Electrical Essays

Motionally Induced Electric Fields—Part V

Relative Motion of Magnetic Field and Body

Jack, the physicist, is continuing his lectures to Alter Ego and his friends on the basic principles underlying dynamo-electric machines.

Jack is trying to determine the motional electric field induced in a moving iron bar. He has declared that the motional electromotive force, or $\mathbf{E}_{mot} \cdot \mathbf{ds}$, induced in a coil which is part iron and part copper and which is being translated in an otherwise uniform magnetic field, will be zero, thus declaring inoperative some extraordinary inventions of Alter Ego. However, Alter Ego had based his expectation of a nonzero motional electromotive force in such a coil in translation in an otherwise uniform magnetic field on the equation

$$\mathbf{E}_{mot} = \frac{1}{c}[\mathbf{v} \times \mathbf{B}] \qquad (1)$$

which Jack had said was universally valid. In this equation, up to this point, Jack had defined $\mathbf{v}$ as the velocity of the material of the moving body, in some suitably chosen material frame of reference.

Jack recognizes the contradiction between his statement of the universal validity of equation 1 and his statement of the inoperativeness of Alter Ego's inventions. He is now trying to resolve the dilemma by changing the meaning to be ascribed to $\mathbf{v}$ in equation 1.

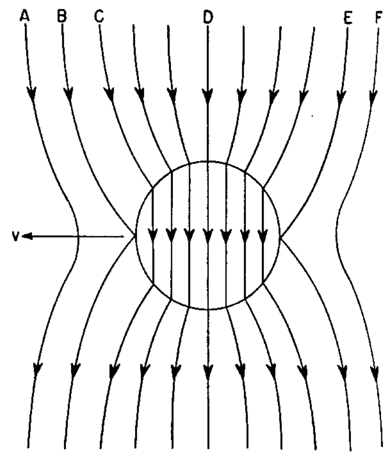
Jack: "Perhaps the way out is to make $\mathbf{v}$ in equation 1 not represent the velocity of the iron bar relative to some material frame of reference which we arbitrarily say is at rest, but to make $\mathbf{v}$ represent the relative velocity of the iron bar and the magnetic lines of force.

"Thus, in Figure 1 we may regard the lines of magnetic force, *A*, *B*, *C*, and so forth, as at rest where they are remote from the iron bar. Then as the bar moves up to any line of force, the line of force will bulge out to meet it, as line *A* in Figure 1. At this bulge which is thus produced, the line of force is not at rest, but is advancing to meet the iron bar. This relative velocity of the bulge and bar increases, until the bulge hits the bar, as is shown for line *B* in Figure 1.

"After the line of force enters the iron, the bulge starts to straighten out, as at *C*. This straightening out of the bulge in the iron makes the lines of force in the iron move along in the same direction as that in which the iron bar is moving, so that the relative velocity of the bar and the magnetic lines is less than the velocity of the bar itself.

"The bulge is straightened out at *D*, but then develops in the opposite direction, keeping the same reduced velocity relative to the bar. At *E*, the line emerges from the iron, and snaps straight again through intermediate form *F*.

Figure 1. Converging field in a moving iron cylinder



"If $\mathbf{B}_0$ is the field strength remote from the bar, where the lines are stationary, and $\mathbf{B}$ is the magnetic field strength in the iron, and if $\mathbf{v}_0$ is the velocity of the bar, and if $\mathbf{v}$ is the relative velocity of the bar and the moving field lines in it, then

$$[\mathbf{v} \times \mathbf{B}] = [\mathbf{v}_0 \times \mathbf{B}_0] \qquad (2)$$

since each side of equation 2 gives the rate of cutting of lines of force by the bar. Hence, the motional field in the iron will be

$$\mathbf{E}_{mot} = \frac{1}{c}[\mathbf{v} \times \mathbf{B}] = \frac{1}{c}[\mathbf{v}_0 \times \mathbf{B}_0] \qquad (3)$$

which is the same as that for the copper bar."

