\mathbf{u}\times\mathbf{B}]\cdot d\mathbf{s}=\left(\mathbf{E}+\frac{1}{c}[\mathbf{v}\times\mathbf{B}]\right)\cdot\mathbf{l} \qquad (15)$$

where the meaning of the vector **l** is sufficiently self-obvious.

Now, since *AB* moves with velocity **v**, keeping always in the same position relative to the slide bar, the magnetic field configuration at that end of *ABCD* remains unchanging. The rate of change of the flux linked by *ABCD* will be its rate of increase of area, *vl* multiplied by $\mathbf{B}_0$, or $[\mathbf{v}\times\mathbf{B}_0]\cdot\mathbf{l}$. Applying equations 14 and 15, paying due attention to algebraic signs, we get this equation

$$\mathbf{E}+\frac{1}{c}[\mathbf{v}\times\mathbf{B}]=\frac{1}{c}[\mathbf{v}\times\mathbf{B}_0] \qquad (16)$$

Now for the iron bar, we have from the constitutive equation

$$\mathbf{i}=\sigma\left(\mathbf{E}+\frac{1}{c}[\mathbf{v}\times\mathbf{B}]\right)=\sigma\left(\frac{1}{c}[\mathbf{v}\times\mathbf{B}_0]\right) \qquad (17)$$

We see then that the current set flowing in the iron bar by its motion in the otherwise uniform magnetic field, $\mathbf{B}_0$, is entirely determined by its conductivity at rest and is independent of its magnetic properties. If the bar causes the magnetic field within it and in its neighborhood to be different from $\mathbf{B}_0$, then its motion **v** will cause an electric field **E** to be set up within it and in its neighborhood given by equation 16, and the current density remains given by equation 17.

We see then that Alter Ego's commutatorless d-c machines are inoperative, but we sympathize with him for being misled by the notion that there exists a "motional electric field." There is but one electric field, **E**, and Maxwell is its prophet!

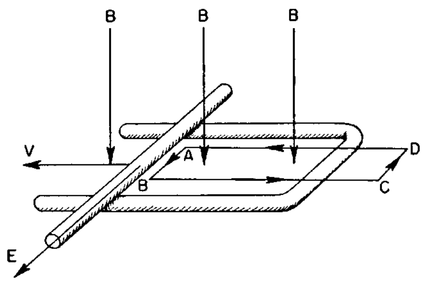

Figure 1. Path of integration, ABCD

We see also how the incorrect treatment of Lorentz' electron theory given by Jack could be corrected.

For the glass rod, if we define its polarization, **P**, by equation 18, and use the constitutive equation 1 and equation 16, we get

$$\mathbf{P}=\frac{1}{4\pi}(\mathbf{D}\quad\mathbf{E})=\frac{1}{4\pi}(\epsilon-1)\left(\mathbf{E}+\frac{1}{c}[\mathbf{v}\times\mathbf{B}]\right)=\frac{1}{4\pi}(\epsilon-1)\left(\frac{1}{c}[\mathbf{v}\times\mathbf{B}_0]\right) \qquad (18)$$

The polarization then is the same as would be produced in the bar at rest by an electric field equal to $\frac{1}{c}[\mathbf{v}\times\mathbf{B}_0]$ although actually the moving glass bar will have in it a magnetic field differing slightly from $\mathbf{B}_0$ by equation 2.

J. SLEPIAN

(Westinghouse Research Laboratories, East Pittsburgh, Pa.)

REFERENCES

1. Electrostatic Space Ship, **J. Slepian.** *Electrical Engineering*, volume 69, March 1950, page 248.
2. Motionally Induced Electromotive Force—Part I, **J. Slepian.** *Electrical Engineering*, volume 69, November 1950, pages 1025–6.
3. Polarization or Charge in a Dielectric? **J. Slepian.** *Electrical Engineering*, volume 69, September 1950, page 792.
4. Motionally Induced Electromotive Force—Part III, **J. Slepian.** *Electrical Engineering*, volume 70, February 1951, pages 161–2.

Three Impedances. The following is the author's answer to his previously published essay (*EE, Feb '51, p 160*).

From equations used to convert networks from star to delta, and from delta to star after substitutions and reductions, the following relationships are obtained:

$$Z_1^2=-Z_2^2=-Z_3^2 \qquad (1)$$
$$Z_2=-Z_3 \qquad (2)$$

These are satisfied by impedances that are equal in magnitude, one being real and positive, and the other two being conjugate imaginary impedances.

$$Z_1=K$$
$$Z_2=jK$$
$$Z_3=-jK$$

A. A. KRONEBERG (F '48)

(Southern California Edison Company, Los Angeles, Calif.)