Alter Ego: "Well, if you are going to interpret $\mathbf{v}$ in equation 1 that way, I concede that it will ruin my iron-copper coil commutatorless d-c generator. But what happens to my space ship and commutatorless d-c motor? Your universal equation for the mechanical force per unit volume on a body carrying current density $\mathbf{i}$ in a magnetic field $\mathbf{B}$ is

$$\mathbf{F}_m = [\mathbf{i} \times \mathbf{B}] \qquad (4)$$

"This equation does not have $\mathbf{v}$ in it at all. Therefore, the relative motion of the magnetic lines does not have any effect, so that with $\mathbf{B}$ twice as great, the force on the iron side of the coil will be twice as great as that on the copper side, and there will be a net force, not zero, tending to translate the coil."

(*To be continued*)

I think Jack will be able to take care of the mechanical forces. Can you, dear reader, also do so?

How about those relatively moving lines of magnetic force? Will Jack be able to get anywhere with that idea?

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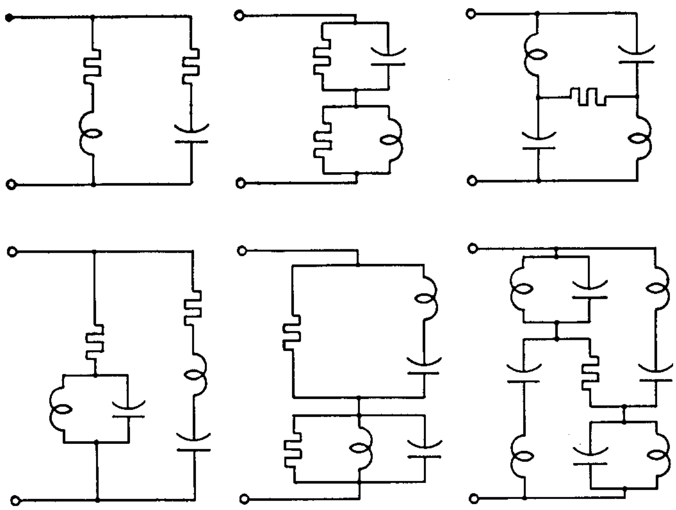

Figure 1 (above). Six networks

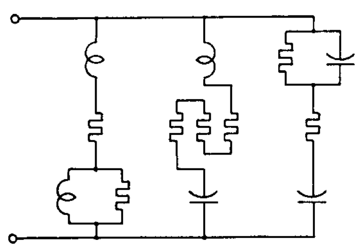

Figure 2 (left). Seventh network

engineering and physics. He uses inductors of L henrys, capacitors of C farads, and resistors of $R=\sqrt{L/C}$ ohms in assembling the six networks shown in Figure 1.

From elements remaining on hand, he then assembles the seventh network illustrated in Figure 2. Does this network belong in the same group with the other six networks?

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Answers to Previous Essays

Motionally Induced Electric Fields—Part V. The following is the author's answer to his previously published essay (*EE, Mar '51, p 254*).

As Part VI of the series shows, Jack is in an awful mess with his idea of moving lines of magnetic force.

The author has several times now stressed that the electric and magnetic fields in Maxwell's Theory are vector fields and nothing more.[1–4] A vector field associates each point of metrical space with a direction and a magnitude, and that is all. Now, we are accustomed to picture electric and magnetic fields by means of assemblages of lines of force, these lines of force at all points of space having the same direction as the vectors they are portraying, and the density of the lines of force being chosen so as to be proportional to the magnitude of the vectors they are portraying. If necessary, in order to keep up this correspondence between line density and vector magnitude, we end lines or start new lines in the various regions of space. The excess of the number of new line beginnings over line endings per unit volumes is thus proportional to the divergence of the vector field being portrayed.

Now, however, if we are not careful, we will assume that certain properties of these lines of force, which we introduce into any particular choice of lines of force which we may make, are also significant or pertinent to the vector fields which they represent even though such properties are not in themselves in the vector field. Such properties are continuity and individuality of the lines. These properties correspond to no properties of the bare vector field, which is all that appears in Maxwell's equations.

Considering continuity first, we may at any place in our line-of-force diagram break off the lines of force we may have been using, and which are approaching the region, and start off a set of completely new lines with the same density. The broken lines represent the vector field just as well as the continuous lines, since they have the same density and direction. The excess of line beginnings over line endings thus introduced is zero, so that the correspondence with the vector field divergence is not altered. Thus, continuity of the lines in a line-of-force representation is an irrelevant property as far as electromagnetic phenomena are concerned. Any deduction concerning electromagnetic phenomena, which depends on the continuity of the lines of force of a field representation, may be false. As a matter of fact, a line-of-force representation with completely continuous lines of force everywhere is not possible, except for very special electric and magnetic fields.[1,2,4]

Similarly for the notion of the individuality of a line, a concept not independent of the property of continuity. In a temporal succession of lines-of-force representations, we may correlate a particular line in one picture with a particular line in a later picture and assert that it is the same line. But such an assertion can have no verifiable significance or pertinence to electromagnetic phenomena. We may, if we like, assign continuing identifiable individuality to lines in such a way that the lines execute a most intricate dance among themselves, provided only that we keep their density and direction always corresponding to the field they are portraying. Such intricately dancing lines will portray the field just as well as more sedate line assemblages. However, any deduction concerning electromagnetic phenomena which depends upon either dancing proclivities or sedateness of the lines of force may be false. The lines of force are related to the vector fields which they portray like the colors on a map to the actual country which is being portrayed. The boundary line between pink Pennsylvania and yellow Ohio on the map is very pleasingly made evident, but any expectation of finding something corresponding to pinkness and yellowness in actual Pennsylvania and Ohio is doomed to disappointment.

Now Jack, in trying to rescue his "universal" motional field $\mathbf{E}_{\text{mot}}=\frac{1}{c}[\mathbf{v}\times\mathbf{B}]$ by the device of moving lines as in part IV of this series,[5] is clearly using the notions of continuity and individuality of lines of force, which have no relevance to electromagnetism. Conclusions which he may draw then may be false. As a matter of fact, his assumption that the ends of the magnetic lines of force remote from the iron bar are stationary is not only without verifiable physical meaning but it is also entirely arbitrary. He might just as well have let these remote line ends dance among themselves as he pleased, provided he kept their density constant. Then he would be seeing quite other motional electric fields in the iron bar, and he would also see "motional electric fields" in stationary parts of the circuit.

A metallic body, for which Ohm's Law, $\mathbf{i}=\sigma\mathbf{E}$ and $\mathbf{D}=0$, holds when it is at rest, will have the constitutive equation when it is moving

$$\mathbf{i}=\sigma\left(\mathbf{E}+\frac{1}{c}[\mathbf{v}\times\mathbf{B}]\right) \tag{1}$$

where $\mathbf{v}$ is the translational velocity of the body itself relative to a suitable material frame of reference. For such a body if it is nonmagnetic, we may say that in motion it behaves, as far as current flow is concerned, as if it were at rest, in the same steady field $\mathbf{E}$, with an additional field $\frac{1}{c}[\mathbf{v}\times\mathbf{B}]$ acting on it. In that sense, and that case, we might talk of a "motional field" $\frac{1}{c}[\mathbf{v}\times\mathbf{B}]$. But for a magnetic body, even in an unchanging field, and for a nonmagnetic body in a varying field, $\mathbf{E}$ for the body at rest cannot stay the same as $\mathbf{E}$ for the body in motion, and so calling $\frac{1}{c}[\mathbf{v}\times\mathbf{B}]$ the "motional field" is no longer justified, since the motion of the body also causes a change in $\mathbf{E}$. For other bodies such as, for example, an insulator with the constitutive equations

$$\mathbf{D}+\frac{1}{c}[\mathbf{v}\times\mathbf{H}]=\epsilon\left(\mathbf{E}+\frac{1}{c}[\mathbf{v}\times\mathbf{B}]\right)$$
$$\mathbf{H}-\frac{1}{c}[\mathbf{v}\times\mathbf{D}]=\mathbf{B}-\frac{1}{c}[\mathbf{v}\times\mathbf{E}] \tag{2}$$

we cannot at all say that the effect of the motion is the same as introducing an additional electric field $\frac{1}{c}[\mathbf{v}\times\mathbf{B}]$. For such other bodies the idea of a universal "motional field" is quite without meaning.

We have seen[6] that for any closed curve, the points of which move with arbitrarily assigned velocity $\mathbf{u}$

$$\int\left(\mathbf{E}+\frac{1}{c}[\mathbf{u}\times\mathbf{B}]\right)\cdot\mathbf{ds}=-\frac{1}{c}\frac{d}{dt}\int\int\mathbf{B}\cdot\mathbf{dS} \tag{3}$$

It follows then that in a metallic circuit of total resistance R, under conditions such that the total current I in it is the same throughout its length (that is, no charging currents)

$$RI=-\frac{1}{c}\frac{d}{dt}\int\int\mathbf{B}\cdot\mathbf{dS} \tag{4}$$

Now, under these same conditions, Jack's revised version of his "motional field" given in the preceding essay,[5] when integrated around the metallic circuit, will equal the right-hand member of equation 4, and therefore will also equal RI. This will also be the case if he lets his lines of force dance around arbitrarily as mentioned before. Thus it would seem as if Jack's device might take care of metallic circuits, whether magnetic or not, moving in fields which would be otherwise constant. It would seem then that Jack's trick might be sufficient for the usual dynamo-electric machine and perhaps also for any modification of such machine that Alter Ego might propose. However there is still a serious difficulty that Alter Ego's next invention will bring out.

REFERENCES

1. Answers to Previous Essays, **J. Slepian.** *Electrical Engineering*, volume 68, September 1949, page 763.
2. Answers to Previous Essays, **J. Slepian.** *Electrical Engineering*, volume 68, October 1949, page 878.
3. Answers to Previous Essays, **J. Slepian.** *Electrical Engineering*, volume 68, November 1949, page 985.
4. Lines of Force in Electric and Magnetic Fields, **Joseph Slepian.** *American Journal of Physics* (New York, N. Y.), February 1951, pages 87–90.
5. Motionally Induced Electric Fields—Part IV, **J. Slepian.** *Electrical Engineering*, volume 70, February 1951, pages 159–60.
6. Answers to Previous Essays, **J. Slepian.** *Electrical Engineering*, volume 70, March 1951, equation 15, page 256.

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Metallurgical Advisory Board

Formation of a board of leading industrial and academic metallurgists to advise the Research and Development Board, Department of Defense, on research aspects of some of the nation's most critical metals problems was announced recently. The new Advisory Board's work will be directly in the interest of national security since metals in large quantities are essential for modern military operations.

The Board was organized by the National Academy of Sciences—National Research Council under a contract with the Research and Development Board.

Within the limits of specific problem assignments, the metallurgists are to advise the Board on the correlation, co-ordination, interpretation, and application of metals' research and development programs conducted or sponsored by the Military Services, suggest new research projects or reorientation of existing research, and collect and distribute such useful metallurgical information as can be gathered by the establishment of close liaison with the professional societies, governmental agencies, and academic and industrial organizations devoted to metals and their use.

Preliminary work by the Metallurgical Advisory Board is already in progress on research and development phases of three of the most urgent metals problems: critical and strategic metals and their substitutes; the application of metals to be used at high temperatures; and the development of the presently small titanium industry.

A permanent secretariat and special project committees are to assist the Metallurgical Advisory Board, which also will have the co-operation of the military services, the Munitions Board, and other Federal agencies.